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SCHOOL EDITION.

THE

PACKARD

COMMERCIAL ARITHMETIC.

BY

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P R E F A C E .

THIS book is prepared expressly for schools having a commercial department and for short course commercial schools, including evening classes. It contains sufficient material for a full arithmetical course such as is required in this class of institutions, and differs from the larger arithmetic by the same authors only in having fewer examples, and in omitting some subjects which would not be appropriate here. The aim has been to put within the prescribed limits the most valuable material for the purpose and to present it in the best order. No space has been thrown away, and nothing has been left out essential to the real aim of the book. It is, in fact as well as in name, a Commercial Arithmetic, symmetrical and complete, and whoever compasses its lessons, whether as a pupil of a school or as a private learner, need not fear to offer his services to an exacting public.

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ARITHMETIC.

NOTATION AND NUMERATION.

Arithmetic is the science of numbers and the art of computation by them.

A **Unit**, or **Unity**, is one, or a single thing : as one, one foot, one dollar.

A **Number** is a unit, or a collection of units : as one, four, three feet, five dollars. Numbers are expressed by words, by letters, and by figures.

Notation is a system of representing numbers by symbols. There are two methods of notation in use, the *Roman* and the *Arabic*.

Numeration is a system of naming or reading numbers.

The Arabic method of notation employs ten characters or figures, viz.:

1	2	3	4	5	6	7	8	9	0
One,	Two,	Three,	Four,	Five,	Six,	Seven,	Eight,	Nine,	Zero.

The first nine of the above are called *significant figures*, because each, standing by itself, represents a value, or denotes some number. They are also called *digits*, from the Latin word *digitus*, which means a finger.

The last one is called *zero*, *naught*, or *cipher*, because when standing alone it has no value, or signifies nothing.

NUMERATION TABLE.

etc., etc.	Hundreds of Quadrillions.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
	6	4	2,	1	9	5,	7	3	2,	4	3	6,	8	0	7,	4	9	3.
	6th period,			5th period,			4th period,			3d period,			2d period,			1st period,		
	Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		

Copy and read the following numbers :

(1)	(2)	(3)	(4)	(5)	(6)
73	102	616	1064	8174	12741
69	333	348	3604	8006	20809
48	570	222	4364	7070	47038
90	895	843	7208	3300	68605

Express by figures the following :

1. Nineteen.	7. One hundred forty-four.
2. Twenty-two.	8. Three thousand sixteen.
3. Forty-six.	9. Four thousand forty-four.
4. Sixty-eight.	10. Six million two thousand six.
5. Ninety-two.	11. Sixteen million eight hundred two.
6. Eighty-seven.	12. Eighty-seven thousand sixty-two.

ROMAN NOTATION.

In the **Roman Notation**, seven capital letters are used to express numbers, as follows :

I	V	X	L	C	D	M
One,	Five,	Ten,	Fifty,	One Hundred,	Five Hundred,	One Thousand.

Other numbers are expressed by combining the letters according to the following principles :

1. If a letter is repeated, its value is repeated. Thus, III represents three ; XX, twenty ; CCC, three hundred.

2. If a letter of *less* value is placed *before* one of greater value, the less is taken from the greater. Thus, IV represents four ; IX, nine ; XL, forty.

3. If a letter of *less* value is placed *after* one of greater value, the less is added to the greater. Thus, VI represents six ; XI, eleven ; LX, sixty.

4. A bar (—) placed over a letter increases its value a thousand times. Thus, \overline{X} represents ten thousand ; \overline{M} , one million.

The Roman Notation is used for numbering dials, chapters, pages, etc.

TABLE OF ROMAN NOTATION.

<i>Roman.</i>	<i>Arabic.</i>	<i>Roman.</i>	<i>Arabic.</i>	<i>Roman.</i>	<i>Arabic.</i>
I,	1.	XIV,	14.	LX,	60.
II,	2.	XV,	15.	LXX,	70.
III,	3.	XVI,	16.	LXXX,	80.
IV,	4.	XVIII,	18.	XC,	90.
V,	5.	XIX,	19.	C,	100
VI,	6.	XX,	20.	CCC,	300.
VIII,	8.	XXI,	21.	D,	500.
IX,	9.	XXX,	30.	DCC,	700.
X,	10.	XLI,	40.	M,	1000.
XIII,	13.	L,	50.	MD,	1500.

Express by Roman notation :

1. Eighteen.	6. Eighty-seven.	11. 584
2. Thirty-six.	7. Three hundred sixty.	12. 777.
3. Forty-eight.	8. Six hundred forty-nine.	13. 1638.
4. Seventy-six.	9. Five hundred eighty-eight.	14. 1886.
5. Sixty-four.	10. Two thousand sixty-two.	15. 8000.

Express by Arabic notation :

1. LXXVII.	6. DCCLXVI.	11. \overline{M} MCCCXXVII.
2. CCXIX.	7. DCXLIV.	12. MMDCCXVIII.
3. XCVIII.	8. \overline{D} CXLIV.	13. MMCCXCIX.
4. CCCLIV.	9. MDCXLVI.	14. MMDCCCLXXV.
5. DCXVI.	10. CCLXXIX.	15. MDCCCLXXXVII.

A D D I T I O N .

The **Sum** or **Amount** of two or more numbers is a number which contains as many units as all the numbers combined.

Addition is the process of finding the sum of two or more numbers.

The sign of addition is $+$, and is read *plus*.

The sign of equality is $=$, and is read *equals*, or *is equal to*.

Thus, $6 + 2 = 8$ is read *6 plus 2 equals 8*, or *the sum of 6 and 2 is equal to 8*.

The sign of dollars is $\$$; of cents ϕ , et., or cts.

To find the sum of two or more numbers.

Ex. Find the sum of 416, 578, 695.

OPERATION.

416	
578	
695	
1689	Sum.
11	

ANALYSIS.—Write the numbers so that like units stand in the same column, and begin to add at the right. The sum of the units ($6 + 8 + 5$) is (14, 19) 19 units, equal to 1 ten 9 units. Write the 9 units under the column of units, and add the 1 ten to the column of tens, obtaining for the sum (2, 9, 18) 18 tens, equal to 1 hundred 8 tens. Write the 8 tens under the column of tens, and add the 1 hundred to the column of hundreds, obtaining for the sum (5, 10, 16) 16 hundreds, equal to 1 thousand 6 hundreds, which write in the hundreds' and thousands' places. Hence, the entire sum is 1689.

NOTES.—1. Think of results and not of the numbers themselves. Thus in the above example, do not say 6 and 8 are 14 and 5 are 19, but 14, 19.

2. To avoid repeating the work, in case of interruption, write the figures to be carried in pencil underneath.

RULE.—Write the numbers to be added so that like units stand in the same column.

Commencing at the right, add each column separately, and if the sum is less than 10, write it under the column added.

If the sum of any column is 10 or more than 10, write the right-hand figure under the column added, and add the remaining figure or figures to the next column.

PROOF.—Find the sum by adding the columns in the opposite direction, thus forming new combinations of figures. If the results agree, the work is probably correct.

EXAMPLES.

Copy or write from dictation and add the following :

(1)	(2)	(3)	(4)	(5)	(6)
789	682	1234	1357	7812	9876
123	109	5678	9135	3625	6789
456	375	9012	8642	4875	9787
<u>246</u>	<u>488</u>	<u>3456</u>	<u>4109</u>	<u>9850</u>	<u>8678</u>
(7)	(8)	(9)	(10)	(11)	(12)
568	431	9672	7812	8796	808
134	866	8738	1357	809	7612
680	219	4126	404	4205	37
419	581	1886	9686	6666	4123
723	49	7143	8072	7777	2264
842	376	8275	9706	8088	7714
906	408	9325	5555	4144	9008
<u>294</u>	<u>792</u>	<u>4444</u>	<u>2009</u>	<u>9995</u>	<u>3348</u>

There is nothing of more importance to the student than the ability to add a column of figures easily, accurately, and rapidly. In order that his labor may be lightened and much valuable time saved, not only in after-life but in his

school work, he should have various kinds of daily drill exercises in addition, especially in the earlier part of his course of study. The following suggestions will be found valuable in securing accuracy and rapidity.

The 45 simple combinations should be used as an exercise in addition. They may be copied on the blackboard in the following or in irregular order, and the sum should be announced by the student at sight :

1	2	3	2	4	3	5	4	3	6	5	4	7	6	5
<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>1</u>	<u>2</u>	<u>3</u>
4	8	7	6	5	9	8	7	6	5	9	8	7	6	9
<u>4</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>3</u>
8	7	6	9	8	7	9	8	7	9	8	9	8	9	9
<u>4</u>	<u>5</u>	<u>6</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>6</u>	<u>7</u>	<u>7</u>	<u>8</u>	<u>8</u>	<u>9</u>

The above should be supplemented by exercises similar to the following :

74	64	44	94	34	24	14	54	84
<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>	<u>8</u>
45	75	35	15	55	65	95	25	85
<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>

Make combinations of 10, 20, 30, or other numbers as often as possible, and add them as single numbers.

Thus, add 9 and 1, 8 and 2, 5 and 5, 4, 3, and 3, etc., as 10 ; 7 and 2, 6, 2, and 1, etc., as 9 ; 2 and 3, 4 and 1, 2, 2, and 1, as 5 ; 8, 7, and 5, 9, 7, and 4, etc., as 20 ; etc., etc.

In Example 4, page 11, think only of the following results : 9, 19, 30, 50.

Drill on the following and similar combinations (10, 20, 30, etc.) until the student can announce the sums at sight.

3	2	1	4	5	3	8	2	2	6	1	2	2	1	3
4	3	4	1	1	1	1	7	6	2	8	1	4	3	3
<u>3</u>	<u>5</u>	<u>5</u>	<u>5</u>	<u>4</u>	<u>6</u>	<u>1</u>	<u>1</u>	<u>2</u>	<u>2</u>	<u>1</u>	<u>7</u>	<u>4</u>	<u>6</u>	<u>4</u>
7	6	6	8	4	8	9	4	3	2	5	8	9	7	9
6	8	5	4	7	3	2	7	8	9	7	7	8	7	7
<u>7</u>	<u>6</u>	<u>9</u>	<u>8</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>9</u>	<u>8</u>	<u>5</u>	<u>3</u>	<u>6</u>	<u>4</u>
1	1	1	3	3	6	8	4	7	7	7	6	9	8	9
2	4	3	3	3	6	4	4	7	4	7	7	6	8	4
3	2	2	1	3	6	1	4	4	4	7	8	7	9	8
<u>4</u>	<u>3</u>	<u>4</u>	<u>3</u>	<u>1</u>	<u>2</u>	<u>7</u>	<u>8</u>	<u>2</u>	<u>5</u>	<u>9</u>	<u>9</u>	<u>8</u>	<u>5</u>	<u>9</u>

EXAMPLES.

Copy or write from dictation and add the following :

(1)	(2)	(3)	(4)	(5)			
4 } 9	15 { 35 } 13	12 85 }	123	382			
5 }		21 96 }	456	648			
3 } 19	30 { 95 } 20	31 { 44 } 21	789	584			
7 }		66 }	462	765			
8 } 30	40 { 75 } 36	43 { 38 } 31	315	406			
3 }		92 }	829	483			
6 } 50	55 { 68 } 46	55 { 75 } 42	918	163			
6 }		56 }	234	852			
8 }		62 77 49	789	574			
<hr/>							
(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
48	71	39	12	77	312	514	376
13	43	34	34	88	123	627	499
82	36	46	56	66	456	842	678
67	94	25	78	99	789	462	437
54	69	83	90	41	987	460	245
87	25	31	89	63	654	329	536
43	38	63	76	74	321	411	984

(14)	(15)	(16)	(17)	(18)	(19)
1234	4121	1728	3416	17642	18114
5678	1865	5280	4725	176	285
9212	3760	2246	8850	20048	28510
3456	4825	4153	4975	248	30048
9753	7145	4839	2137	24800	400
8642	3333	2437	8910	1149	17512
7531	7163	4627	2048	1216	8
1594	4943	7342	175	385	14150
<u>7777</u>	<u>7289</u>	<u>8916</u>	<u>1075</u>	<u>19175</u>	<u>30032</u>

20. Find the sum of the following numbers: Forty-five thousand forty-five; sixteen thousand three hundred sixty; one hundred sixty-seven thousand; eight hundred fifty thousand ninety-two; nine million twenty-four.

21. $46 + 72 + 89 + 93 + 75 + 31 + 58 + 45 + 52 = ?$

22. $376 + 416 + 287 + 123 + 456 + 789 + 916 = ?$

23. $42 + 175 + 287 + 56 + 63 + 324 + 189 + 172 = ?$

24. $365 + 1728 + 64 + 172 + 89 + 38 + 9 + 5280 = ?$

25. A bushel of corn weighs 56 pounds, a bushel of rye 56 pounds, a bushel of wheat 60 pounds, a bushel of barley 45 pounds, a bushel of oats 32 pounds, and a bushel of buckwheat 48 pounds. What would be the total weight of one bushel of each of the above grains?

26. Find the total distance around a rectangular field 1728 feet long and 1683 feet wide.

27. An exporter of provisions buys 187 barrels hams, 428 barrels shoulders, 475 barrels pork, 229 barrels beef, and 392 barrels bacon. How many barrels in all?

28. In an orchard there are 375 apple trees, 416 pear trees, 37 quince trees, 98 cherry trees, 238 peach trees, and 276 plum trees. How many trees in all?

29. A man pays for a house and lot \$6375. For repairs as follows: mason-work, \$68; plumbing, \$78; carpenter-work, \$164; painting and decorating, \$277. For how much must he sell it to gain \$567 on the total cost?

30. A manufacturer sells on Monday 2387 barrels flour, on Tuesday 2618 bbls., on Wednesday 2178 bbls., on Thursday 2125 bbls., on Friday 2348 bbls., and on Saturday 2496 bbls. How many does he sell during the week?

34. Find the total number of pounds of tobacco produced in the following states in 1879: Kentucky, 171,121,134; Virginia, 80,099,838; Pennsylvania, 36,957,772; Ohio, 34,725,405; Tennessee, 29,365,052; North Carolina, 26,986,448; Maryland, 26,082,147; Connecticut, 14,044,652; Missouri, 11,994,077; Wisconsin, 10,878,463.

32. Find the total value of the following crops for the year 1884: Corn, \$640,735,589; wheat, \$330,861,254; rye, \$14,855,255; oats, \$161,528,470; barley, \$29,781,155.

33. Find the total number of bales of cotton produced in the following states in 1879 : Mississippi, 955,808 ; Georgia, 814,441 ; Texas, 803,642 ; Alabama, 699,654 ; Arkansas, 608,256 ; South Carolina, 522,548 ; Louisiana, 508,569 ; North Carolina, 389,598.

34. Find the total number of bushels of wheat produced in the following states in 1879 : Illinois, 51,136,455 ; Indiana, 47,288,989 ; Ohio, 46,014,869 ; Michigan, 35,537,097 ; Minnesota, 34,625,657 ; Iowa, 31,177,225 ; California, 28,787,132 ; Missouri, 24,971,727 ; Wisconsin, 24,884,689.

Complete the following statements by adding downwards and from left to right. The sums of the totals should be equal.

35. SALES FOR THE WEEK ENDING JAN. 21, 1887.

[illegible]

SUBTRACTION.

The **difference** between two numbers is a number which, added to the smaller, will produce a result equal to the greater.

Subtraction is the process of finding the *difference* between two numbers.

The greater of two numbers whose difference is required is called the *minuend*, and the smaller the *subtrahend*. The result is called the *remainder*.

The sign of subtraction is —, and is read *minus* or *less*.

Thus, $8 - 5$ is read 8 minus 5, or 8 less 5, and means that 5 is to be taken from 8.

To find the difference between two numbers.

Ex. Find the difference between 967 and 384.

OPERATION.	ANALYSIS.—
967 Minuend.	Write the smaller number under the greater so that units are under units, tens under tens, etc. Commence to subtract at the right. 4 units from 7 units are 3 units, which write below the line under the column of units.
384 Subtrahend.	
583 Remainder.	

Since 8 tens cannot be taken from 6 tens, take 1 hundred from 9 hundreds, leaving 8 hundreds, and add it (1 hundred = 10 tens) to the 6 tens, making 16 tens. 8 tens from 16 tens are 8 tens, which write under the column of tens. 3 hundreds from 8 (9—1) hundreds are 5 hundreds. Hence the result is 583.

Instead of subtracting 1 from the figure of the *upper number* of the next higher order when it has been necessary to add 10 to the figure of the *minuend*, some persons add 1 to the figure of the *lower number* of the next higher order. This method depends on the principle that adding equivalent numbers to both minuend and subtrahend does not affect the remainder.

In practice, do not think of explanations, nor say 4 from 7 is 3, etc., but think only of results and write them at once. Thus, in the above example, say or think only 3, 8, 5.

RULE.—*Write the subtrahend under the minuend so that units of the same order stand in the same column.*

Commencing at the right, subtract each figure in the lower number from the one above it, and write the difference in the line below.

If any figure is greater than the one above it, add 10 to the latter, perform the subtraction, and then consider the next figure in the upper number decreased by 1 (or, consider the next figure in the lower number increased by 1).

EXAMPLES.

Find the difference between

- | | |
|--------------------------|----------------------------|
| 1. 8716 and 4379. | 11. 80706040 and 23456789. |
| 2. 917642 and 9819. | 12. 76483672 and 87132191. |
| 3. 64321 and 23456. | 13. 123456789 and 9897960. |
| 4. 428165 and 317618. | 14. 72081099 and 87643229. |
| 5. 9371641 and 876543. | 15. 16417528 and 90716801. |
| 6. 7642878 and 6789119. | 16. 43184296 and 37529510. |
| 7. 8090403 and 7090508. | 17. 100010001 and 9890978. |
| 8. 6380912 and 5270937. | 18. 30040050 and 29917168. |
| 9. 7654321 and 1234567. | 19. 20103040 and 19181746. |
| 10. 7060509 and 6987969. | 20. 40020003 and 20807064. |

21. There were 50017 post-offices in the United States in 1884 and 51252 in 1885. What was the increase during the year?

22. In 1880, the population of the United States was 50,152,866, and in 1870, 38,558,371. What was the increase during the decade?

23. The area of Alaska is 369,529,600 acres. How much greater is it than Texas, whose area is 175,587,840 acres?

24. The public debt of the United States was \$1,447,657,-568, Nov. 1, 1885, and \$1,354,347,947, Nov. 1, 1886. What was the reduction of the debt during the year?

25. The gross weights (weights of barrels and sugar) and tares (weights of barrels) of ten barrels of sugar are as follows: 326-19, 332-19, 307-18, 321-18, 324-19, 330-19, 313-18, 313-19, 317-17, 327-19. Find the total net weight.

NOTE.—Find the total gross weight and total tare, and then the difference, or the total net weight.

Population of the following cities of the United States in 1880:

New York,	-	1,206,590	Buffalo,	-	-	155,137
Philadelphia,	-	846,984	Washington,	-	-	147,307
Brooklyn,	-	580,370	Newark,	-	-	136,400
Chicago,	-	503,304	Louisville,	-	-	123,645
Boston,	-	362,535	Jersey City,	-	-	120,728
St. Louis,	-	350,522	Detroit,	-	-	116,342
Baltimore,	-	332,190	Milwaukee,	-	-	115,578
Cincinnati,	-	255,708	Providence,	-	-	104,850
San Francisco,	-	233,956	Albany,	-	-	90,903
New Orleans,	-	216,140	Rochester,	-	-	89,363
Cleveland,	-	160,142	Allegheny,	-	-	78,681
Pittsburg,	-	156,381	Indianapolis,	-	-	75,074

26. What is the total population of the first column of the above cities? Of the second column? What is the total population of all?

27. What is the difference between the sums of the first and second columns?

28. How much does the total population of New York, Brooklyn, Newark, Jersey City, Hoboken (30,999), Yonkers (18,892), and Long Island City (17,117) lack of being 2,500,000?

29. How much does the total population of Pittsburg and Allegheny exceed that of San Francisco?

MULTIPLICATION.

Multiplication is the operation of taking one number as many times as there are units in another.

The number taken or multiplied is called the *Multiplicand*. The number which indicates how many times the multiplicand is taken or multiplied, is called the *Multiplier*. The result obtained is called the *Product*.

The sign of multiplication is \times , and is read *times*, or *multiplied by*.

Thus, $5 \times 4 = 20$, is read 5 times 4 equals 20, or 5 multiplied by 4 equals 20.

Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

To find the product of two numbers when the multiplier does not exceed 12.

Ex. Multiply 456 by 7.

OPERATION. **ANALYSIS.**—7 times 6 units are 42 units = 4 tens and 2 units. Write the 2 units under the figure of the multiplier (the column of units) and add the 4 tens to the next product (the column of tens). 7 times 5 tens are 35 tens, plus 4 tens from the preceding product, are 39 tens = 3 hundreds and 9 tens. Write the 9 tens under the column of tens, and add the 3 hundreds to the next product (the column of hundreds). 7 times 4 hundreds are 28 hundreds, plus 3 hundreds from the preceding product are 31 hundreds = 3 thousands and 1 hundred. Write the 1 hundred under the column of hundreds and the 3 thousands in the column of thousands.

$$\begin{array}{r} 456 \\ 7 \\ \hline 3192 \end{array}$$

RULE.—*Commencing at the right, multiply each figure of the multiplicand by the multiplier, writing the result and carrying as in addition.*

EXAMPLES.

Multiply

1. 23456 by 7 ; by 8.
2. 37804 by 9 ; by 6.
3. 24687 by 2 ; by 4.
4. 36925 by 3 ; by 8.
5. 48716 by 5 ; by 9.
6. 90809 by 9 ; by 8.
7. 26048 by 5 ; by 7.

Multiply

8. 789123 by 2 ; by 3.
9. 123567 by 4 ; by 5.
10. 781693 by 6 ; by 9.
11. 417009 by 8 ; by 7.
12. 509048 by 8 ; by 7.
13. 637485 by 6 ; by 9.
14. 748596 by 7 ; by 5.

15. There are 5280 feet in one mile. How many feet in 11 miles ?

16. There are 4 gills in one pint, 2 pints in one quart, and 4 quarts in one gallon. How many gills in 63 gallons ?

17. There are 2 pints in one quart, 8 quarts in one peck, and 4 pecks in one bushel. How many pints in 379 bushels ?

18. There are 12 inches in one foot, and 3 feet in one yard. How many inches in 1760 yards ?

To find the product of two numbers, when the multiplier is more than 12.

Ex. Multiply 456 by 237.

OPERATION.	ANALYSIS.—Write the multiplier under the multipli-
456	cand so that their right-hand figures are in the same ver-
237	tical line. Since the multiplier consists of 7 units, 3 tens,
3192	and 2 hundreds, the multiplicand is repeated or multiplied
1368	by 7, by 30, and by 200. 7 times 456 is 3192, the first
912	partial product; 30 times 456 is 13680, the second partial
108072	product; 200 times 456 is 91200, the third partial product.
	The sum of these partial products is 108072, the entire
	product. In practice, the ciphers are omitted. In the
	operation, observe that the first or right-hand figure of each partial
	product is directly under the figure of the multiplier used.

RULE.—Write the multiplier under the multiplicand so that their right-hand figures are in the same vertical line.

Multiply the multiplicand by each significant figure of the multiplier, writing the first or right-hand figure of each partial product under the figure of the multiplier used.

Add the partial products. The sum will be the desired product.

EXAMPLES.

Multiply

1. 1728 by 37; by 481.
2. 2893 by 26; by 506.
3. 3904 by 18; by 624.
4. 5107 by 41; by 375.
5. 6079 by 59; by 208.
6. 8125 by 67; by 567.

Multiply

7. 23456 by 294; by 3742.
8. 40607 by 144; by 4803.
9. 32738 by 176; by 5964.
10. 91609 by 201; by 6075.
11. 24135 by 345; by 7186.
12. 38246 by 678; by 8297.

13. Find the cost of 375 barrels pork at \$14 per barrel?

14. There are 5280 feet in one mile. How many feet in 96 miles? In 208 miles?

15. How many pounds in 471 bushels corn, if there are 56 pounds in one bushel?

16. In a bushel of timothy seed, there are 45 pounds. How many pounds in 2367 bushels? In 3416 bushels?

17. A certain building has 192 windows, and each window contains 24 panes of glass. How many panes in all?

18. How many feet of wire will be required to fence a field 209 feet square, the fence being 6 wires high and on all sides of the field?

To find the product of two numbers when there are ciphers at the right of the significant figures of one or both.

Ex. Multiply 37600 by 47000.

OPERATION.

```

37600
 47000
-----
2632
1504
-----
1767200000

```

ANALYSIS.—Write the numbers so that the right-hand significant figures are in the same vertical line. $37600 = 376 \times 100$, and $47000 = 47 \times 1000$. Since the product of two or more numbers is the same in whatever order they are multiplied, multiply 376 by 47, and their product by 100000 (100×1000), by annexing 5 ($3+2$) ciphers to the right.

RULE.—*Write the numbers so that their right-hand significant figures are in the same vertical line. Multiply the significant figures together as if there were no ciphers, and to their product annex as many ciphers as are found on the right of both numbers.*

EXAMPLES.

Multiply

1. 3600 by 40; by 300.
2. 1728 by 80; by 500.
3. 3456 by 70; by 420.
4. 3710 by 50; by 360.
5. 4000 by 30; by 800.
6. 2800 by 90; by 370.
7. 4200 by 20; by 500.

Multiply

8. 48400 by 200; by 1400.
9. 37000 by 500; by 2500.
10. 12345 by 600; by 3600.
11. 28000 by 420; by 4700.
12. 19700 by 340; by 5800.
13. 14320 by 560; by 6900.
14. 96000 by 900; by 4800.

DIVISION.

Division is the operation of finding how many times one number is contained in another.

The number divided is called the *dividend*. The number by which it is divided is called the *divisor*. The result obtained is called the *quotient*. The part of the dividend which remains after the operation is completed is called the *remainder*.

The sign of division is \div , and is read *divided by*.

Thus $16 \div 2 = 8$ is read, sixteen divided by two equals eight.

To divide when the divisor does not exceed 12.

NOTE.—When the work is performed mentally, as in the following operation, the process is called *Short Division*.

Ex. Divide 1859 by 4.

OPERATION.

$$\begin{array}{r} 4 \overline{) 1859} \\ 464\frac{3}{4} \end{array}$$

ANALYSIS.—Write the divisor at the left of the dividend, as in the operation, and begin to divide at the left. 4 is not contained in 1 thousand, the highest order of the dividend, therefore, divide 18

hundreds by 4. 4 is contained in 18 (hundreds), 4 (hundred) times, and 2 hundreds remain. Write the 9 hundreds under the line in hundreds' place, and reduce the 2 hundreds remaining to tens, making 20 tens, which added to the 5 tens of the dividend, make 25 tens. 4 is contained in 25 (tens), 6 (tens) times and 1 ten remains. Write the 6 tens under the line in tens' place, and reduce the 1 ten remaining to units, making 10 units, which added to the 9 units of the dividend, make 19 units. 4 is contained in 19 (units), 4 (units) times, and 3 units remain. Write the 4 units in units' place, and write the remainder over the divisor, with a line between them in the form of a fraction, thus, $\frac{3}{4}$ (three-fourths). The complete result is $464\frac{3}{4}$.

Observe that each quotient figure is placed directly under the last figure of the dividend used.

RULE.—Write the divisor at the left of the dividend with a curved line between them.

Beginning at the left, divide each figure of the dividend by the divisor, and place the quotient beneath the figure divided. Whenever a remainder occurs, prefix it to the following figure of the dividend, and divide as before.

Continue the operation until all the figures of the dividend have been divided, and place the remainder, if any, over the divisor at the right of the quotient.

PROOF.—Multiply the quotient by the divisor, and to the product add the remainder. If the result equals the dividend, the work is probably correct.

EXAMPLES.

Divide

1. 78912348 by 2 ; by 3.

2. 97652464 by 4 ; by 6.

3. 16327620 by 5 ; by 6.

4. 78070804 by 4 ; by 7.

5. 12345678 by 6 ; by 9.

6. 988654320 by 5 ; by 8.

7. 234568836 by 4 ; by 9.

8. 357212254 by 2 ; by 7.

Divide

9. 103050709 by 2 ; by 5.

10. 214161810 by 3 ; by 6.

11. 425262728 by 4 ; by 7.

12. 123456789 by 3 ; by 8.

13. 246801234 by 6 ; by 11.

14. 789123650 by 7 ; by 10.

15. 287236450 by 5 ; by 12.

16. 176111888 by 6 ; by 11.

17. In one square yard there are 9 square feet. How many square yards in 41652 square feet ?

18. There are 12 pence in one shilling. How many shillings in 124656 pence ?

19. In a barrel containing 1068 eggs, how many dozen ? What is their value at 23 cents per dozen ?

20. In one foot there are 12 inches. How many feet in 63360 inches ?

21. There are 2 pints in one quart, and 4 quarts in one gallon. How many gallons in 160048 pints ?

To divide by any divisor greater than 12.

NOTE.—When the work is all written, as in the following operation, the process is called *Long Division*.

Ex. Divide 13218 by 43.

OPERATION.			ANALYSIS.—Since 43 is not contained
Divisor.	Dividend.	Quotient.	
43)	13218 ($307\frac{17}{43}$	in 13 (thousands), we take 132 (hundreds) for the first partial dividend.
	129		43 is contained in 132 (hundreds), 3
	<hr/> 318		(hundred) times. 43×3 (hundreds) =
	103		129 (hundreds), which write under the
	<hr/> 17	Remainder.	132 (hundreds), and subtract. The re-
			mainder is 3 (hundreds), to which
			annex the 1 (ten) of the dividend, and
			the second partial dividend is 31 (tens). 43 is not contained in 31
			(tens), therefore write 0 as the next figure of the quotient. Annex to
			the partial dividend, 31 (tens), the 8 (units) of the dividend, and the
			next partial dividend is 318 (units). 43 is contained in 318 (units), 7
			(units) times. 43×7 (units) = 301 (units), which write under the 318
			(units) and subtract. The remainder is 17 (units). Indicate the
			division of this remainder in the form of a fraction, thus: $\frac{17}{43}$, and
			annex it to the quotient, producing $307\frac{17}{43}$ for the complete quotient.

RULE.—Write the divisor at the left of the dividend, with a curved line between them.

Take for the first partial dividend the least number of figures on the left that will contain the divisor, and write the quotient figure at the right.

Multiply the divisor by the quotient, write the product under the partial dividend, and subtract. To the remainder, annex the next figure of the dividend, for the second partial dividend.

Divide as before, and thus continue until all the figures of the dividend have been used.

Write the remainder, if any, over the divisor in the form of a fraction, and annex it to the quotient. The result will be the complete quotient.

PROOF.—*Multiply the divisor by the quotient, and to the product add the remainder. If the sum equals the dividend, the work is probably correct.*

EXAMPLES.

Divide

1. 307845 by 26 ; by 143.
2. 248916 by 19 ; by 249.
3. 375428 by 38 ; by 375.
4. 481369 by 48 ; by 116.
5. 423706 by 25 ; by 208.
6. 306402 by 18 ; by 429.
7. 128943 by 64 ; by 567.
8. 709080 by 73 ; by 432.

Divide

9. 8712460 by 73 ; by 817.
10. 1428716 by 84 ; by 365.
11. 2893429 by 69 ; by 144.
12. 7364128 by 14 ; by 128.
13. 2125639 by 70 ; by 320.
14. 3756425 by 64 ; by 231.
15. 4183691 by 36 ; by 365.
16. 3804072 by 96 ; by 729.

17. How many days in 8766 hours ?

18. In 20000 pens, how many gross ? (1 gross = 144.)

19. How many bushels in 21674 pounds of oats, if there are 32 pounds in one bushel ?

20. There are 56 pounds in a bushel of rye. How many bushels in 19958 pounds ?

21. There were 31392893 gallons of molasses imported into the United States in 1885. How many hogsheads of 63 gallons each ?

22. How many cords in 47164 cubic feet, if there are 128 cubic feet in one cord ?

23. How many miles in 49164 rods, if there are 320 rods in one mile ?

24. The expenditures of the United States for the year 1886 were \$287,034,182. How much was that per day (365 days in the year) ?

25. During the year 1882, 738992 immigrants arrived in the United States. What was the average number per day ?

26. The exports of cotton during the year 1885 were 1,889,514,368 pounds. How many bales averaging 476 pounds each ?

UNITED STATES MONEY.*

United States Money is the legal currency of the United States.

TABLE.

10 Mills	= 1 Cent	<i>c.</i> , <i>ct.</i>	NOTE.—The currency of the Canadian Provinces is nominally the same as that of the United States.
10 Cents	= 1 Dime	<i>d.</i>	
10 Dimes	= 1 Dollar	<i>§.</i>	
10 Dollars	= 1 Eagle	<i>E.</i>	

To write United States money.

In writing U. S. money, the decimal notation is used. *Dollars* are written at the left of the decimal point and form the integral part. *Cents* are written as *hundredths* of a dollar, and occupy the first two places at the right of the decimal point. *Mills* are written as *thousandths* of a dollar, and occupy the third decimal place.

Thus, two dollars, eight cents, and six mills, is written \$2.086.

When the number of cents is less than ten, a cipher must be written in the first place at the right of the point. Thus, eight dollars and six cents is written, \$8.06.

Express the following amounts by figures :

1. Eighty-six dollars, nineteen cents, five mills.
2. Fourteen dollars, seventy-five cents, three mills.
3. Five hundred twenty-six dollars, seventy cents.
4. Two thousand dollars, thirty cents, two mills.
5. Seven hundred dollars, nine cents.
6. Fifty thousand dollars, seven mills.
7. Four hundred eight dollars, two cents, five mills.
8. Two hundred fifty dollars, sixty cents, three mills.

To reduce dollars to cents and mills, or to reduce cents and mills to dollars.

Dollars may be reduced to cents by multiplying by 100 or by annexing two ciphers. Dollars may be reduced to mills by multiplying by 1000 or by annexing three ciphers.

Thus, \$64 = 6400 cents, or 64000 mills.

If the amount consists of dollars and cents, reduce to cents by removing the decimal point 2 places to the right ; to mills, three places to the right. Thus, \$17.28 = 1728 cents, or 17280 mills ; \$34.658 = 34658 mills.

Cents may be reduced to dollars by dividing by 100, or by pointing off two decimal places. Mills may be reduced to dollars by dividing by 1000, or by pointing off three decimal places.

Thus, 12345 cents = \$123.45 ; 37560 mills = \$37.56.

Reduce the following to cents :

1. \$345.	4. \$17.04.	7. \$148.19.
2. \$2376.	5. \$28.37.	8. \$204.40.
3. \$2004.	6. \$49.75.	9. \$317.04.

Reduce the following to dollars :

1. 148 cents.	4. 705 cents.	7. 18000 mills.
2. 2300 cents.	5. 4212 cents.	8. 9370 mills.
3. 4617 cents.	6. 13409 cents.	9. 12375 mills.

To add or subtract in United States money.

RULE.—Write dollars under dollars, and cents under cents. Add or subtract as in simple numbers, and place the point in the result directly under the points in the numbers added or subtracted.

NOTE.—In subtraction of U. S. money, if there are cents in the subtrahend and none in the minuend, suppose ciphers to be added to the subtrahend in cents' place.

EXAMPLES.

1. Add 5 dollars, 16 cents; 18 dollars, 5 cents; 404 dollars, 75 cents; 25 dollars, 8 cents; 2376 dollars, 40 cents; 8 dollars, 2 cents.

2. Add \$170, \$106.40, \$240, \$200.40, \$70, \$.70, \$234.75.

3. Add \$108.25, \$2345, \$6.04, \$7.10, \$192.43, \$117.05.

4. Add \$.06, \$6, \$108.16, \$500.64, \$564, \$5.64, \$117.10, \$2081.48.

5. From \$124.16 subtract \$109.25.

6. From \$117 subtract \$98.49.

7. From \$575 subtract 575 cents.

8. Add \$1875.24, \$416, \$234.70, \$558.96, \$437.10, \$442.57, \$120.92, \$316.75, \$242.71, \$195.

To multiply United States money.

Ex. Find the cost of 9 desks at \$2.45 each.

OPERATION.

$\begin{array}{r} 2.45 \\ 9 \\ \hline 22.05 \end{array}$	<p>ANALYSIS.—Disregarding the decimal point, multiply as in ordinary multiplication. 9 times 245 cents is 2205 cents = \$22.05.</p>
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RULE.—*Multiply as in simple numbers, and from the right of the product point off as many figures as there are figures to the right of the decimal points in both numbers.*

NOTE.—If, as in Examples 6 and 7, the number expressing cents would make a convenient multiplier, use it as such, considered as an abstract number, and point off the results according to the rule.

Multiply

1. 12 dollars and 18 cents by 8.
2. 4 dollars and 25 cents by 12.
3. 16 dollars and 9 cents by 17.
4. 27 dollars and 8 cents by 25.
5. 43 dollars and 50 cents by 76.
6. 8 cents by 2345; by 3456.

Multiply

7. \$.07 by 1239; by 13416.
8. \$20.04 by 20; by 108.
9. \$176 by 18; by 144.
10. \$36.25 by 36; by 117.
11. \$48.19 by 48; by 288.
12. \$50.08 by 75; by 192.

To divide in United States money.

Ex. If 9 desks are worth \$22.05, what is one worth ?

OPERATION.	ANALYSIS.—
$\begin{array}{r} 9 \overline{) 22.05} \\ \$2.45 \end{array}$	If 9 desks are worth 2205 cents (\$22.05), 1 desk is worth one-ninth of 2205 cents or 245 cents. 245 cents = \$2.45.

Ex. If 8 chairs are worth \$18, what is one worth ?

OPERATION.	ANALYSIS.—
$\begin{array}{r} 8 \overline{) 18.00} \\ \$2.25 \end{array}$	If the dividend consists of dollars only, and does not contain the divisor an exact number of times, reduce it to cents by annexing two ciphers.

Ex. At \$6.25 each, how many sheep can be bought for \$50 ?

OPERATION.	ANALYSIS.—
$\begin{array}{l} \$6.25 \overline{) \$50.00} \text{ (8} \\ \text{Or,} \\ 625 \overline{) 5000} \text{ (8} \end{array}$	If 1 sheep costs \$6.25, as many sheep can be bought for \$50 as \$6.25 is con- tained times in \$50. \$50 = 5000 cents. \$6.25 = 625 cents. 5000 cents \div 625 cents = 8 times. Hence the result is 8 sheep.

RULE.—*Divide as in simple numbers, and point off from the right of the quotient as many decimal places as those in the dividend exceed those in the divisor.*

NOTE.—If the divisor alone contains cents, make the dividend cents by annexing two ciphers ; or, reduce both divisor and dividend to cents by annexing ciphers, omit the decimal points, and divide as in simple numbers.

EXAMPLES.

1. If 12 books are sold for \$41.40, what is the price of one book ?

2. How many pounds of tea at 65 cents per pound can be bought for \$9.75 ?

NOTE.—In the following examples, if the quotient is in U. S. money and the result is not an exact number of dollars, continue the division to cents.

Divide

3. \$25.44 by 48; by 106.

4. \$476 by 25; by 35.

5. \$1728 by 36; by 48.

6. \$73.08 by 84; by 87.

7. \$106.56 by 72; by 576.

8. \$1884 by 75; by 1535.

Divide

9. \$130.38 by \$2.46; by \$1.06.

10. \$149.04 by \$0.36; by \$2.07.

11. \$156.24 by \$0.72; by \$4.34.

12. \$1728 by \$0.75; by \$6.75.

13. \$3456 by \$2.25; by \$13.50.

14. \$7154 by \$1.75; by \$25.55.

15. A drover received \$4134.72 for 59 head of cattle at 8c. per pound. What was their average weight?

PROPERTIES OF NUMBERS.

DEFINITIONS.

1. A **Unit**, or **Unity**, is one, or a single thing; as *one*, *one* foot, *one* dollar.

2. A **Number** is a unit, or a collection of units; as *one*, *four*, *three* feet, *five* dollars.

3. All numbers are either *integral* or *fractional*, *abstract* or *concrete*.

4. An **Integral Number**, or **Integer**, is a number which expresses whole things; as *two*, *four* gallons.

5. A **Fractional Number**, or **Fraction**, is a number which expresses one or more equal parts of a unit; as *one-half*, *three-fourths*.

6. An **Abstract Number** is a number which does not refer to any particular object; as *one*, *six*, *ten*.

7. A **Concrete Number** is a number applied to an object, or quantity; as *three* apples, *five* pounds, *ten* cents.

8. Integral numbers are either *odd* or *even*, *prime* or *composite*.

9. An **Odd Number** is a number whose unit figure is 1, 3, 5, 7, or 9; as 1, 21, 39.

10. An Even Number is a number whose unit figure is 0, 2, 4, 6, or 8; as 6, 40, 74.

11. A Prime Number is a number which can be exactly divided only by itself and unity; as 1, 7, 13, 29.

12. Numbers are prime to each other when no integer greater than 1 will divide each without a remainder.

Numbers that are prime to each other are not necessarily prime numbers. Thus, 25 and 28 are prime to each other, but they are not prime numbers.

13. A Composite Number is a number which can be exactly divided by other integers besides itself and unity.

Thus 28, the product of 4 and 7, is a composite number. It is exactly divisible by 4 and 7.

DIVISIBILITY OF NUMBERS.

14. An Exact Divisor of a number is any number that will divide it without a remainder.

15. A number is said to be *divisible* by another when the latter will divide the former without a remainder. Any number is divisible

1. By 2, if it is an even number; as 6, 28, and 32.

2. By 3, if the sum of its digits is divisible by 3; as 849 ($8+4+9=21$, 21 is divisible by 3), 732, and 4782.

3. By 4, if the two right-hand figures are ciphers, or express a number divisible by 4; as 1100, 216, and 7328.

4. By 5, if the right hand figure is 0 or 5; as 40 and 135.

5. By 6, if it is an even number and the sum of its digits is divisible by 3; as 216, 840, and 732.

6. By 8, if the three right-hand figures are ciphers, or express a number divisible by 8; as 3000 and 7168.

7. By 9, if the sum of its digits is divisible by 9; as 216, 783, and 12348.

PRIME FACTORS.

16. The **Factors** of a number are those numbers which multiplied together will produce the number.

Thus 4 and 7; 2 and 14; 2, 2, and 7 are factors of 28.

The *factors* of a number are also the *exact divisors* of it.

17. A **Prime Factor** is a prime number used as a factor.

Thus 2, 2, and 7 are the prime factors of 28. 4 is a factor of 28, but not a *prime* factor.

18. To find all the prime factors of a composite number.

Ex. What are the prime factors of 6930 ?

OPERATION.

$$2 \overline{) 6930}$$

$$3 \overline{) 3465}$$

$$3 \overline{) 1155}$$

$$5 \overline{) 385}$$

$$7 \overline{) 77}$$

$$11$$

ANALYSIS.—Any prime number that is an exact divisor of the given number is a prime factor of it. Divide the given number by 2 (**15**, 1), the least prime divisor of it, obtaining the quotient 3465. Next, divide this quotient successively by 3 (**15**, 2), 3, 5 (**15**, 4), and 7. The last quotient 11 is a prime number and therefore a prime factor. The several divisors 2, 3, 3, 5, 7 and the last quotient 11 are the prime factors required.

$$2 \times 3 \times 3 \times 5 \times 7 \times 11 = 6930.$$

19. RULE.—*Divide by the least prime number which will divide the given number without a remainder. In like manner divide the resulting quotient, and continue the division until the quotient is a prime number. The several divisors and the last quotient are the prime factors.*

EXAMPLES.

20. Find the prime factors of the following numbers:

1. 3465.

4. 3150.

7. 6552.

10. 5368.

2. 3003.

5. 3675.

8. 7826.

11. 3825.

3. 4158.

6. 2310.

9. 6006.

12. 5324.

COMMON DIVISORS.

21. A Common Divisor of two or more numbers is any number that will divide each without a remainder; hence it is a common factor of each of them.

22. The Greatest Common Divisor of two or more numbers is the *greatest* number that will divide each without a remainder; hence it is their greatest common factor.

Thus, 2, 3, 4, and 12 are common divisors of 36, 48, and 60; 12 is their greatest common divisor.

23. PRINCIPLE.—*The greatest common divisor of two or more numbers is the product of all their common prime factors.*

24. To find the greatest common divisor of two or more numbers.

Ex. What is the greatest common divisor of 168, 252, and 420?

FIRST OPERATION.

$$168 = 2 \times 2 \times 2 \times 3 \times 7$$

$$252 = 2 \times 2 \times 3 \times 3 \times 7$$

$$420 = 2 \times 2 \times 3 \times 5 \times 7$$

$$2 \times 2 \times 3 \times 7 = 84.$$

ANALYSIS.—Resolve the numbers into their prime factors. The product, 84, of the common factors 2, 2, 3, and 7 is the greatest common divisor (*Prin.*)

SECOND OPERATION.

$$4 \overline{) 168, 252, 420}$$

$$7 \overline{) 42, 63, 105}$$

$$3 \overline{) 6, 9, 15}$$

$$2, 3, 5$$

$$4 \times 7 \times 3 = 84.$$

ANALYSIS.—Divide the given numbers by any number that will divide them all without a remainder, and divide the quotients in the same manner until the last quotients have no common divisor. Since 4 will divide all the given numbers, and 3 and 7 will divide successively the resulting quotients, their product, 84, is a common

divisor of the given numbers. Since the last quotients have no common divisor or factor, 84 is the *greatest* common divisor.

25. RULE.—*Resolve the numbers into their prime factors. The product of the factors common to all the numbers will be the greatest common divisor. Or,*

Divide the given numbers by any factor that will divide all of them without a remainder. In like manner divide the resulting quotients, and continue the division until the quotients have no common factor. The product of the several divisors will be the greatest common divisor.

EXAMPLES.

26. Find the greatest common divisor of the following numbers :

- | | |
|----------------------|------------------------|
| 1. 24, 36, and 48. | 9. 108, 144, and 360. |
| 2. 35, 56, and 70. | 10. 144, 336, and 240. |
| 3. 42, 56, and 28. | 11. 165, 550, and 220. |
| 4. 30, 60, and 75. | 12. 792, 144, and 216. |
| 5. 64, 96, and 128. | 13. 405, 243, and 324. |
| 6. 66, 198, and 330. | 14. 378, 126, and 252. |
| 7. 90, 150, and 210. | 15. 375, 625, and 250. |
| 8. 84, 420, and 126. | 16. 288, 720, and 864. |

27. To find the greatest common divisor of two numbers when they are not readily factored.

28. PRINCIPLES.—1. *If the smaller of two numbers is a divisor of the greater, it is their greatest common divisor.*

2. *A common divisor of two numbers is a divisor of their sum, and also of their difference.*

3. *A divisor of a number is a divisor of any multiple of that number.*

29. RULE.—*Divide the greater number by the smaller, and divide the last divisor by the remainder; and so continue until there is no remainder. The last divisor will be the greatest common divisor.*

NOTES.—1. When the greatest common divisor of more than two numbers is required, find the greatest common divisor of the smallest two first, and of this greatest common divisor and the next greater, and so on, until all the numbers are used. The last divisor will be the greatest common divisor of all the given numbers.

2. If, at any step in the process, a prime factor appear that is not common to all the numbers, it may be rejected. (See second operation.)

3. If the remainder at any time is a prime number, and it is not contained in the last divisor, there is no common divisor greater than 1; it will therefore be useless to farther continue the division.

Ex. Find the greatest common divisor of 391 and 437.

OPERATIONS.		DEMONSTRATION.—
391) 437 (1		Since 23 is a divisor of
<u>391</u>	Or,	46, it is a divisor of 368
46) 391 (8	2) <u>46</u>	a multiple of 46 (<i>Prin.</i>
<u>368</u>	23) 391 (17	3). Since 23 is a divi-
23) 46 (2	<u>23</u>	sor of itself and 368, it
<u>46</u>	161	is a divisor of their sum,
0	<u>161</u>	391 (<i>Prin.</i> 2). Since 23
	0	is a divisor of 46 and 391,
		it is a divisor of their
		sum, 437. 23 is there-
		fore a <i>common divisor</i>
		of 391 and 437, the given
		numbers.

The greatest common divisor of 391 and 437, whatever it may be, is a divisor of their difference, 46 (*Prin.* 2); also of 368, a multiple of 46 (*Prin.* 3); also of 23, 391 — 368 (*Prin.* 2). Since the divisor of a number cannot be greater than itself, the greatest common divisor of the given numbers cannot be greater than 23. 23 is therefore the *greatest common divisor*.

30. Find the greatest common divisor of the following numbers :

- | | |
|-----------------------|------------------------|
| 1. 319 and 377. | 6. 744, 984, and 522. |
| 2. 259 and 629. | 7. 391, 667, and 920. |
| 3. 589 and 713. | 8. 451, 481, and 737. |
| 4. 903 and 989. | 9. 504, 756, and 252. |
| 5. 611, 799, and 987. | 10. 425, 748, and 561. |

COMMON MULTIPLES.

31. A **Multiple** of a number is a number that is exactly divisible by it; or, it is any product of which the given number is a factor.

32. A **Common Multiple** of two or more numbers is a number that is exactly divisible by each of them.

33. The **Least Common Multiple** of two or more numbers is the *least* number that is exactly divisible by each of them.

Thus, 12, 24, 36, and 48 are common multiples of 4 and 6; 12 is their least common multiple.

34. PRINCIPLES.—1. *A multiple of a number contains all the prime factors of that number.*

2. *A common multiple of two or more numbers contains all the prime factors of each of those numbers.*

3. *The least common multiple of two or more numbers contains all the prime factors of each of the numbers, and no other factors.*

35. To find the least common multiple of two or more numbers.

Ex. What is the least common multiple of 12, 18, 20, and 40?

FIRST OPERATION.

$$12 = 2 \times 2 \times 3$$

$$18 = 2 \times 3 \times 3$$

$$40 = 2 \times 2 \times 2 \times 5$$

$$2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$$

ANALYSIS.—Since 40, a multiple

of 20, contains all the prime factors of 20, the number 20 may be omitted in the operation. Resolve the numbers into their prime factors.

The least common multiple must contain 2 as a factor 3 times in order

to be divisible by 40; it must contain 3 as a factor twice in order to be divisible by 18; and it must contain 5 as a factor, in order to be divisible by 40. 360, the product of the factors 2, 2, 2, 3, 3, and 5, is the least common multiple of the given numbers, since it contains the different factors the greatest number of times that they occur in the given numbers, and no other factors (*Prin. 3*).

SECOND OPERATION.

$$2 \overline{) 12, 18, 40}$$

$$2 \overline{) 6, 9, 20}$$

$$3 \overline{) 3, 9, 10}$$

$$1, 3, 10$$

$$2 \times 2 \times 3 \times 3 \times 10 = 360$$

ANALYSIS.—The factors of the required multiple may be selected by the following process. Divide the given numbers by any prime number that will divide two or more of them, writing the quotients and the undivided numbers beneath. Treat the resulting numbers in like manner, and continue the process until no two of the numbers have a common factor or divisor. The product of the several divisors and the remaining quotients and undivided numbers will be the least common multiple.

36. RULE.—Resolve the given numbers into their prime factors. The product of the different prime factors, taking each factor the greatest number of times it appears in any of the numbers, will be the least common multiple. Or,

Divide the given numbers by any prime number (See Note 2) that will exactly divide two or more of them, writing the quotients and undivided numbers beneath. Repeat the operation with the resulting numbers until there is no exact divisor of any two of them. The product of the divisors and the last quotients will be the least common multiple.

NOTES.—1. In the operation, reject such of the smaller numbers as are divisors of the larger; also reject such of the quotients and undivided numbers as are divisors of the others.

2. Divide by composite numbers when they are exact divisors of all the numbers.

EXAMPLES.

37. Find the least common multiple of the following :

1. 2, 3, 4, 5, and 6.

6. 5, 8, 15, and 18.

2. 8, 10, 12, and 15.

7. 6, 9, 21, and 24.

3. 12, 15, 18, and 24.

8. 32, 48, and 60.

4. 6, 10, 15, and 30.

9. 24, 32, and 40.

5. 14, 21, and 28.

10. 10, 24, and 32.

CANCELLATION.

38. Cancellation is a method of shortening an operation by rejecting equal factors from both dividend and divisor.

39. PRINCIPLES.—1. *Canceling or rejecting a factor from a number, divides the number by that factor.*

2. *Dividing both dividend and divisor by the same number does not affect the value of the quotient.*

Ex. Divide 84×36 by 27×14 .

OPERATIONS.

$$\begin{array}{r} 2 \qquad 4 \\ 84 \times 36 \\ 27 \times 14 \\ \hline 3 \end{array} = 8$$

Or,

$$\begin{array}{r|l} 3 & 27 \quad 84 \quad 6 \quad 2 \\ & 14 \quad 36 \quad 4 \\ \hline & 8 \end{array}$$

ANALYSIS.—Indicate the operations to be performed as in the margin.

It is seen by inspection that 36 and 27 contain the common factor 9; therefore cancel or reject it from both, retaining the factors 4 and 3 respectively. 14 and 84 contain the common factor 14;

therefore reject it, retaining the factor 6 in the dividend. [Since cancellation is a process of division, the rejecting of 14 does not destroy it, but divides it, leaving 1 as a quotient. It is unnecessary to write 1 as a quotient, except when there are no other factors in the dividend.] 3 is a common factor of 6 and 3; therefore reject it from both, retaining the factor 2 in the dividend. The product of the remaining factors, 2 and 4, is the required quotient.

40. RULE.—*Indicate the operations to be performed by writing the numbers denoting multiplication above a horizontal line, and the numbers denoting division below. The numbers above the line will form a dividend, and the numbers below a divisor. Cancel or reject the factors common to both dividend and divisor. The product of the remaining factors of the dividend divided by the product of the remaining factors of the divisor will be the required quotient.*

NOTE.—In many examples, it is more convenient to write the dividend or its factors on the right of a vertical line, and the divisor or its factors on the left. (See second operation above.)

EXAMPLES.

41. 1. Divide $27 \times 48 \times 60$ by $54 \times 36 \times 40$.

Find the value of the following expressions :

$$2. \quad \frac{40 \times 36 \times 42 \times 18}{9 \times 35 \times 30 \times 8}.$$

$$3. \quad \frac{24 \times 30 \times 54 \times 35}{14 \times 15 \times 21 \times 64}.$$

$$4. \quad \frac{360 \times 28 \times 27 \times 5}{25 \times 42 \times 18 \times 12}.$$

$$5. \quad \frac{1760 \times 6 \times 145}{100 \times 365}.$$

$$6. \quad \frac{1760 \times 175 \times 6}{4 \times 9 \times 100 \times 10}.$$

$$7. \quad \frac{144 \times 625 \times 37 \times 12}{288 \times 375 \times 185}.$$

8. Multiply 72 by 3×18 , divide the product by 8 times 9, multiply the quotient by 7×20 , divide the product by 360, multiply the quotient by 6 times 8.

9. A merchant bought 375 barrels of flour at \$5.50 per barrel, and paid in cloth at \$2.75 per yard ; how many yards did it require ?

10. Sold 28 bushels of apples for \$21 ; what should I receive for 42 bushels ?

11. How many cows worth \$35 each must be given in exchange for 84 tons of hay at \$15 per ton ?

12. What should be paid for the transportation of 3600 pounds of cheese at the rate of 47 cents per 100 pounds ?

13. What must be paid for transporting 31600 pounds of iron at \$5 per ton of 2000 pounds each ?

14. What will 7840 pounds of coal cost, at \$6 per ton of 2240 pounds each ?

15. If 3 men eat 7 pounds of meat in one week, how much would 6 men eat in 4 weeks ?

16. How many times can 16 bottles, each holding 3 pints, be filled from 6 demijohns, each containing 10 gallons of 8 pints each ?

17. A man exchanged 275 barrels of potatoes, each containing 3 bushels, at 54 cents per bushel, for a certain number of pieces of muslin each containing 45 yards, at 11 cents per yard. How many yards did he receive ?

REVIEW EXAMPLES.

42. 1. Write in figures each of the following numbers, add them, and express in words (or numerate) their sum : Forty-five thousand and forty-five ; sixteen thousand three hundred and sixty ; one hundred and sixty-seven thousand ; eight hundred and fifty thousand and ninety-two ; nine million and twenty-four.

2. Subtract eight hundred and fourteen thousand nine hundred and sixteen from four million and nine thousand.

3. Multiply five hundred and sixty thousand seven hundred and eight by eighteen hundred and sixty.

4. A quantity of merchandise was bought for \$27618.75, and sold for \$32418.25. What was the gain ?

5. What is the sum of 2817, 273, 30006, 97, 7285, 2700576, 7000781, and 27 ?

6. If I sell goods for \$23876, and gain \$5389, what did the goods cost me ?

7. What is the sum of the prime numbers from 20 to 50 ?

Add the following numbers as they stand, from left to right, and from right to left. [In making out bills, and in other commercial operations, a great deal of time can be saved by adding in this manner, without re-arranging the numbers.]

8. 17, 27, 36, 14, 43, 42, 65, 73, 81, 35.

9. 137, 414, 528, 345, 678, 975, 864, 357, 121, 234.

10. 67.16, 5.12, 3.75, 75, 38.42, 59.27, 38.75, 175.25.

11. 2345, 16, 375, 4218, 376, 7, 8475, 247, 39.

12. 1234.27, 348.25, 775, 7.16, 89.76, 374.12, 5673.56.

✓ 13. One factor of a certain number is 217 and the other 5280 ; what is the number ?

14. If the quotient is 375 and the divisor 246, what is the dividend ?

15. If the product of two factors is 450072, and one of the factors is 987, what is the other factor ?

Find the difference between the numbers in each of the following groups. [In all of these cases the subtrahend is placed above the minuend, the purpose being to give the student practice in subtracting *downward* rather than upward, as the general custom is. It is often requisite in business to perform the work in this way, and the accountant should practice both methods.]

(16.)	(17.)	(18.)	(19.)	(20.)
76534	19827	26347	72016	12345
<u>81279</u>	<u>84362</u>	<u>71356</u>	<u>99385</u>	<u>54321</u>

- ✓ 21. Find the prime factors of 108108.
22. What is the sum of the composite numbers from 60 to 90 inclusive?
23. Divide 76432801 by 783. Prove that your solution is correct.
24. A clerk receiving a salary of \$1256, pays \$468 a year for board, \$180 for clothing, and \$150 for other expenses. What amount has he left?
25. What is the least number that can be exactly divided by each of the following numbers: 32, 80, 48, and 90?
26. If I take 24889 from the sum of 9872 and 24967, divide the remainder by 50, and multiply the quotient by 18, what is the product?
- ✓ 27. If 160 acres of land cost \$10720, how many acres can be bought for \$8844?
28. What is the least common multiple of the nine digits?
29. If 75 head of cattle cost \$2550, what will 59 head cost?
30. A merchant sold 426 barrels of flour for \$2556, which was \$639 more than it cost. What did it cost a barrel?
31. What is the greatest number that will exactly divide each of the following numbers: 246, 744, and 522?
- ✓ 32. What is the smallest sum of money with which horses can be bought at \$96 each, cows at \$30 each, or sheep at \$5 each, using the same amount in each case?

33. A merchant bought 387 yards of cloth at 79 cts. per yard; he sold 298 yards at \$1.16 per yard, and the remainder at 97 cts. per yard; how much did he gain?

✓ 34. Cash on hand at beginning of the day, \$6492.75; cash received, \$11456.75; cash paid out, \$13285.26. Required the cash balance at the end of the day.

35. Mr. A has three farms, the first of which contains 158 acres, the second 32 acres less than the first, and the third as many as the other two. What is the value per acre, if all are worth \$26128?

✓ 36. A drover bought a number of cattle for \$12204, and sold the same for \$13560, by which he gained \$4 per head. How many cattle were purchased?

37. A farmer raised in one year 512 bushels of wheat, the next year twice as much as he raised the first year, and the third year four times as much as he did the second year. What was the value of the three crops at \$1.65 per bushel?

✓ 38. How many pounds of tea at 78 cts. per pound must be given for 375 bushels of wheat at \$1.56 per bushel?

✓ 39. Bought 75 tons of hay at \$16 per ton; gave in payment 56 sheep at \$3.75 each, and the remainder I paid in butter at 33 cts. per pound. How many pounds of butter were required?

✓ 40. Bought 225 acres of land for \$12600, and sold 116 acres at \$65 per acre, and the remainder at cost; how much did I gain?

✓ 41. In 1878 there were 39258 postmasters in the United States, and their total salaries were \$7,977,852; what was the average salary paid?

42. A sold to B 175 acres of land at \$135 an acre, and by so doing gained \$1925; B sold the land at a loss of \$1750. What did A pay per acre, and what was B's selling-price per acre?

✓ 43. A merchant sold 800 barrels of flour for \$5867.144 barrels of which he sold at \$7 per barrel, and 225 barrels at \$6.75. At how much per barrel did he sell the remainder?

FRACTIONS.

DEFINITIONS.

43. A **Fraction** is one or more of the equal parts of a unit ; as *one-half* ($\frac{1}{2}$), *two-thirds* ($\frac{2}{3}$), *seven-eighths* ($\frac{7}{8}$).

If a unit be divided into four equal parts, each part is called a fourth. If one of these parts be taken, the expression will be one-fourth ($\frac{1}{4}$); if three parts, three-fourths ($\frac{3}{4}$), etc.

44. The greater the number of equal parts into which a unit is divided, the less will be each part ; the less the number of parts, the greater will be each part.

One-half ($\frac{1}{2}$) is greater than one-third ($\frac{1}{3}$); one-fourth ($\frac{1}{4}$) is less than one-third ($\frac{1}{3}$).

45. A fraction is usually expressed by two numbers, one written above the other, with a line between. Fractions written in this form are usually called **Common Fractions**.

46. The number below the line is called the **Denominator**, because while indicating the number of equal parts into which the unit is divided, it *denominates* or names those parts.

47. The number above the line is called the **Numerator**, because it shows how many of the parts are taken to form the fraction.

48. The numerator and denominator, taken together, are called the **Terms** of the fraction.

In the fraction three-fourths ($\frac{3}{4}$), 3 and 4 are the terms; 4 is the denominator, and shows that the unit is divided into four equal parts, called fourths; 3 is the numerator, and shows that three of these parts are taken to constitute the fraction.

49. A fraction is an expression of unperformed division. The numerator is the dividend, the denominator is the divisor, and the value of the fraction is the quotient.

50. A **Simple Fraction** is a single fraction, both of whose terms are integers.

51. Simple fractions are *proper* or *improper*.

52. A **Proper Fraction** is one that is less than a unit : the numerator being less than the denominator. Thus $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$ are proper fractions.

53. An **Improper Fraction** is one that is equal to, or greater than a unit : hence the numerator must be equal to, or greater than the denominator. Thus, $\frac{3}{2}$, $\frac{5}{4}$, $\frac{3}{3}$, and $\frac{11}{8}$ are improper fractions.

54. A **Mixed Number** is an integer and a fraction united ; as $2\frac{1}{2}$, $4\frac{3}{4}$, $18\frac{7}{8}$.

55. PRINCIPLES.—1. *Multiplying the numerator or dividing the denominator by a number multiplies the fraction by that number.*

2. *Dividing the numerator or multiplying the denominator by a number divides the fraction by that number.*

3. *Multiplying or dividing both numerator and denominator by the same number does not change the value of the fraction.*

EXERCISES.

56. 1. Read the following fractions, and copy separately : 1, the simple fractions : 2, the proper fractions ; 3, the improper fractions : 4, the mixed numbers.

$\frac{13}{16}$; $4\frac{1}{4}$; $\frac{17}{4}$; $\frac{5}{10}$; $\frac{7}{8}$; $175\frac{3}{4}$; $13\frac{2}{3}$; $\frac{7}{9}$; $\frac{16}{5}$; $7\frac{3}{4}$; $8\frac{7}{9}$; $46\frac{3}{8}$; $\frac{141}{8}$; $\frac{18}{5}$; $\frac{3}{5}$.

2. Write the following fractions : three fourths ; seven eighths ; nineteen sixteenths ; five, and one half ; one hundred and three thirty-seconds ; one hundred, and three

thirty-seconds; forty-eight, and five twelfths; eleven tenths, nine forty-fifths; thirty-six twenty-eighths; sixty-five forty-eighths; thirteen, and two-thirds; sixteen twenty-fourths; ten tenths; fourteen, and forty-six hundredths; nineteen one hundred nineteenth; thirty-six four hundred thirty-seconds.

REDUCTION.

57. Reduction of Fractions is the changing their form without changing their value.

58. A fraction is reduced to *lower terms* when the numerator and denominator are expressed in smaller numbers.

59. A fraction is in its *lowest terms* when its numerator and denominator have no common divisor.

60. A fraction is reduced to *higher terms* when the numerator and denominator are expressed in larger numbers.

61. To reduce a fraction to its lowest terms.

Ex. Reduce $\frac{84}{126}$ to its lowest terms.

<p>OPERATION.</p> $\frac{84}{126} = \frac{2}{3}$	<p>ANALYSIS.—Dividing both terms of the fraction, $\frac{84}{126}$, by the common divisor, 6, the result is $\frac{14}{21}$; dividing both terms of $\frac{14}{21}$ by the common divisor, 7, the result is $\frac{2}{3}$. Since 2 and 3 have no common divisor, the fraction is reduced to its lowest terms (59).</p>
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The value of the fraction has not been changed, because both terms have been divided by the same number (55, 3).

The same result is often more readily obtained by dividing both terms by the greatest common divisor.

62. RULE.—*Divide the terms of the fraction by any number that will divide both without a remainder, and continue the operation with the resulting fractions until they have no common divisor.* Or,

Divide the terms of the fraction by their greatest common divisor.

EXAMPLES.

✓ **63.** Reduce to their lowest terms,

1. $\frac{32}{48}$.	8. $\frac{18}{144}$.	15. $\frac{528}{1760}$.	22. $\frac{125}{625}$.
2. $\frac{45}{60}$.	9. $\frac{96}{108}$.	16. $\frac{512}{728}$.	23. $\frac{125}{1000}$.
3. $\frac{48}{64}$.	10. $\frac{132}{168}$.	17. $\frac{648}{1024}$.	24. $\frac{6125}{10000}$.
4. $\frac{63}{81}$.	11. $\frac{105}{120}$.	18. $\frac{325}{450}$.	25. $\frac{288}{512}$.
5. $\frac{72}{128}$.	12. $\frac{144}{180}$.	19. $\frac{375}{1000}$.	26. $\frac{1056}{1784}$.
6. $\frac{85}{105}$.	13. $\frac{1296}{1416}$.	20. $\frac{625}{2000}$.	27. $\frac{136}{3088}$.
7. $\frac{90}{135}$.	14. $\frac{275}{625}$.	21. $\frac{86}{946}$.	28. $\frac{127}{3474}$.

64. To reduce a fraction to higher terms.

Ex. Reduce $\frac{3}{4}$ to a fraction whose denominator is 32.

OPERATION.

$$32 \div 4 = 8$$

$$\frac{3}{4} = \frac{24}{32}$$

ANALYSIS.—The fraction $\frac{3}{4}$ is reduced to *thirty* seconds, without changing its value, by multiplying the terms by the number that will cause its denominator 4 to become 32 (55, 3). By divid-

ing the required denominator 32 by the given denominator 4, this number is found to be 8. Multiplying both terms of $\frac{3}{4}$ by 8, the result is $\frac{24}{32}$.

65. RULE.—*Divide the required denominator by the denominator of the given fraction, and multiply both terms of the given fraction by the quotient.*

EXAMPLES.

✓ **66.** 1. Reduce $\frac{3}{4}$ to 48ths.

2. Change $\frac{7}{12}$ to an equivalent fraction having 60 for its denominator.

3. Reduce $\frac{2}{3}$, $\frac{5}{6}$, $\frac{9}{16}$ each to 48ths.

4. Reduce $\frac{5}{7}$, $\frac{3}{5}$, $\frac{21}{15}$ each to 105ths.

5. Reduce $\frac{4}{4}$, $\frac{5}{8}$, $\frac{1}{2}$ each to 56ths.

6. Reduce $\frac{7}{16}$, $\frac{11}{12}$, $\frac{13}{32}$ each to 96ths.

7. Reduce $\frac{7}{8}$, $\frac{8}{9}$, $\frac{3}{10}$ each to 360ths.

8. Reduce $\frac{17}{24}$, $\frac{5}{9}$, $\frac{11}{18}$ each to 72ds.

9. Reduce $\frac{1}{9}$, $\frac{20}{27}$, $\frac{17}{12}$ each to 108ths.

67. To reduce two or more fractions to equivalent fractions having their least common denominator.

68. A Common Denominator of two or more fractions is a denominator to which they can all be reduced, and is the common multiple of their denominators.

69. The Least Common Denominator of two or more fractions is the least denominator to which they can be reduced, and is the least common multiple of their denominators.

Ex. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{9}{10}$ to equivalent fractions having their least common denominator.

	OPERATION.	
$\frac{2}{3} = \frac{40}{60}$	2) 3, 4, 6, 10	
$\frac{3}{4} = \frac{45}{60}$	2 3 5	
$\frac{5}{6} = \frac{50}{60}$		
$\frac{9}{10} = \frac{54}{60}$	$2 \times 2 \times 3 \times 5 = 60$	

ANALYSIS.—The least common multiple of the denominators is found to be 60 (**35**), which we take as the least common denominator. By Art. **65**, $\frac{2}{3}$ is reduced to $\frac{40}{60}$. We proceed in the same manner

with each of the other fractions. The value of each fraction remains unchanged, since both terms have been multiplied by the same number. In many cases, the least common denominator can be readily found by inspection.

70. RULE.—*Find the least common multiple of the given denominators for the least common denominator, and reduce the given fractions to this denominator.*

EXAMPLES.

✓ **71.** Reduce the following fractions to equivalent fractions having their least common denominator:

- | | | |
|---|--|---|
| 1. $\frac{2}{3}, \frac{7}{10}, \frac{4}{15}$. | 5. $\frac{20}{21}, \frac{25}{28}, \frac{11}{14}$. | 9. $\frac{17}{12}, \frac{13}{18}, \frac{10}{15}$. |
| 2. $\frac{3}{3}, \frac{1}{2}, \frac{5}{4}$. | 6. $\frac{3}{5}, \frac{7}{16}, \frac{13}{20}$. | 10. $\frac{13}{20}, \frac{19}{25}, \frac{3}{10}$. |
| 3. $\frac{11}{12}, \frac{8}{15}, \frac{27}{16}$. | 7. $\frac{4}{7}, \frac{13}{25}, \frac{4}{5}$. | 11. $\frac{13}{18}, \frac{14}{21}, \frac{43}{27}$. |
| 4. $\frac{7}{8}, \frac{13}{16}, \frac{9}{18}$. | 8. $\frac{23}{24}, \frac{8}{9}, \frac{17}{18}$. | 12. $\frac{27}{25}, \frac{18}{20}, \frac{16}{15}$. |

72. To reduce an integer or a mixed number to an improper fraction.

Ex. In 18 units, how many fourths?

OPERATION.

18 ANALYSIS.—In 1 there are 4 fourths ($\frac{1}{4}$), and in 18,
 $\frac{4}{1}$ eighteen times 4 fourths, or 72 fourths ($\frac{18}{4}$). Hence,
 72 fourths. $18 = \frac{72}{4}$.

Ex. Reduce $16\frac{7}{8}$ to an improper fraction.

OPERATION.

$16\frac{7}{8}$. ANALYSIS.—In 1 there are 8 eighths ($\frac{1}{8}$), and in 16,
 $\frac{8}{1}$ sixteen times 8 eighths, or 128 eighths ($\frac{128}{8}$). 128
 128 eighths. eighths and 7 eighths are 135 eighths. Hence,
 $\frac{7}{1}$ eighths. $16\frac{7}{8} = \frac{135}{8}$.
 135 eighths.

73. RULE.—Multiply the integer by the required denominator, and to the product add the numerator of the fraction, and under the result write the denominator.

NOTE.—When the numerator of the fraction is a small number, add it mentally to the product of the integer and the denominator.

EXAMPLES.

74. 1. In 27, how many ninths?

2. Reduce $46\frac{1}{2}$ to halves.

3. How many eighths of a peck in $37\frac{1}{3}$ pecks?

Reduce the following to improper fractions :

- | | |
|--|---|
| 4. $37\frac{3}{4}$; $19\frac{7}{8}$; $208\frac{9}{16}$. | 10. $116\frac{11}{5}$; $456\frac{4}{11}$; $87\frac{3}{8}$. |
| 5. $56\frac{2}{3}$; $49\frac{5}{6}$; $182\frac{7}{8}$. | 11. $24\frac{3}{4}$; $179\frac{13}{16}$; $176\frac{3}{4}$. |
| 6. $375\frac{1}{2}$; $94\frac{9}{10}$; $46\frac{5}{8}$. | 12. $87\frac{3}{8}$; $490\frac{5}{12}$; $168\frac{1}{8}$. |
| 7. $44\frac{3}{4}$; $37\frac{5}{12}$; $19\frac{3}{16}$. | 13. $384\frac{5}{8}$; $161\frac{3}{4}$; $175\frac{5}{8}$. |
| 8. $12\frac{1}{2}$; $48\frac{7}{10}$; $45\frac{5}{12}$. | 14. $172\frac{1}{3}$; $216\frac{1}{3}$; $387\frac{1}{2}$. |
| 9. $81\frac{2}{3}$; $196\frac{1}{2}$; $375\frac{3}{4}$. | 15. $216\frac{5}{9}$; $333\frac{4}{9}$; $435\frac{3}{8}$. |

75. To reduce an improper fraction to an integer or a mixed number.

Ex. Reduce $\frac{27}{4}$ to a mixed number.

ANALYSIS.— $1 = \frac{4}{4}$; hence in $\frac{27}{4}$, there are as many units as 4 fourths are contained times in 27 fourths, or $6\frac{3}{4}$.

76. RULE.—*Divide the numerator by the denominator.*

EXAMPLES.

✓ **77. 1.** Change $\frac{317}{8}$ to a mixed number.

2. Reduce $\frac{64}{4}$ of a dollar to dollars.

Reduce to integers or mixed numbers :

3. $\frac{315}{4}$; $\frac{416}{8}$.

7. $\frac{542}{16}$; $\frac{873}{4}$.

11. $\frac{511}{25}$; $\frac{937}{80}$.

4. $\frac{138}{2}$; $\frac{376}{3}$.

8. $\frac{3861}{12}$; $\frac{1248}{16}$.

12. $\frac{111}{5}$; $\frac{1385}{36}$.

5. $\frac{518}{9}$; $\frac{111}{5}$.

9. $\frac{445}{9}$; $\frac{185}{48}$.

13. $\frac{387}{16}$; $\frac{1416}{32}$.

6. $\frac{303}{16}$; $\frac{127}{12}$.

10. $\frac{387}{16}$; $\frac{441}{32}$.

14. $\frac{517}{8}$; $\frac{2387}{40}$.

ADDITION.

78. Addition of Fractions is the process of finding the sum of two or more fractions.

79. PRINCIPLE.—*In order that fractions may be added, they must have like denominators and be parts of like units.*

Ex. What is the sum of $\frac{5}{12}$, $\frac{9}{12}$, and $\frac{1}{12}$?

OPERATION.

$$\frac{5}{12} + \frac{9}{12} + \frac{1}{12} = \frac{15}{12} = \frac{5}{4} = 1\frac{1}{4}$$

ANALYSIS.—As these frac-

tions have a common denominator, we add their numera-

tors, and write their sum, 15, over the common denominator, 12 $\frac{15}{12} = 1\frac{1}{4}$, the required result.

Ex. Add $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

OPERATION.

$$\frac{2}{3} + \frac{3}{4} + \frac{5}{6} = \frac{8 + 9 + 10}{12} = \frac{27}{12} = \frac{9}{4} = 2\frac{3}{4} = 2\frac{1}{4}.$$

ANALYSIS.—Reduce the given fractions to equivalent fractions having the least common denominator, 12 (70). Then proceed as in previous example.

Ex. Find the sum of $29\frac{1}{6}$, $38\frac{3}{4}$, $17\frac{5}{8}$, and $42\frac{1}{3}$.

OPERATION.

$$\begin{array}{r} 29\frac{1}{6} \\ 38\frac{3}{4} \\ 17\frac{5}{8} \\ 42\frac{1}{3} \\ \hline 127\frac{1}{2} \end{array} \quad \begin{array}{r} \frac{4}{24} \\ \frac{18}{24} \\ \frac{15}{24} \\ \frac{8}{24} \\ \hline \frac{45}{24} = 1\frac{3}{8} \end{array}$$

ANALYSIS.—The sum of the fractions is $\frac{45}{24} = 1\frac{3}{8}$, which added to the sum of the integers, gives $127\frac{1}{2}$, the required result.

Ex. How many yards in 12 pieces of prints containing 46^1 , 48^2 , 51^2 , 49^3 , 44^1 , 48^2 , 47^1 , 49 , 47^3 , 50^3 , 48^1 , 48^2 yards respectively.

OPERATION.

$$\begin{array}{r} 46^1 \\ 48^2 \\ 51^2 \\ 49^3 \\ 44^1 \\ 48^2 \\ \hline \end{array} \quad \begin{array}{r} 47^1 \\ 19 \\ 17^3 \\ 50^3 \\ 48^1 \\ 48^2 \\ \hline 580^1 \end{array}$$

NOTE.—The small figures represent fourths (quarters).

ANALYSIS.—The sum of the fourths is $\frac{24}{4} = 6$, which added to the sum of the integers gives $580\frac{1}{4}$, the total number of yards.

80. RULE.—Reduce the given fractions to equivalent fractions having the least common denominator. Write the sum of the numerators over the common denominator, and reduce the resulting fraction to its simplest form.

When there are mixed numbers or integers, add the integers and fractions separately, and then add the results.

EXAMPLES.



81. Add the following fractions:

1. $\frac{5}{18}$, $\frac{11}{18}$, $\frac{7}{18}$, and $\frac{15}{18}$.
2. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$.
3. $12\frac{1}{2}$, $7\frac{3}{8}$, $16\frac{9}{16}$, and $38\frac{3}{4}$.
4. $48\frac{1}{4}$, $46\frac{7}{8}$, $31\frac{2}{3}$, and $17\frac{1}{6}$.
5. $18\frac{8}{9}$, $27\frac{1}{3}$, $42\frac{2}{3}$, and $51\frac{5}{9}$.
6. $127\frac{7}{8}$, $\frac{23}{2}$, $175\frac{2}{3}$, and $\frac{5}{8}$.
7. $141\frac{9}{10}$, $197\frac{3}{4}$, and $43\frac{7}{8}$.
8. $75\frac{2}{3}$, $\frac{5}{9}$, $1028\frac{2}{3}$, and $\frac{13}{18}$.
9. $\frac{7}{8}$, $119\frac{1}{2}$, $240\frac{3}{4}$, and $17\frac{3}{16}$.
10. $146\frac{3}{4}$, $1\frac{1}{4}$, $53\frac{5}{14}$, and $68\frac{1}{2}$.

Add the following fractions :

11. $1172\frac{5}{8}$, $19\frac{2}{3}$, $440\frac{1}{8}$, and $6\frac{3}{4}$.
12. $\frac{7}{16}$, $106\frac{5}{12}$, $37\frac{8}{9}$, and $7\frac{1}{6}$.
13. 175 , $116\frac{7}{10}$, $143\frac{3}{5}$, and $27\frac{5}{4}$.
14. $20\frac{2}{3}$, $164\frac{3}{7}$, $\frac{1}{2}1$, and $43\frac{5}{6}$.
15. $44\frac{1}{3}$, $16\frac{7}{8}$, $29\frac{7}{16}$, and $13\frac{3}{4}$.
16. 31^1 , 48^3 , 6^21 , 19^3 , 2^72 , 48^1 , and 37^3 .
17. 61^3 , 48^1 , 47^3 , 48 , 48^2 , 49^1 , and 45^3 .
18. $19\frac{3}{8}$, $444\frac{5}{16}$, $737\frac{1}{4}$, and $385\frac{1}{2}$.
19. 46^1 , 48^3 , 40^2 , 49 , 47^3 , and 46^2 .
20. 40^3 , 41^1 , 48^2 , 44^1 , 49^3 , 49^1 , 47^3 , and 48^3 .

SUBTRACTION.

82. Subtraction of Fractions is the process of finding the difference between two fractions.

83. PRINCIPLE.—*In order that fractions may be subtracted, they must have like denominators and be parts of like units.*

Ex. From $\frac{8}{9}$ take $\frac{5}{9}$.

OPERATION. ANALYSIS.—As these fractions have a common denominator, we take the difference of the numerators, and place it over the common denominator. $\frac{8}{9} - \frac{5}{9} = \frac{3}{9} = \frac{1}{3}$ is the result required.

Ex. What is the difference between $\frac{3}{4}$ and $\frac{2}{3}$?

OPERATION. ANALYSIS.—Reduce the given fractions to equivalent fractions having the least common denominator (70). Then proceed as in the previous example.

$$\frac{3}{4} - \frac{2}{3} = \frac{9 - 8}{12} = \frac{1}{12}$$

Ex. From $176\frac{3}{8}$ subtract $89\frac{3}{4}$.

OPERATION. ANALYSIS.— $\frac{3}{8}$ from $\frac{3}{4}$ we cannot take; we therefore take $1 = \frac{8}{8}$ from 176 , leaving 175 . $\frac{8}{8} + \frac{3}{8} = \frac{11}{8}$. $\frac{11}{8} - \frac{3}{8} = \frac{8}{8}$. $175 - 89 = 86$. $86 + \frac{8}{8} = 86\frac{8}{8}$.

$$\begin{array}{r} 176\frac{3}{8} \\ 89\frac{3}{4} \\ \hline 86\frac{8}{8} \end{array}$$

84. RULE.—Reduce the given fractions to equivalent fractions having the least common denominator. Write the difference of the numerators over the common denominator, and reduce the resulting fraction to its simplest form.

When there are mixed numbers, subtract the integers and fractions separately, and add the results.

EXAMPLES.

85. Find the difference between

- | | | |
|---|---|--|
| 1. $\frac{3}{4}$ and $\frac{5}{9}$. | 10. $149\frac{1}{6}$ and $18\frac{5}{9}$. | 19. $248\frac{5}{9}$ and $129\frac{1}{3}$. |
| 2. $\frac{1}{8}$ and $\frac{5}{12}$. | 11. $416\frac{1}{8}$ and $49\frac{3}{4}$. | 20. $764\frac{1}{6}$ and $375\frac{1}{8}$. |
| 3. $\frac{2}{3}$ and $\frac{1}{24}$. | 12. $512\frac{3}{4}$ and $53\frac{1}{8}$. | 21. $764\frac{1}{8}$ and $375\frac{3}{16}$. |
| 4. $2\frac{1}{2}$ and $1\frac{9}{16}$. | 13. 100 and $13\frac{3}{4}$. | 22. $827\frac{1}{8}$ and $737\frac{3}{8}$. |
| 5. $\frac{7}{10}$ and $\frac{8}{15}$. | 14. $116\frac{5}{6}$ and $48\frac{2}{3}$. | 23. $919\frac{3}{4}$ and $447\frac{5}{16}$. |
| 6. $\frac{5}{7}$ and $\frac{3}{5}$. | 15. $381\frac{2}{3}$ and $17\frac{3}{4}$. | 24. 376^1 and 287^3 . |
| 7. $17\frac{1}{2}$ and $9\frac{1}{4}$. | 16. $157\frac{5}{8}$ and $19\frac{7}{5}$. | 25. 445^2 and 318^3 . |
| 8. $175\frac{1}{4}$ and $86\frac{1}{2}$. | 17. 118^3 and 48^2 . | 26. 737^3 and 438^2 . |
| 9. $138\frac{2}{3}$ and $17\frac{1}{4}$. | 18. $387\frac{3}{8}$ and $116\frac{3}{4}$. | 27. 648^1 and 526^3 . |

MULTIPLICATION.

86. To multiply a fraction by an integer.

Ex. What will 4 pounds of tea cost @ $\$ \frac{7}{8}$ a pound?

OPERATIONS.

$$\frac{7}{8} \times 4 = \frac{7 \times 4}{8} = \frac{28}{8} = 3\frac{1}{2}$$

Or,

$$\frac{7}{8} \times 4 = \frac{7}{8 \div 4} = \frac{7}{2} = 3\frac{1}{2}$$

Or,

$$\frac{7}{8} \times \frac{4}{1} = \frac{7}{2} = 3\frac{1}{2}$$

ANALYSIS.—If 1 pound costs $\$ \frac{7}{8}$, 4 pounds will cost 4 times $\$ \frac{7}{8}$, or $\$ \frac{28}{8}$, equal to $\$ 3\frac{1}{2}$. Hence, 4 pounds of tea @ $\$ \frac{7}{8}$ will cost $\$ 3\frac{1}{2}$.

To multiply $\frac{7}{8}$ by 4, multiply the numerator 7 by 4, or divide the denominator 8 by 4; either operation will give $3\frac{1}{2}$, the required product (55, 1).

By cancellation (38), the operation is shortened, and the result is obtained in its lowest terms.

Ex. Multiply $227\frac{3}{4}$ by 175.

OPERATIONS.	
$227\frac{3}{4}$	Or,
175	$227\frac{3}{4}$
$4 \overline{) 525}$	175
$131\frac{1}{4}$	$87\frac{1}{2}$
1135	$43\frac{3}{4}$
1589	1135
227	1589
$39856\frac{1}{4}$	227
	$39856\frac{1}{4}$

ANALYSIS.—As in preceding example.

Or, by aliquot parts, when the fractions are fourths, eighths, etc., the fractions generally used in commercial operations.

$$\frac{3}{4} = \frac{1}{2} + \frac{1}{4} \left(\frac{1}{2} \text{ of } \frac{1}{2} \right).$$

$$\frac{1}{2} \text{ of } 175 = 87\frac{1}{2}.$$

$$\frac{1}{4} \text{ of } 175, \text{ or } \frac{1}{2} \text{ of } 87\frac{1}{2} = 43\frac{3}{4}.$$

87. RULE.—Multiply the numerator or divide the denominator of the fraction by the integer.

When the multiplicand is a mixed number, multiply the fraction and integer separately, and add the results.

EXAMPLES.

88. 1. Find the cost of 20 yards of silk at $\$1$ a yard?

2. How much grain in 12 bins, each containing $76\frac{3}{8}$ bushels?

3. If a ton of hay cost $\$16\frac{3}{4}$, how much will 22 tons cost?

4. Required the cost of 60 yards of muslin at $35\frac{3}{8}$ cents a yard?

Multiply

5. $\frac{9}{16}$ by 7.	16. $412\frac{3}{8}$ by 47.	27. $234\frac{1}{2}$ by 318.
6. $\frac{11}{16}$ by 8.	17. $148\frac{3}{8}$ by 40.	28. $678\frac{3}{8}$ by 427.
7. $\frac{29}{8}$ by 3.	18. $412\frac{3}{8}$ by 89.	29. $625\frac{3}{4}$ by 516.
8. $110\frac{1}{2}$ by 12.	19. $775\frac{1}{8}$ by 65.	30. $718\frac{1}{4}$ by 542.
9. $117\frac{3}{4}$ by 16.	20. $119\frac{9}{16}$ by 20.	31. $275\frac{3}{8}$ by 287.
10. $248\frac{5}{8}$ by 3.	21. $772\frac{3}{4}$ by 17.	32. $813\frac{5}{8}$ by 319.
11. $146\frac{2}{3}$ by 3.	22. $338\frac{5}{8}$ by 30.	33. $444\frac{1}{4}$ by 412.
12. $197\frac{1}{8}$ by 7.	23. $550\frac{5}{9}$ by 27.	34. $555\frac{5}{8}$ by 875.
13. $420\frac{9}{16}$ by 8.	24. $643\frac{3}{4}$ by 121.	35. $817\frac{3}{4}$ by 416.
14. $381\frac{5}{8}$ by 12.	25. $875\frac{3}{8}$ by 234.	36. $913\frac{1}{4}$ by 375.
15. $375\frac{1}{2}$ by 48.	26. $916\frac{1}{2}$ by 275.	37. $787\frac{3}{4}$ by 525.

89. To multiply an integer by a fraction, or to find a fractional part of an integer.

90. PRINCIPLE.—*Multiplying by a fraction is taking such part of the multiplicand as the fraction is of a unit.*

Ex. If 1 ton of hay cost \$18, what will $\frac{3}{4}$ of a ton cost?

OPERATIONS.

$$\begin{array}{r} 4 \overline{) 18} \\ \underline{4\frac{1}{2}} \\ 3 \\ \underline{13\frac{1}{2}} \end{array} \qquad \begin{array}{r} 18 \\ 3 \\ \underline{4 \overline{) 54}} \\ 13\frac{1}{2} \end{array} \qquad \frac{3}{4} \text{ of } \frac{18}{1} = \frac{27}{2} = 13\frac{1}{2}$$

ANALYSIS.—If 1 ton cost \$18, $\frac{3}{4}$ of a ton will cost $\frac{3}{4}$ of \$18. $\frac{3}{4}$ of \$18 is 3 times $\frac{1}{4}$ of \$18. $\frac{1}{4}$ of \$18 is \$4 $\frac{1}{2}$ (taking $\frac{1}{4}$ is the same as dividing by 4), and 3 times \$4 $\frac{1}{2}$ is \$13 $\frac{1}{2}$.

Or, $\frac{3}{4}$ of \$18 is $\frac{1}{4}$ of 3 times \$18. 3 times \$18 is \$54. $\frac{1}{4}$ of \$54 is \$13 $\frac{1}{2}$.

Ex. Multiply 275 by 47 $\frac{3}{8}$.

FIRST OPERATION.	SECOND OPERATION.	THIRD OPERATION.
275	275	275
$47\frac{3}{8}$	$47\frac{3}{8}$	$47\frac{3}{8}$
$S \overline{) 825}$	$34\frac{3}{8}$	$68\frac{3}{4}$
$\underline{103\frac{1}{8}}$	$\underline{3}$	$\underline{34\frac{3}{8}}$
1925	103 $\frac{1}{8}$	1925
1100	1925	1100
$\underline{13028\frac{1}{8}}$	$\underline{1100}$	$\underline{13028\frac{1}{8}}$
	13028 $\frac{1}{8}$	

ANALYSIS.—Multiply by the fraction $\frac{3}{8}$ and by the integer 47 separately, and add the products.

When the fractions are fourths, eighths, etc., multiply by means of aliquot parts. (See third operation.)

$$2 = \frac{1}{4} + \frac{1}{8} \left(\frac{1}{2} \text{ of } \frac{1}{4} \right).$$

$$\frac{1}{4} \text{ of } 275 = 68\frac{3}{4}.$$

$$\frac{1}{8} \text{ of } 275, \text{ or } \frac{1}{2} \text{ of } 68\frac{3}{4} = 34\frac{3}{8}.$$

91. RULE.—*Multiply by the numerator of the fraction, and divide the product by the denominator. Or,*

Divide by the denominator of the fraction and multiply the quotient by the numerator.

When the multiplier is a mixed number, multiply by the fraction and integer separately, and add the results.

EXAMPLES.

92. 1. Find the cost of $8\frac{3}{4}$ yds. of ribbon at 25 cts. a yard.

2. What is the cost of $42\frac{7}{8}$ pounds of butter at 26 cts. a pound?

3. Required the value of $48\frac{7}{8}$ yards of flannel at 75 cts. a yard.

Multiply

- | | | |
|------------------------------|------------------------------|-------------------------------|
| 4. 84 by $\frac{3}{4}$. | 11. 375 by $24\frac{7}{8}$. | 18. 611 by $87\frac{1}{2}$. |
| 5. 126 by $\frac{4}{7}$. | 12. 375 by $22\frac{2}{3}$. | 19. 625 by $92\frac{3}{8}$. |
| 6. 49 by $\frac{5}{8}$. | 13. 146 by $28\frac{3}{4}$. | 20. 937 by $75\frac{3}{4}$. |
| 7. 128 by $9\frac{1}{2}$. | 14. 184 by $16\frac{4}{5}$. | 21. 575 by $81\frac{7}{8}$. |
| 8. 156 by $8\frac{1}{4}$. | 15. 110 by $41\frac{7}{8}$. | 22. 643 by $99\frac{4}{5}$. |
| 9. 187 by $10\frac{3}{4}$. | 16. 780 by $64\frac{5}{6}$. | 23. 787 by $67\frac{2}{3}$. |
| 10. 216 by $14\frac{3}{8}$. | 17. 512 by $37\frac{1}{4}$. | 24. 438 by $78\frac{3}{16}$. |

93. To multiply a fraction by a fraction.*

Ex. At $\$7\frac{1}{9}$ a pound, what will $\frac{3}{4}$ of a pound of tea cost?

OPERATION.

$$\frac{3}{4} \times \frac{7}{9} = \frac{2}{3} \frac{1}{6} = \frac{7}{12}$$

$$\text{Or, } \frac{3}{4} \times \frac{7}{9} = \frac{7}{12}$$

ANALYSIS.—If 1 pound cost $\$7\frac{1}{9}$, $\frac{3}{4}$ of a pound will cost $\frac{3}{4}$ of $\$7\frac{1}{9}$. $\frac{3}{4}$ of $\$7\frac{1}{9}$ is 3 times $\frac{1}{4}$ of $\$7\frac{1}{9}$. $\frac{1}{4}$ of $\$7\frac{1}{9}$ is $\$7\frac{1}{36}$, and 3 times $\$7\frac{1}{36}$ is $\$7\frac{1}{12}$, or $\$7\frac{1}{12}$.

Ex. What is the value of $8 \times 8\frac{1}{3} \times \frac{7}{10} \times \frac{1}{14}$?

OPERATION.

$$8^2 \times \frac{3}{3} \times \frac{7}{10} \times \frac{1}{14} = \frac{10}{3} = 3\frac{1}{3}$$

ANALYSIS.—Reduce the integer 8 and the mixed number $8\frac{1}{3}$ to improper fractions, and multiply as in the preceding example.

By canceling the factors common to the numerators and denominators before multiplying, the operation is shortened and the result is obtained in its lowest terms.

* The practical methods of multiplying one mixed number by another are given under Art. 103.

94. RULE.—Reduce integers and mixed numbers to improper fractions.

Cancel all factors common to the numerators and denominators.

Multiply the remaining numerators together for the numerator, and the remaining denominators for the denominator.

EXAMPLES.

95. Find the product of

- | | | |
|--|--|---|
| 1. $\frac{3}{8}$ and $\frac{2}{5}$. | 5. $\frac{5}{8}$ and $1\frac{2}{5}$. | 9. $\frac{1}{8}$, $13\frac{1}{3}$, and $\frac{1}{5}$. |
| 2. $\frac{2}{3}$ and $\frac{2}{7}$. | 6. 6 , $3\frac{1}{3}$, and $\frac{4}{5}$. | 10. $26\frac{1}{4}$, $\frac{5}{7}$, and $\frac{2}{3}$. |
| 3. $\frac{3}{4}$ and $1\frac{5}{12}$. | 7. $5\frac{2}{3}$, $\frac{6}{7}$, and $\frac{2}{3}\frac{1}{4}$. | 11. $\frac{3}{4}$, $\frac{8}{9}$, and $16\frac{4}{7}$. |
| 4. $\frac{2}{5}$ and $1\frac{0}{7}$. | 8. $12\frac{1}{2}$, $10\frac{2}{3}$, and $1\frac{5}{12}$. | 12. $13\frac{1}{3}$, $\frac{4}{5}$, and $\frac{7}{8}$. |

Reduce the following compound fractions to simple ones.

NOTE.—A **Compound Fraction** is a fraction of a fraction; as $\frac{1}{2}$ of $\frac{2}{3}$, $\frac{4}{5}$ of $7\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{6}$.

The word “of” is equivalent to the sign \times .

- | | | |
|---|--|--|
| 13. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$. | 17. $\frac{5}{6}$ of $\frac{8}{9}$ of 18 . | 21. $\frac{5}{8}$ of $\frac{40}{9}$ of $1\frac{1}{3}$. |
| 14. $\frac{2}{3}$ of $31\frac{1}{4}$ of $\frac{4}{5}$. | 18. $\frac{3}{8}$ of $11\frac{2}{3}$ of $\frac{5}{7}$. | 22. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{8}{9}$. |
| 15. $\frac{3}{8}$ of $\frac{5}{6}$ of $7\frac{1}{2}$. | 19. $\frac{1}{2}\frac{6}{1}$ of $1\frac{1}{5}$. | 23. $\frac{1}{4}$ of $12\frac{1}{2}$ of $6\frac{2}{3}$. |
| 16. $\frac{3}{4}$ of $\frac{7}{8}$ of $5\frac{1}{3}$. | 20. $1\frac{7}{2}$ of $\frac{8}{9}$ of $1\frac{5}{12}$. | 24. $\frac{5}{6}$ of $1\frac{8}{9}$ of $4\frac{3}{4}$. |

Find the value of the following expressions :

- | | |
|--|---|
| 25. $\frac{2}{3}$ of 1728 . | 30. $(\frac{4}{5} + \frac{9}{10}) \times (\frac{3}{4} + \frac{5}{14})$. |
| 26. $\frac{3}{4} \times 375$. | 31. $(\frac{3}{4} - \frac{2}{3}) \times (\frac{1}{8} + \frac{3}{4})$. |
| 27. $\frac{7}{8}$ times 864 . | 32. $(1\frac{5}{12} + \frac{3}{4}) \times (1\frac{1}{4} - \frac{1}{5})$. |
| 28. $\frac{2}{3}$ of $75 \times \frac{3}{4}$ of $16\frac{2}{3}$. | 33. $37\frac{1}{2}$ times $\frac{2}{3}$ of $1\frac{0}{10}$. |
| 29. $\frac{7}{8} \times \frac{3}{4}$ of $1\frac{5}{12} \times \frac{8}{9}$. | 34. $\frac{3}{8}$ of $\frac{5}{6}$ by $\frac{5}{6}$ of $\frac{2}{3}$. |

DIVISION.

96. To divide a fraction by an integer.

Ex. What cost 1 pound of tea, if 5 pounds cost \$3 $\frac{1}{3}$?

OPERATIONS.

$$\frac{10}{3} \div 5 = \frac{10 \div 5}{3} = \frac{2}{3}$$

$$\text{Or, } \frac{10}{3} \div 5 = \frac{10}{3 \times 5} = \frac{10}{15} = \frac{2}{3}$$

$$\text{Or, } \frac{10}{3} \times \frac{1}{5} = \frac{2}{3}$$

ANALYSIS.—If 5 pounds cost \$3 $\frac{1}{3}$, 1 pound will cost $\frac{1}{5}$ of \$3 $\frac{1}{3}$, or \$ $\frac{2}{3}$.

To divide $\frac{10}{3}$ ($3\frac{1}{3}$) by 5, divide the numerator 10 by 5, or multiply the denominator 3 by 5; either operation will give $\frac{2}{3}$, the required quotient (55, 2).

Ex. Divide 86 $\frac{3}{4}$ by 4.

OPERATION.

$$\begin{array}{r} 4 \overline{) 86\frac{3}{4}} \\ \underline{216\frac{15}{16}} \end{array} \quad \begin{array}{l} 3\frac{3}{4} = \frac{15}{4} \\ 1\frac{5}{4} \div 4 = \frac{15}{16} \end{array}$$

ANALYSIS.—Dividing as in simple numbers, 4 is contained in 86 $\frac{3}{4}$, 216 times and a remainder of 3 $\frac{3}{4}$. 3 $\frac{3}{4}$ equals $\frac{15}{4}$, which divided by 4 is $\frac{15}{16}$.

97. RULE.—*Divide the numerator or multiply the denominator of the fraction by the integer.*

When the dividend is a mixed number, divide the integer and the fraction separately, and add the results.

EXAMPLES.

98. Divide

- | | | |
|-----------------------------|-------------------------------|-------------------------------|
| 1. $\frac{3}{4}$ by 6. | 11. 637 $\frac{1}{2}$ by 9. | 21. 5316 $\frac{2}{3}$ by 4. |
| 2. $\frac{5}{6}$ by 3. | 12. 875 $\frac{5}{11}$ by 12. | 22. 7144 $\frac{1}{2}$ by 5. |
| 3. $\frac{8}{9}$ by 6. | 13. 1716 $\frac{2}{3}$ by 8. | 23. 1729 $\frac{3}{4}$ by 3. |
| 4. $\frac{5}{12}$ by 4. | 14. 1729 $\frac{1}{2}$ by 3. | 24. 1749 $\frac{1}{4}$ by 9. |
| 5. $\frac{8}{17}$ by 4. | 15. 2418 $\frac{3}{4}$ by 5. | 25. 8763 $\frac{1}{2}$ by 6. |
| 6. 16 $\frac{3}{4}$ by 5. | 16. 3516 $\frac{2}{3}$ by 5. | 26. 7385 $\frac{3}{4}$ by 8. |
| 7. 172 $\frac{1}{2}$ by 3. | 17. 2428 $\frac{3}{4}$ by 3. | 27. 4255 $\frac{3}{4}$ by 9. |
| 8. 875 $\frac{2}{3}$ by 6. | 18. 6375 $\frac{2}{3}$ by 4. | 28. 7134 $\frac{3}{4}$ by 7. |
| 9. 935 $\frac{3}{4}$ by 8. | 19. 4287 $\frac{1}{8}$ by 2. | 29. 9727 $\frac{1}{2}$ by 12. |
| 10. 729 $\frac{1}{2}$ by 9. | 20. 3281 $\frac{1}{4}$ by 8. | 30. 6345 $\frac{3}{4}$ by 16. |

99. To divide by a fraction.

100. PRINCIPLE. 1 divided by a fraction is the fraction inverted.

Thus, 1 divided by $\frac{3}{4}$ is $\frac{4}{3}$. This principle may be demonstrated as follows: In 1 there are 4 fourths. 1 fourth is contained in 4 fourths 4 times. Since $\frac{3}{4}$ is 3 times $\frac{1}{4}$, $\frac{3}{4}$ is contained in 1 $\frac{4}{3}$ as many times as 1. Hence, $\frac{3}{4}$ is contained in 1 $\frac{4}{3}$ of 4 times, or $\frac{4}{3}$ times.

Ex. At \$ $\frac{3}{4}$ a yard, how many yards of cloth can be bought for \$5?

OPERATIONS.

$$5 \div \frac{3}{4} = \frac{20}{4} \div \frac{3}{4} = 6\frac{2}{3}$$

$$\text{Or, } 5 \div \frac{3}{4} = \frac{5}{1} \times \frac{4}{3} = \frac{20}{3} = 6\frac{2}{3}$$

Or, \$ $\frac{3}{4}$ is contained in \$1 $\frac{4}{3}$ times (*Prin.*), and in \$5, 5 times $\frac{4}{3}$ or $\frac{20}{3}$, equal to $6\frac{2}{3}$ times.

ANALYSIS.—Since 1 yard cost \$ $\frac{3}{4}$, as many yards can be bought for \$5 as \$ $\frac{3}{4}$ is contained times in \$5. 5 is equal to $\frac{20}{4}$, and 3 fourths is contained in 20 fourths $6\frac{2}{3}$ times.

Ex. At \$ $\frac{2}{4}$ a yard, how many yards of cloth can be bought for \$ $\frac{5}{6}$?

OPERATIONS.

$$\frac{5}{6} \div \frac{2}{4} = \frac{10}{12} \div \frac{2}{12} = 1\frac{1}{2}$$

$$\text{Or, } \frac{5}{6} \div \frac{2}{4} = \frac{5}{6} \times \frac{4}{2} = \frac{20}{12} = 1\frac{1}{2}$$

$$\text{Or, } \frac{5}{6} \div \frac{2}{4} = \frac{5}{6} \times \frac{2}{2} = \frac{10}{6} = 1\frac{1}{2}$$

ANALYSIS.—Since 1 yard cost \$ $\frac{2}{4}$, as many yards can be bought for \$ $\frac{5}{6}$ as \$ $\frac{2}{4}$ is contained times in \$ $\frac{5}{6}$. $\frac{2}{4}$ is equal to $\frac{10}{12}$, and $\frac{5}{6}$ is equal to $\frac{10}{12}$. $\frac{10}{12}$ is contained in $\frac{10}{12}$ 1 $\frac{1}{2}$ times.

Or, \$ $\frac{2}{4}$ is contained in \$1 $\frac{2}{2}$ times (*Prin.*), and in \$ $\frac{5}{6}$, $\frac{5}{6}$ times $\frac{2}{2}$ or $\frac{10}{6}$, equal to $1\frac{1}{2}$ times.

Ex. If $6\frac{2}{3}$ yards of cloth cost \$5, what will 1 yard cost?

OPERATIONS.

$$5 \div \frac{20}{3} = (5 \div 20) \times 3 = \frac{3}{4}$$

$$\text{Or, } 5 \div \frac{20}{3} = \frac{5}{1} \times \frac{3}{20} = \frac{15}{20} = \frac{3}{4}$$

$$\text{Or, } 5 \div \frac{20}{3} = \frac{5}{1} \times \frac{3}{20} = \frac{3}{4}$$

ANALYSIS. $6\frac{2}{3}$ yards are equal to $\frac{20}{3}$ yards. Since $\frac{20}{3}$ yards cost \$5, $\frac{1}{3}$ of a yard will cost $\frac{1}{20}$ of \$5 or \$ $\frac{1}{4}$, and $\frac{3}{3}$ or 1 yard will cost 3 times \$ $\frac{1}{4}$ or \$ $\frac{3}{4}$.

Or, the price per yard equals the cost, divided by the quantity as an abstract number. 5 divided by $\frac{20}{3}$ equals 5 times 1 divided by $\frac{20}{3}$, or 5 times $\frac{3}{20}$ (*Prin.*), equal to $\frac{3}{4}$.

Ex. Divide 7378 by $175\frac{2}{3}$.

OPERATION.

$$\begin{array}{r}
 175\frac{2}{3} \overline{) 7378} \\
 \underline{3 3} \\
 527) 22134 (42 \\
 \underline{2108} \\
 1054 \\
 \underline{1054} \\
 0
 \end{array}$$

ANALYSIS.—Reduce both divisor and dividend to improper fractions, and divide as in the preceding example.

Or, multiplying both divisor and dividend by the same number does not affect the quotient. Multiply both divisor and dividend by the denominator of the fraction, and divide as in simple numbers.

If both divisor and dividend are mixed numbers, multiply by the least common denominator of the fractions.

101. RULE.—Reduce the divisor and dividend to equivalent fractions having a common denominator, and divide the numerator of the dividend by the numerator of the divisor. Or,

Invert the terms of the divisor and proceed as in multiplication.

In dividing mixed numbers, multiply both divisor and dividend by the least common denominator of the fractions, and divide as in simple numbers.

EXAMPLES.

102. Divide

- | | | |
|--------------------------------------|---|---|
| 1. 16 by $\frac{4}{5}$. | 13. 45 by $7\frac{3}{4}$. | 25. 720 by $43\frac{1}{3}$. |
| 2. 28 by $\frac{3}{4}$. | 14. $8\frac{1}{2}$ by $3\frac{3}{4}$. | 26. 700 by $37\frac{1}{3}$. |
| 3. 49 by $\frac{7}{9}$. | 15. $6\frac{2}{3}$ by $3\frac{1}{2}$. | 27. 560 by 264. |
| 4. 88 by $\frac{3}{4}$. | 16. $4\frac{5}{7}$ by $3\frac{2}{3}$. | 28. $682\frac{1}{2}$ by $45\frac{1}{2}$. |
| 5. $\frac{3}{4}$ by $\frac{7}{8}$. | 17. $7\frac{1}{2}$ by $8\frac{1}{3}$. | 29. $847\frac{7}{8}$ by $89\frac{1}{4}$. |
| 6. $\frac{5}{6}$ by $\frac{2}{3}$. | 18. $9\frac{7}{9}$ by $18\frac{1}{3}$. | 30. 984^3 by 75^3 . |
| 7. $\frac{9}{10}$ by $\frac{3}{5}$. | 19. 875 by $33\frac{1}{3}$. | 31. 862^2 by 18^3 . |
| 8. $\frac{7}{12}$ by $\frac{5}{8}$. | 20. 625 by $83\frac{1}{3}$. | 32. 731^1 by 56^1 . |
| 9. $\frac{3}{4}$ by $\frac{2}{3}$. | 21. 516 by $34\frac{2}{3}$. | 33. $431\frac{1}{4}$ by $18\frac{3}{4}$. |
| 10. 28 by $4\frac{1}{2}$. | 22. 917 by $43\frac{2}{3}$. | 34. $983\frac{1}{3}$ by $29\frac{1}{2}$. |
| 11. 33 by $3\frac{2}{3}$. | 23. 864 by $86\frac{2}{3}$. | 35. $504\frac{1}{6}$ by $36\frac{2}{3}$. |
| 12. 64 by $5\frac{2}{3}$. | 24. 702 by $30\frac{1}{2}$. | 36. $583\frac{1}{3}$ by $43\frac{3}{4}$. |

Find the value of the following complex fractions and expressions of division :

NOTE.—A Complex Fraction is one whose numerator is a fraction or mixed number ; as, $\frac{\frac{3}{4}}{3}$, $\frac{105\frac{2}{3}}{12}$, $\frac{75\frac{3}{4}}{16}$, $\frac{3\frac{2}{3}}{5}$.

The expression $\frac{3\frac{3}{4}}{5\frac{1}{2}}$ indicates division, and is not properly a fraction. A unit cannot be divided into $5\frac{1}{2}$ equal parts.

$$\begin{array}{ll}
 37. \quad \frac{5\frac{1}{6}}{9}; \quad \frac{4\frac{2}{3}}{35}; \quad \frac{24\frac{3}{4}}{36}. & 40. \quad \frac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{9} \text{ of } 2\frac{1}{4}}; \quad \frac{\frac{1}{4} + 3\frac{1}{2}}{5\frac{2}{3} - 3\frac{1}{6}}. \\
 38. \quad \frac{3\frac{1}{8}}{40}; \quad \frac{8\frac{2}{3}}{13}; \quad \frac{16\frac{2}{3}}{20}. & 41. \quad \frac{18\frac{1}{2} \div 12\frac{1}{3}}{16\frac{1}{2} - 15\frac{5}{6}}; \quad \frac{12\frac{1}{2} \times 11\frac{1}{5}}{68\frac{3}{4} + 1\frac{1}{4}}. \\
 39. \quad \frac{5\frac{1}{2}}{7\frac{1}{3}}; \quad \frac{\frac{3}{4}}{\frac{7}{8}}; \quad \frac{\frac{9}{10}}{\frac{5}{6}}. & 42. \quad \frac{175\frac{3}{4} - 16\frac{7}{8}}{187\frac{1}{8} - 186\frac{7}{8}}; \quad \frac{38\frac{2}{3} - 30\frac{1}{3}}{16\frac{1}{3} + 8\frac{5}{6}}.
 \end{array}$$

103. To multiply mixed numbers together.*

Ex. What cost $1016\frac{1}{2}$ pounds of cotton, at $12\frac{3}{8}$ cents per pound ?

Instead of reducing the mixed numbers to improper fractions, use the following methods. The second method (by aliquot parts) is preferable, and is well adapted to commercial operations, in which the fractions are usually halves, fourths, eighths, etc.

In business transactions, it is customary to omit the fraction in the result, if it is less than $\frac{1}{2}$, and to add 1 to the cents if it is more than $\frac{1}{2}$. Unless otherwise stated, the exact answers will be given to examples.

FIRST OPERATION.

$$\begin{array}{r}
 1016\frac{1}{2} \\
 \times 12\frac{3}{8} \\
 \hline
 8) 3049\frac{1}{2} \\
 \underline{381\frac{3}{16}} \\
 12198 \\
 \hline
 125.79\frac{3}{16}
 \end{array}$$

ANALYSIS.—Multiply $1016\frac{1}{2}$ by the fraction $\frac{3}{8}$ by multiplying by the numerator 3 and dividing by the denominator 8 (**89**); then multiply $1016\frac{1}{2}$ by the integer 12 (**86**), and add the results.

* The multiplication of mixed numbers is purposely put in this connection, as it appropriately comes here, a knowledge of division of fractions being a prerequisite to a fair understanding of the process.

SECOND OPERATION.

$$\begin{array}{r}
 1016\frac{1}{2} \\
 \underline{.12\frac{3}{8}} \\
 254\frac{1}{8} \\
 127\frac{1}{16} \\
 \hline
 12198 \\
 \hline
 125.79\frac{3}{16}
 \end{array}$$

ANALYSIS. $\frac{3}{8} = \frac{1}{4} + \frac{1}{8}$. Multiply $1016\frac{1}{2}$ by $\frac{1}{4}$ by dividing by 4. Multiply $1016\frac{1}{2}$ by $\frac{1}{8}$ by taking $\frac{1}{2}$ of $254\frac{1}{8}$, the product by $\frac{1}{4}$. Multiply $1016\frac{1}{2}$ by 12 (**86**), and add the results.

EXAMPLES.

104. (1.)

$$\begin{array}{r}
 1675\frac{1}{2} \\
 \underline{9\frac{1}{2}} \\
 837\frac{3}{4} \\
 15079\frac{1}{2} \\
 \hline
 15917\frac{1}{4}
 \end{array}$$

$$\begin{array}{r}
 1675\frac{1}{2} \\
 \underline{347\frac{3}{4} [\frac{1}{2} + \frac{1}{4}]} \\
 837\frac{3}{4} [\text{prod. by } \frac{1}{2}] \\
 418\frac{1}{8} [\text{prod. by } \frac{1}{4} (\frac{1}{2} \text{ of } \frac{1}{2})] \\
 \hline
 173\frac{1}{2} \\
 11725 \\
 6700 \\
 \hline
 5025 \\
 \hline
 582655\frac{1}{8}
 \end{array}$$

(2.)

$$\begin{array}{r}
 \text{Or,} \quad 1675\frac{1}{2} \\
 \underline{347\frac{3}{4}} \\
 4 \overline{) 5026\frac{1}{2}} \\
 \underline{1256\frac{2}{8}} \\
 173\frac{1}{2} \\
 11725 \\
 6700 \\
 \hline
 5025 \\
 \hline
 582655\frac{1}{8}
 \end{array}$$

(3.)

$$\begin{array}{r}
 864\frac{3}{4} [\frac{1}{2} + \frac{1}{4}] \\
 126\frac{5}{8} [\frac{1}{2} + \frac{1}{8}] \\
 \underline{432\frac{3}{8} [\text{prod. by } \frac{1}{2}]} \\
 108\frac{3}{2} [\text{prod. by } \frac{1}{8} (\frac{1}{4} \text{ of } \frac{1}{2})] \\
 63 [126 \times \frac{1}{2}] \\
 31\frac{1}{2} [126 \times \frac{1}{4} (\frac{1}{2} \text{ of } \frac{1}{2})] \\
 \hline
 5184 \\
 10368 \\
 \hline
 109498\frac{3}{2}
 \end{array}$$

$$\begin{array}{r}
 \text{Or,} \quad 864\frac{3}{4} \\
 \underline{126\frac{5}{8}} \\
 8 \overline{) 4323\frac{3}{4}} \\
 \underline{540\frac{1}{2}} \\
 94\frac{1}{2} \\
 4 \overline{) 377} \\
 \hline
 5184 \\
 10368 \\
 \hline
 109498\frac{3}{2}
 \end{array}$$

√ Multiply in like manner,

4. $875\frac{1}{2}$ by $8\frac{1}{2}$; by $37\frac{1}{4}$; by $26\frac{3}{4}$.

5. $737\frac{1}{4}$ by $10\frac{1}{2}$; by $12\frac{1}{4}$; by $44\frac{3}{8}$.

6. $512\frac{3}{4}$ by $7\frac{1}{2}$; by $27\frac{1}{2}$; by $64\frac{3}{4}$.

7. $449\frac{3}{8}$ by $16\frac{1}{4}$; by $36\frac{1}{8}$; by $45\frac{1}{8}$.

REVIEW EXAMPLES.

- ✓ 105. 1. Reduce $\frac{288}{512}$ to its lowest terms.
- ✓ 2. Reduce $\frac{7}{8}$ to forty-eighths.
- ✓ 3. Reduce $72\frac{3}{8}$ to an improper fraction.
- ✓ 4. Reduce $13\frac{75}{8}$ to a mixed number.
- ✓ 5. Add $17\frac{1}{2}$, $31\frac{3}{4}$, $18\frac{2}{3}$, $49\frac{5}{6}$, $13\frac{3}{8}$, and $56\frac{5}{12}$.
- ✓ 6. From $1728\frac{1}{2}$ take $865\frac{3}{4}$.
- ✓ 7. Multiply $\frac{1}{2} \times 3\frac{1}{2} \times \frac{5}{14} \times \frac{3}{10} \times 16\frac{2}{3}$.
- ✓ 8. Multiply $1727\frac{3}{4}$ by 175.
- ✓ 9. Multiply 1727 by $175\frac{3}{4}$.
- ✓ 10. Divide $1\frac{7}{8}$ by $\frac{3}{16}$.
- ✓ 11. Divide 1736 by $144\frac{2}{3}$.
- ✓ 12. Divide $5779\frac{2}{3}$ by $275\frac{2}{3}$.
- ✓ 13. Divide $12346\frac{1}{4}$ by 7; by 35.
- ✓ 14. What is the cost of 1583 lbs. sugar @ $11\frac{3}{4}$ cts. per lb.?
- ✓ 15. Add $\frac{3}{5}$ of $\frac{7}{8}$ of $4\frac{1}{6}$, $\frac{5}{8}$, $136\frac{2}{3}$, and $\frac{5\frac{3}{5}}{8}$.
- ✓ 16. A merchant sold a quantity of goods for \$7344, which was $\frac{3}{4}$ of the cost. What was the loss?
- ✓ 17. Required the value of 2993 pounds of sugar @ $9\frac{3}{8}$ cts. per pound?
- ✓ 18. If $\frac{7}{8}$ of a ship is worth \$42430 $\frac{1}{2}$, what is the value of the whole?
- 19. Bought $47\frac{3}{4}$ yards of cloth at $\$4\frac{1}{2}$ per yard, and paid for it in wheat at $\$2\frac{1}{4}$ per bushel; how many bushels were required?
- ✓ 20. Find the value of $314\frac{3}{8}$ pounds snuff @ 72 cts. per pound.
21. The less of two numbers is $777\frac{3}{4}$ and their difference 117 $\frac{3}{8}$; what is the greater number?
22. A and B together have \$1728; if A's money is equal to $\frac{2}{3}$ of B's, how much have each?
23. A merchant having $2146\frac{3}{4}$ yards of cloth, sold $\frac{5}{8}$ of it at $\$1\frac{1}{4}$ a yard, and the remainder at $\$2\frac{1}{2}$ a yard; how much did he receive?

24. A number being increased by $\frac{5}{8}$ of itself, the sum is 546; what is the number?

25. A man had \$5280; he bought goods with $\frac{3}{8}$ of it, and then lent $\frac{1}{4}$ of the balance to a friend; how much had he left?

26. Find the selling price of goods sold at a profit of \$75, being $\frac{2}{3}$ of the cost.

27. A bought $117\frac{3}{4}$ acres of land at one time, and $87\frac{5}{8}$ at another; after selling $110\frac{1}{8}$ acres, how much remained?

28. If $8\frac{3}{4}$ tons of coal cost \$30 $\frac{5}{8}$, what will $27\frac{1}{2}$ tons cost? How many tons can be bought for \$127 $\frac{3}{4}$?

29. A man paid \$1145 $\frac{5}{8}$ for a horse and carriage. What was the value of each, the carriage being valued at $\frac{5}{8}$ as much as the horse?

30. If $\frac{3}{4}$ of a farm is valued at \$2253 $\frac{1}{2}$, what is the value of $\frac{2}{3}$ of it?

31. What is the value of 2102 $\frac{1}{2}$ yards prints at 7 $\frac{1}{2}$ cents per yard?

NOTE.—The small figures represent fourths (quarters).

32. What number must be taken from $96\frac{3}{4}$, and the remainder multiplied by $16\frac{2}{3}$, that the product shall be 770 $\frac{5}{8}$?

33. What is the value of $164\frac{1}{2}$ yards muslin at $5\frac{3}{4}$ cents per yard?

34. If 7 barrels of oil contain $313\frac{1}{4}$ gallons, how many gallons will $2\frac{3}{4}$ barrels contain?

35. An executor collects \$12724.84. He pays out \$4096.48 and the residue he disburses to the widow and her four children as follows: The widow receives a third part, and the remainder is divided equally among the children. What was the share of each?

36. What number increased by $\frac{3}{4}$ of itself will produce 2456 $\frac{1}{8}$?

37. Find the selling price of goods, bought at \$144, and sold at $\frac{1}{3}$ above cost.

38. A invests $\frac{6}{8}$ of his capital in real estate, and has \$1725 remaining; what is his capital?

39. A merchant sold $12\frac{1}{8}$ yards of silk to one customer, $21\frac{3}{4}$ to another, $20\frac{3}{8}$ to another, and $28\frac{1}{2}$ to another; at $\$2\frac{3}{8}$ per yard, how many dollars did he receive?

40. An army loses $\frac{3}{16}$ of its number in battle and has 16042 remaining; how many did it originally contain?

41. What is the cost of 34 pieces prints, containing 1604^2 yards, at 5^1 cents per yard?

42. What is the cost of 12 pieces prints containing 48, 48^1 , 48^2 , 48, 49^2 , 48^3 , 48, 49^3 , 49^2 , 48^3 , 49^2 , 48^3 yards respectively at 4^3 cents per yard?

43. A merchant purchased 24 pieces prints containing 48^3 , 48^2 , 41^2 , 48^2 , 48^3 , 47 , 49, 49^2 , 52^1 , 57^3 , 48^3 , 48^2 , 38, 48^2 , 48^2 , 48^2 , 47^3 , 48^2 , 48, 51, 48, 44^1 , 51^2 , and 48 yards respectively; what was the cost at 5^2 cents per yard?

44. There are 5280 feet in one mile, and $16\frac{1}{2}$ feet in one rod; how many rods in one mile?

45. A market-woman bought 120 oranges at the rate of 5 for 2 cents, and sold $\frac{1}{2}$ of them at the rate of 3 for 1 cent, and the remainder at the rate of 2 for 1 cent. Did she gain or lose, and how much?

46. What is the duty on 22375 pounds sugar, at $24\frac{3}{8}$ cts. per pound?

47. A farmer sold $1276\frac{1}{3}\frac{2}{3}$ bushels oats at 44 cts. per bushel, $876\frac{2}{5}\frac{2}{5}$ bushels corn at $52\frac{3}{4}$ cts., and $3381\frac{4}{5}\frac{8}{5}$ bushels wheat at $\$1.32$; how much did he receive?

48. How many bushels of corn at $54\frac{1}{4}$ cts. per bushel must a farmer exchange for 62 yards of sheeting at $8\frac{3}{4}$ cts. per yard, and 31 yards broadcloth at $\$1.75$ per yard?

49. What is the value of 45^3 yards damask at 77^2 cts. per yard?

50. $1\frac{4}{16}$ pounds of beef and $1\frac{6}{16}$ pounds of flour are allowed to a ration; how much will 617 rations cost, if the price of beef is $11\frac{3}{8}$ cts. per pound, and of flour $3\frac{1}{4}$ cts. per pound?

51. What is the value of 36385 pounds of corn at $48\frac{3}{4}$ cents per bushel, each bushel containing 56 pounds?

DECIMALS

DEFINITIONS.

106. A **Decimal** (from the Latin *decem*, ten) **Fraction** is a fraction whose denominator is 1 followed by one or more ciphers ; as $\frac{3}{10}$, $\frac{16}{100}$, $\frac{7}{1000}$.

107. Decimal fractions arise from dividing a unit into 10 equal parts, and then dividing these parts into 10 other equal parts, and so on.

Thus, if a unit be divided into 10 equal parts, each part is called a *tenth*. If a unit be divided into 100 equal parts, or 1 tenth into 10 equal parts, the parts are called *hundredths*. If a unit be divided into 1000 equal parts, or 1 hundredth into 10 equal parts, the parts are called *thousandths*.

108. All the rules, principles, operations, etc., of common fractions may be applied to decimal fractions. Since decimal fractions increase and decrease uniformly according to the scale of ten, a more simple notation, similar to that of integers, has been devised for them.

A hundred is written 100; a tenth part of a hundred (ten) is written 10, the 1 being written one place to the right; a tenth part of one ten (one unit) is written 1, the 1 being written one place to the right; in like manner, a tenth part of one unit (one-tenth) is written .1, the 1 being written one place to the right; the tenth part of one-tenth (one hundredth) is written .01, the 1 being written one place to the right, etc., etc.

Decimal fractions, like integers, decrease from left to right in a ten-fold ratio, and increase from right to left in the same ratio.

109. In the decimal notation, the numerator only is written, the denominator being indicated by the position of a point (.) called the *decimal point*. The decimal point separates the integral from the fractional part.

114. A Complex Decimal has a fraction in its right-hand place.

Thus, $.16\frac{2}{3}$ ($\frac{16\frac{2}{3}}{100}$) is a complex decimal, and is read $16\frac{2}{3}$ hundredths, the fraction not being counted as a decimal place.

115. PRINCIPLES.—1. *Annexing ciphers to a decimal does not alter its value.*

Annexing a cipher multiplies both the numerator and denominator by 10, and hence does not alter the value of the decimal (**55**, 3). Thus, $.7$ ($\frac{7}{10}$) = $.70$ ($\frac{70}{100}$) = $.700$ ($\frac{700}{1000}$).

2. *Each removal of the decimal point one place to the right multiplies the value of the decimal by 10.*

Removing the point one place to the right does not change the numerator, but divides the denominator by 10, and hence multiplies the value of the decimal (**55**, 1). Thus, $.072$ ($\frac{72}{1000}$) becomes $.72$ ($\frac{72}{100}$); $\frac{72}{100} = \frac{72}{1000} \times 10$.

3. *Each removal of the decimal point one place to the left divides the value of the decimal by 10.*

Removing the point one place to the left does not change the numerator, but multiplies the denominator by 10, and hence divides the value of the fraction by 10 (**55**, 2). Thus, $.72$ ($\frac{72}{100}$) becomes $.072$ ($\frac{72}{1000}$); $\frac{72}{1000} = \frac{72}{100} \div 10$.

NUMERATION OF DECIMALS.

116. RULE.—*Read the decimal as if it were an integer, and give it the name of its right-hand order.*

EXERCISES.

117. Write in words, or read orally the following numbers:

1. .6.	7. .1007.	13. 3000.0045.
2. .008.	8. 17.6.	14. .3045.
3. .27.	9. 8.029.	15. 375.18 $\frac{3}{4}$.
4. .0375.	10. 24.000488.	16. 19.0033 $\frac{1}{2}$.
5. .6108.	11. 400.000088.	17. 6.148 $\frac{2}{3}$.
6. .775.	12. 76.7071.	18. 648.6 $\frac{2}{3}$.

NOTATION OF DECIMALS.

118. Write in the form of a decimal, sixty-four thousandths.

ANALYSIS.—Since there are only two figures in the numerator 64, and the right-hand figure of the decimal must occupy the third decimal place to express thousandths, it is necessary to prefix a cipher to bring the right-hand figure into its proper place. Therefore write *point, naught, six, four* (.064) in the order named.

119. RULE.—*Prefix the decimal point, and decimal ciphers if necessary, to the numerator written as an integer, so that the right-hand figure will occupy the order named.*

NOTE.—Before writing, determine mentally the place of the right-hand figure and the number of ciphers required. Write in all cases from left to right.

EXERCISES.

120. 1. What is the name of the third decimal order? The sixth? The first? The fourth?

2. How many decimal places are required to express hundredths? Millionths? Ten-thousandths? Tenths? Hundred-millionths?

3. How many ciphers must be written after the decimal point in writing 375 millionths? 27 hundredths? 875 thousandths? 446 ten-millionths? 37 ten-thousandths?

4. Write the following as decimals, so that the decimal points stand in the same vertical line: 8 tenths; 16 hundredths; 175 thousandths; 1804 millionths; 56 ten-thousandths; 3004 ten-millionths; 1728 ten-thousandths.

NOTE.—In the following exercises, the comma is used to separate the integral and decimal parts.

5. Seventeen, and seventy-five hundredths.

6. Twenty-six, and twenty-six thousandths.

7. Two hundred and forty-six ten-millionths.

8. Two hundred, and forty-six ten-millionths.

9. Three hundred and seventy-five, and eighteen hundred thousandths.

10. Eight thousand, and sixty-five ten-thousandths.

11. Eight thousand and sixty-five ten-thousandths.

12. $\frac{7}{10}$, $\frac{37}{100}$, $19\frac{3}{10}$, $218\frac{325}{1000}$, $1728\frac{8}{10000}$.

13. $16\frac{75}{100}$, $19\frac{37}{1000}$, $345\frac{12}{10000}$, $1005\frac{5}{10000}$, $1000\frac{31}{10000}$.

14. $28\frac{16}{1000}$, $37\frac{27}{1000}$, $316\frac{2008}{1000000}$, $44\frac{1725}{1000000}$, $10000\frac{7}{1000000}$.

15. $170\frac{1256}{1000000}$, $16000\frac{225}{1000000}$, $38\frac{3}{1000}$, $16000\frac{25}{1000000}$.

16. $\frac{4}{10}$, $300\frac{75}{1000}$, $\frac{375}{1000}$, $1635\frac{216}{10000}$, $500\frac{63}{1000}$.

REDUCTION.

121. To reduce a fraction to a decimal.

Ex. Reduce $\frac{3}{4}$ to a decimal.

OPERATION.

$$\begin{array}{r} 4 \overline{) 3.00} \\ \underline{.75} \end{array}$$

ANALYSIS. $\frac{3}{4}$ equals $\frac{1}{4}$ of 3 units. 3 units equal 300 hundredths. $\frac{1}{4}$ of 300 hundredths equals 75 hundredths.

122. RULE.—*Annex decimal ciphers to the numerator, and divide by the denominator, pointing off as many decimal places in the quotient as there are ciphers annexed.*

123. A fraction in its lowest terms can be reduced to a pure decimal only when its denominator contains no prime factors but 2 and 5. If the denominator or divisor contain any prime factor other than 2 and 5, the division will not end. The decimals thus produced are called **Interminate** or **Repeating Decimals**, and the figures repeated, **Repetends**.

When a fraction is in its lowest terms, its numerator and denominator have no common factors (59). Annexing ciphers to the numerator introduces the factors 2 and 5 only; hence, if the denominator is an exact divisor of the numerator with the ciphers annexed, it must contain these prime factors and none others.

EXAMPLES.

✓ **124.** Reduce to equivalent decimals :

- | | | | | |
|--------------------|----------------------|----------------------|----------------------|-----------------------|
| 1. $\frac{1}{2}$. | 4. $\frac{3}{8}$. | 7. $\frac{17}{40}$. | 10. $\frac{7}{12}$. | 13. $16\frac{5}{8}$. |
| 2. $\frac{1}{8}$. | 5. $\frac{7}{16}$. | 8. $\frac{2}{3}$. | 11. $\frac{5}{7}$. | 14. $27\frac{1}{2}$. |
| 3. $\frac{3}{4}$. | 6. $\frac{25}{32}$. | 9. $\frac{5}{6}$. | 12. $\frac{4}{9}$. | 15. $36\frac{3}{4}$. |

125. To reduce a decimal to a fraction.

Ex. Reduce .075 to an equivalent fraction.

OPERATION. ANALYSIS.—A decimal is changed to a fraction by writing its denominator, and omitting the decimal point and prefixed ciphers. $\frac{75}{1000} = \frac{3}{40}$ (61).

Ex. Change $.83\frac{1}{3}$ to a simple fraction.

OPERATION. ANALYSIS. — Reduce $.83\frac{1}{3} = \frac{83\frac{1}{3}}{100} = \frac{83\frac{1}{3} \times 3}{100 \times 3} = \frac{250}{300} = \frac{5}{6}$ the complex fraction $\frac{83\frac{1}{3}}{100}$ to a simple fraction by multiplying both terms by the denominator 3. (55, 3.)

126. RULE.—*Omit the decimal point, supply the proper denominator, and reduce the fraction to its lowest terms.*

EXAMPLES.

127. Reduce to equivalent fractions :

- | | | | |
|----------|------------------------|--------------------------|----------------------------|
| 1. .25. | 8. .128. | 15. $.33\frac{1}{3}$. | 22. $.44\frac{1}{9}$. |
| 2. .75 | 9. .00144. | 16. $.41\frac{1}{3}$. | 23. $.14285\frac{1}{7}$. |
| 3. .375. | 10. .512. | 17. $.066\frac{2}{3}$. | 24. $.0833\frac{1}{3}$. |
| 4. .625. | 11. .5625. | 18. $.37\frac{1}{2}$. | 25. 28.0375 . |
| 5. .875. | 12. .1875. | 19. $.104\frac{1}{8}$. | 26. $107.166\frac{2}{3}$. |
| 6. .125. | 13. $.12\frac{1}{4}$. | 20. $.097\frac{1}{8}$. | 27. 175.096 . |
| 7. .016. | 14. $.16\frac{2}{3}$. | 21. $.0053\frac{1}{3}$. | 28. 6.0175 . |

ADDITION.

✓ **128.** Since decimals, like integers, increase and decrease uniformly according to a scale of ten, with the exception of placing the decimal point in the result (usually called *pointing off*), they may be *added, subtracted, multiplied, and divided* in the same manner as integers.

Ex. What is the sum of 28.7, 175.28, .037, 25.0045, and 4.08?

OPERATION.

28.7
175.28
.037
25.0045
4.08

233.1015

ANALYSIS.—Write the numbers so that units of the same order stand in the same column.

If the decimal points are in the same vertical line, tenths will necessarily be under tenths, hundredths under hundredths, etc. Add as in integers, and place the point in the result directly under the points of the numbers.

Ex. Add $.6$, $.37\frac{3}{4}$, $16.048\frac{1}{3}$, $8.1234\frac{2}{7}$, and 24.125 .

OPERATION.

.6 = .6
 $.37\frac{3}{4}$ = .3775
 $16.048\frac{1}{3}$ = $16.0483\frac{1}{3}$
 $8.1234\frac{2}{7}$ = $8.1234\frac{2}{7}$
 24.125 = 24.125

49.2742 $\frac{13}{7}$

ANALYSIS. — Reduce the complex decimals as far as the decimal places extend in the other numbers. Since the fractions now express parts of the same fractional unit, they may be added.

In practice, the fractions may be rejected if the decimals are carried one place, at least, farther than accuracy is required.

129. RULE.—Write the numbers so that their decimal points are in the same vertical line. Add as in integers, and place the decimal point in the result directly under the points in the numbers added.

EXAMPLES.

✓ **130.** 1. Add ninety-seven hundredths; three hundred and forty-seven thousandths; sixteen, and seventy-five hundred-thousandths; four hundred and seventy-five, and two thousand and thirty-seven millionths.

2. Add seventy-five, and eight hundredths; eighteen thousandths; ninety-six, and eighty-four hundredths; four, and four tenths; and eight hundred and seven ten-millionths.

3. Add four, and eighty-one thousandths; thirty-seven, and two hundred and one ten-thousandths; seven thousand and eight hundred-thousandths; seven thousand, and eight hundred-thousandths; nineteen hundredths; sixty-four, and nine tenths; and fifty-six, and fifty-four thousandths.

4. What is the sum of 18 hundredths; 716 hundred-thousandths; 6342 millionths; 11567 ten-millionths; 625 ten-thousandths; 99 hundredths; and 512 thousandths?

5. Add 81.86; 12.593; 4.004; 18.00129; .443; 400.043; .12875; 175.00175; 17.3008; 9000.0016; and .9016.

6. Required, the sum of 99 ten-thousandths; $157\frac{1}{2}$ thousandths; $789\frac{3}{4}$ millionths; 6 tenths; $18\frac{3}{4}$ hundredths; 1728 ten-millionths; and 88 hundredths.

7. What is the sum of \$12.37 $\frac{1}{2}$; \$144.18 $\frac{3}{4}$; \$6.62 $\frac{1}{2}$; \$175.06 $\frac{1}{4}$; \$40.17 $\frac{1}{3}$; and \$398?

8. Add .26 $\frac{1}{2}$; $4.18\frac{3}{4}$; .0017 $\frac{2}{3}$; .0086 $\frac{1}{3}$; .04 $\frac{2}{5}$; 17.387 $\frac{1}{3}$; and .0102075.

SUBTRACTION.

131. Ex. From 12.75 subtract 8.125.

OPERATION.	ANALYSIS.—Write the subtrahend under the minuend
12.75	so that units of the same order stand in the same column.
8.125	Subtract as in integers, and place the point in the result
4.625	directly under the points of the numbers.

If, as in this example, the minuend has not as many decimal places as the subtrahend, suppose decimal ciphers to be annexed until the right hand figures are of the same order. (115.)

Reduce complex decimals as in addition. (128.)

132. RULE.—Write the numbers so that their decimal points are in the same vertical line. Subtract as in integers, and place the point in the remainder directly under the points in the minuend and subtrahend.

EXAMPLES.

133. 1. From four, and sixty-five thousandths, subtract eight hundred and forty-seven ten-thousandths.

2. From twenty-seven hundredths take twenty-nine hundred-thousandths.

3. From nine thousand, and thirty-four ten-thousandths, subtract nine thousand and thirty-four ten-thousandths.

Find the difference between

- | | |
|--------------------------|-----------------------------|
| 4. 8.3644 and 7.8996. | 10. \$510.60 and \$389.45½. |
| 5. 17.4586 and .785. | 11. \$1728 and \$.06¾. |
| 6. 1.010101 and .999999. | 12. 17.864⅔ and 16.94. |
| 7. \$173.46 and \$87.29. | 13. 144.43⅓ and 113.3875. |
| 8. 3 and .873845. | 14. 54.37⅔ and .98⅓. |
| 9. 17.24½ and 18.973¼. | 15. 117.48¾ and 49.43¾. |

MULTIPLICATION.

134. Ex. Multiply .144 by .12.

OPERATION.

$$\begin{array}{r} 144 \\ .12 \\ \hline .01728 \end{array}$$

ANALYSIS.—For convenience, write the right-hand figures of the factors in the same vertical line.

.144 \times .12 = $\frac{144}{1000} \times \frac{12}{100} = \frac{1728}{100000}$. Multiply the numerators of the two factors for the numerator of the product, as in multiplication of fractions. In the above multiplication of fractions, it will be observed that the number of ciphers in the denominator of the product equals the sum of the ciphers in the denominators of the two factors. Since each cipher represents a decimal place, the product should have as many decimal places as both factors. If the number of figures in the product is less than the number of decimal places in the two factors, supply the deficiency by prefixing ciphers.

135. RULE.—*Multiply as in integers, and from the right point off as many decimal places in the product as there are decimal places in the two factors.*

NOTE.—To multiply a decimal by 10, 100, 1000, etc., remove the decimal point as many places to the right as there are ciphers in the multiplier, annexing ciphers to the multiplicand, if necessary.

EXAMPLES.

136. 1. Multiply three hundred and forty-four thousandths by twelve thousandths.

2. Multiply one hundred and ninety-two thousandths by four, and nineteen hundredths.

3. What is sixteen hundredths of six hundred and thirty-two millionths?

4. What is five hundredths of \$864.32? Of 3645.75 francs?

5. What is $.058\frac{1}{2}$ of 784.65? Of 943.25?

6. What is $.99 \times 1.106 \times .25$? $4.105 \times .625 \times .512$?

Multiply

Multiply

7. 8.716 by .39; by .047. 12. 17.28 by $.016\frac{2}{3}$; by $2.55\frac{1}{4}$.

8. .00865 by .625; by 97.75. 13. $64.32\frac{1}{2}$ by $1.44\frac{2}{3}$; by $.06\frac{1}{4}$.

9. .00128 by 8756.8; by 7.865. 14. 86.75 by $1.33\frac{1}{3}$; by $5.76\frac{5}{6}$.

10. 387.25 by $.0147\frac{1}{3}$; by $.087\frac{5}{6}$. 15. 5.78 by .0885; by $.66\frac{2}{3}$.

11. 58.625 by $.488\frac{2}{3}$; by .375. 16. 237.5 by $.345\frac{1}{3}$; by $4.468\frac{1}{3}$.

17. Multiply .01837 by 1000; .00145 by 100000; $.6874$ by 100; 5.375 by 10; 17.056 by 10000. What is the sum of the products?

DIVISION.

137. Ex. Divide .01728 by 1.44.

OPERATION.

1.44) .01728 (.012

144

288

288

0

ANALYSIS.—Dividing as in integers, without reference to the decimal points and prefixed ciphers, the quotient is 12. Since the dividend is the product of the divisor and quotient, it must contain as many decimal places as both of them.

Hence the number of decimal places in the quotient must equal the number in the dividend less the number in the divisor.

If, as in this example, the number of figures in the quotient is less than the number of decimal places to be pointed off, supply the deficiency by prefixing ciphers.

138. RULE.—*Divide as in integers, and point off from the right of the quotient as many decimal places as the number in the dividend exceed those in the divisor*

NOTES.—1. If the divisor contains more decimal places than the dividend, before dividing make them equal by annexing ciphers to the dividend. If necessary to continue the division, more ciphers may be added.

2. If, after dividing all the figures of the dividend, there is a remainder, the division may be continued by annexing ciphers (**115**). The ciphers thus annexed must be regarded as decimal places of the dividend.

3. To divide a decimal by 10, 100, 1000, etc., remove the decimal point as many places to the left as there are ciphers in the divisor, prefixing ciphers to the dividend, if necessary.

EXAMPLES.

139. 1. Divide three thousand four hundred and fifty-six hundred-thousandths by seventy-two hundredths.

2. Divide six, and twenty-five hundredths by twenty-five thousandths.

Divide

Divide

3. 35.88 by .345 ; by 4.16. 8. .0648 by .00425 ; by .0288.

4. .89958 by .47 ; by .319. 9. .31752 by .648 ; by .00384.

5. 12.6 by 14.4 ; by .125. 10. .1898 by $.33\frac{1}{3}$; by $.0048\frac{2}{3}$.

6. 96.3 by .20 ; by .25. 11. 85.2451 by $4.56\frac{5}{8}$; by $8.27\frac{1}{2}$.

7. 5.27 by 1.24 ; by .85. 12. 45.367 by $.016\frac{2}{3}$; by $1.080\frac{1}{6}$.

13. Divide 17.28 by .20 ; by .25 ; by $.33\frac{1}{3}$; by .125 ; by $.66\frac{2}{3}$.

14. 321 is $.178\frac{1}{3}$ of what number ?

15. 186 is five hundredths of what number ?

16. What must 37.375 be multiplied by to produce 448.5 ?

17. What must 631.25 be divided by to produce 250 ?

18. Divide 176.824 by 100 ; 876.35 by 1000 ; 17380.5 by 10000 ; 2886.57 by 10 ; 375 by 1000000. Find the sum of the quotients.

19. \$12.52 is how many hundredths of \$375.60 ?

20. \$273.60 is how many thousandths of \$1728 ?

REVIEW EXAMPLES.

140. 1. Add 16 hundredths, 137 millionths, 48 ten-thousandths, and 2016 ten-millionths.

2. Add 16.07 , $240.127\frac{2}{3}$, $6.04\frac{1}{4}$, 27.1234 .

3. Reduce $1\frac{2}{6}$ to a decimal.

4. Reduce $.083\frac{1}{3}$ to a fraction.

5. From 175 take $16.083\frac{1}{3}$.

6. From $375.16\frac{2}{3}$ take $198.888\frac{8}{9}$.

7. Change $.8375$ to a fraction.

8. Multiply 117.084 by $7.37\frac{2}{3}$.

9. Divide 43.75 by $.0125$.

10. Divide $.06\frac{2}{3}$ by $1.66\frac{2}{3}$.

11. 1.75 is $\frac{7}{8}$ of what number?

12. What is $\frac{2}{3}$ of $\$175.75$?

13. What is $.33$ of 187.5 ?

14. What is $.33\frac{1}{3}$ times 1728 ?

15. $\$86.40$ is how many hundredths of $\$2592$?

16. 16.56 is $.05$ of what number?

17. What will 17280 bricks cost at $\$3.25$ per M.?

18. If 278 barrels of pork cost $\$4378.50$, what is the cost of 100 barrels?

19. If $31\frac{1}{4}$ bushels of corn cost $\$17.50$, how many bushels can be bought for $\$616$?

20. What cost 12456 feet of plank at $\$8.75$ per M.?

21. What is the value of 5 bbls. sugar, containing 312 , 304 , 301 , 305 , 304 pounds respectively, at $9\frac{3}{8}$ cents per pound?

22. A miller wishes to purchase an equal quantity of wheat, corn, and rye; he pays for wheat, $\$2.22\frac{1}{2}$ a bushel; for corn, $98\frac{1}{2}$ cents a bushel; and for rye $\$1.16\frac{1}{2}$ a bushel. How many bushels of each can he buy for $\$92776.50$?

23. A merchant paid for merchandise during the year $\$137618.75$, and sold merchandise to the amount of $\$146347.87$. What was the gain, if the net market value of the merchandise remaining unsold was $\$24378$?

24. A quartermaster has \$8345 on hand, and receives \$4379.62 from each of six sales of property; he turns over to quartermaster A \$2875.28, and pays \$120 for corn. Upon being relieved from duty, he turns over to quartermaster B one-third of the residue, and divides the remainder equally among three others, C, D, and E. What was paid over to each?

25. Merchandise on hand, Jan. 1, 1879, \$46312.85; merchandise sold during the year, \$317829.32; merchandise purchased in the same time, \$301449.72; merchandise on hand, Dec. 31, 1879, \$61378.12. What was the net gain or loss?

26. A farmer sold land for \$22.50 an acre, as follows: to A, $98\frac{3}{5}$ acres; to B, $\frac{3}{8}$ of the number sold to A; and to C, $\frac{1}{2}$ the number sold to A and B both. How much land was sold, how much did B and C each receive, and what was the amount realized?

27. At \$28.75 per thousand, how many feet of lumber should be given for 2816 pounds of sugar at $7\frac{3}{16}$ cts. per pound?

28. Mr. A offered to sell his horse for $\frac{3}{5}$ more than it cost him, but afterward sold it for \$504, which was $\frac{1}{10}$ less than his first asking price. How much did his horse cost him?

29. A man bequeaths $\frac{1}{3}$ of his property to his wife, $\frac{1}{4}$ to his son, $\frac{1}{6}$ to his daughter, and the remainder, which is \$36375, to charitable institutions. What is the amount bequeathed to each, and the total amount?

30. If a person traveling $3\frac{7}{8}$ miles per hour completes a journey in $16\frac{1}{2}$ hours, what time would it require if he traveled $4\frac{1}{8}$ miles per hour?

31. B after spending $\frac{1}{3}$ of all his money, and $\frac{3}{4}$ of the remainder, had \$177.50 remaining; how much had he at first?

32. A merchant bought 100 yards of cloth at \$3.62 $\frac{1}{2}$ per yard, and $87\frac{1}{2}$ yards at \$4.12 $\frac{1}{2}$ per yard. At what average price per yard should he sell the whole, to realize a profit equal to $\frac{1}{5}$ of the cost?

DENOMINATE NUMBERS.

DEFINITIONS.

141. A **Denominate Number** is a concrete number (**7**), and may be either *simple* or *compound*.

142. A **Simple Denominate Number** refers to units of the same name and value; as 7 inches, 4 pounds.

143. A **Compound Denominate Number** refers to units of different names, but of the same nature; as 3 feet 6 inches, 4 pounds 8 ounces.

144. Denominate numbers are used to express divisions of time, weights, measures, and moneys of different countries.

145. The scale of integers and decimals is uniform; that of most denominate numbers is varying.

The moneys of the United States, Canada, France, Italy, Spain, Germany, Norway and Sweden, Denmark, Brazil, Japan, and of some other countries, and the metric system of weights and measures, have a uniform decimal scale.

DIVISIONS OF TIME.

146. The natural divisions of time are the *year* and the *day*, the other divisions being artificial. The year is the time in which the earth makes one revolution around the sun. The day is the time in which the earth makes one revolution on its axis.

147. The **Solar Day** is the interval between two consecutive returns of the sun to the meridian. On account of the varying motion of the earth around the sun, the solar days are of unequal length. For civil purposes in measuring time the average of all the days in the year is taken as the unit.

TABLE.

60 Seconds (<i>sec.</i>)	= 1 Minute	<i>min.</i>
60 Minutes	= 1 Hour	<i>hr.</i>
24 Hours	= 1 Day	<i>da.</i>
7 Days	= 1 Week	<i>wk</i>
365 Days, 52 Weeks, 1 day, or 12 Calendar Months.	} = 1 Common Year . . .	<i>yr.</i>
366 Days		
100 Years	= 1 Leap Year	<i>yr.</i>
	= 1 Century	<i>C.</i>

NOTE.—In many business transactions the year is regarded as 360 days, or 12 months of 30 days each.

148. The **Calendar Months** with the number of days they contain are as follows :

Season.	Days.	Season.	Days.
WINTER.	{ 1. January (Jan.) 31	SUMMER.	{ 6. June 30
	{ 2. February (Feb.) 28		{ 7. July 31
	“ in leap year 29		{ 8. August (Aug.) 31
SPRING.	{ 3. March (Mar.) 31	AUTUMN.	{ 9. September (Sep.) 30
	{ 4. April (Apr.) 30		{ 10. October (Oct.) 31
	{ 5. May 31		{ 11. November (Nov.) 30
		WINTER.	12. December (Dec.) 31

149. The **Solar Year** is the time between two consecutive returns of the sun to the vernal equinox. Its exact length is 365 *da.* 5 *hr.* 48 *min.* 50 *sec.* in mean solar time. For civil purposes, the year consists of 365 or 366 days.

In the calendar established by Julius Cæsar, B. C. 46, and thence called the Julian calendar, three successive years were made to consist of 365 days each; and the fourth, of 366 days. According to the Julian calendar, the average length of the year was $365\frac{1}{4}$ days, thus making an error of 11 min. 10 sec. each year; which in 400 years would amount to 73 hours, or about 3 days. In the sixteenth century, in consequence of the excess of the Julian year above the true solar year, the error in the calendar was 10 days. To correct the calendar, and to prevent any error in the future, Pope Gregory XIII. decreed that 10 days should be omitted in the month of October, 1582, and that all centennial years not divisible by 400 should be common years. Thus, the years 1700, 1800, and 1900, which according to the Julian calendar would be leap years, would according to the reformed calendar be common years. This calendar is sometimes called the Gregorian calendar. It is now used in all civilized countries except Russia.

The Julian and Gregorian calendars are also designated by the terms Old Style and New Style. In consequence of the years 1700 and 1800 being common years by the Gregorian calendar, the difference between the two styles is now 12 days. Thus, when it is July 4 in Russia, it is July 16 in other countries.

150. RULE FOR LEAP YEARS.—*All years divisible by 4, except centennial years, are leap years. All centennial years divisible by 400 are leap years.*

LINEAR MEASURE.

151. Linear or Long Measure is used in measuring distances, also the length, breadth, and height of bodies, or their linear dimensions.

In measuring length, the yard derived from the standard yard of England is the standard unit, the yards of the United States and England being identical. Theoretically, the yard is equal to $\frac{3600000}{391393}$ of the length of a pendulum that vibrates seconds in a vacuum, at the level or the sea in the latitude of London; that is, a pendulum that vibrates seconds under the above conditions is 39.1393 inches in length. The standard yard is, in fact, the distance between two points on a brass bar, preserved at Washington, the distance to be taken when the bar is at a temperature of 62° Fahrenheit. This bar was obtained from England in 1827.

TABLE.

		<i>mi.</i>	<i>fur.</i>	<i>rd.</i>	<i>yd.</i>	<i>ft.</i>	<i>in.</i>
12 Inches (<i>in.</i>)	= 1 Foot . . <i>ft.</i>	1	= 8	= 320	= 1760	= 5280	= 63360
3 Feet	= 1 Yard . . <i>yd.</i>	1	= 40	= 220	= 660	= 7920	
5½ Yards	= 1 Rod . . <i>rd.</i>		1	= 5½	= 16½	= 198	
40 Rods	= 1 Furlong <i>fur.</i>			1	= 3	= 36	
8 Furlongs	= 1 Mile . . <i>mi.</i>				1	= 12	

NOTES.—1. The inch is usually divided into halves, quarters, eighths, and sixteenths.

2. The foot and inch are divided by civil engineers and others into tenths, hundredths, thousandths, etc.

3. In measuring cloth, ribbon, and other goods sold by the yard, the yard is divided into halves, quarters, eighths, and sixteenths.

4. At the U. S. Custom Houses the yard is divided into tenths and hundredths.

5. The mile (5280 *ft.*) of the above table is the legal mile of the United States and England, and hence it is sometimes called the statute mile.

152. Other denominations.—The following denominations are also used :

1 Size	= ½ Inch.	Used by shoemakers
1 Hand	= 4 Inches	Used in measuring the height of horses.
1 Fathom	= 6 Feet.	Used in measuring depths at sea.
1 Cable-length	= 120 Fathoms, or 240 yards.	
1 Geographic Mile	= 1.15 + Statute Miles	Used in measuring distances at sea.
1 Knot	= 1 Geographic Mile.	Used in determining the speed of vessels.
60 Geo. Miles, or } 69.16 Stat. Miles }	= 1 Degree	{ of latitude on a meridian, or of longitude on the equator.
360 Degrees	= the Circumference of the Earth.	

SURVEYORS' LINEAR MEASURE.

153. Surveyors' Linear Measure is used in measuring lands, roads, etc.

The unit of this measure is a chain, 4 rods or 66 feet in length, called Gunter's Chain. It is divided into 100 parts called *links*, each link being 7.92 inches in length.

TABLE.

	<i>mi.</i>	<i>ch.</i>	<i>ft.</i>	<i>l.</i>	<i>in.</i>
100 Links (<i>l.</i>) = 1 Chain . . <i>ch.</i>	1	80	= 5280	= 8000	= 63360
50 Chains = 1 Mile . . <i>mi.</i>		1	= 66	= 100	= 792
			.66	= 1	= 7.92

- NOTES.—1. Links are written decimally as hundredths of a chain.
 2. 1 rod = 25 links.
 3. Engineers for railroad and other purposes use a chain or tape 100 feet long, the feet being divided into tenths.

S Q U A R E M E A S U R E .

154. Square Measure is used in measuring surfaces.

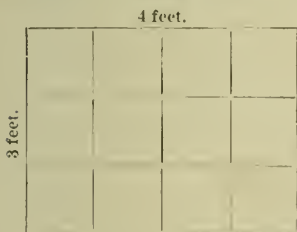
The unit of square measure is a square bounded by lines of some known length. Thus, a square inch is a square whose sides are one inch long; a square foot, a square whose sides are one foot long; etc.

TABLE.

144 Square Inches (<i>sq. in.</i>) = 1 Square Foot . . <i>sq. ft.</i>	
9 Square Feet = 1 Square Yard . . <i>sq. yd.</i>	
30 $\frac{1}{4}$ Square Yards = 1 Square Rod . . <i>sq. rd.</i>	
160 Square Rods = 1 Acre <i>A.</i>	

NOTE.—1 Rood = 40 *sq. rds.* = $\frac{1}{4}$ *A.* The rood has practically gone out of use.

155. The Area of a surface is an expression for that surface in terms of square units.



In the diagram each small square represents a square foot. Since there are 3 rows, and 4 square feet in each row, there are 3 times 4 square feet, or 12 square feet, in the rectangle. Hence, the *area* of any rectangle may be found by multiplying together the numbers denoting its length and breadth, in the same denomination; or, more briefly,

To find the area of a rectangle, multiply its length by its breadth.

$$43560 \text{ sq. ft.} = 1 \text{ acre}$$

$$208.71' \times 208.71' = 1 \text{ acre}$$

SURVEYORS' SQUARE MEASURE.

156. Surveyors' Square Measure is used in measuring land.

TABLE.

10000 Square Links (<i>sq. l.</i>)	= 1 Square Chain . . .	<i>sq. ch.</i>
10 Square Chains	= 1 Acre	<i>A.</i>
640 Acres	= 1 Square Mile . . .	<i>sq. mi.</i>

NOTE.—1 Pole or Perch = 1 *sq. rd.* = $\frac{1}{16}$ *sq. ch.* = $\frac{1}{160}$ *A.*

157. U. S. Public Lands are divided by north and south lines, and by others crossing them at right angles, so as to form *townships* of six miles square.

Townships are subdivided into *sections*, containing, as nearly as may be, 640 acres each, or 1 square mile.

Sections are subdivided into *half-sections*, *quarter-sections*, *half-quarter-sections*, and *quarter-quarter-sections*.

TABLE.

1 Township	= 6 <i>mi.</i> \times 6 <i>mi.</i> = 36 <i>sq. mi.</i> = 23040 <i>A.</i>
1 Section	= 1 " \times 1 " = 1 " = 640 "
1 Half-Section	= 1 " \times $\frac{1}{2}$ " = $\frac{1}{2}$ " = 320 "
1 Quarter-Section	= $\frac{1}{2}$ " \times $\frac{1}{2}$ " = $\frac{1}{4}$ " = 160 "
1 Half-Quarter-Section	= $\frac{1}{2}$ " \times $\frac{1}{4}$ " = $\frac{1}{8}$ " = 80 "
1 Quarter-Quarter-Section	= $\frac{1}{4}$ " \times $\frac{1}{4}$ " = $\frac{1}{16}$ " = 40 "

The following diagrams show the method of numbering the sections of a township, as also that of naming the subdivisions of sections.

A TOWNSHIP.

N					
6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36
S					

A SECTION.

N		
N. $\frac{1}{2}$ 320 <i>A.</i>		
N.W. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ 40 <i>A.</i>	E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ 80 <i>A.</i>	S.E. $\frac{1}{4}$ 160 <i>A.</i>
S.W. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ 40 <i>A.</i>		
S		

SOLID OR CUBIC MEASURE.

158. Solid or Cubic Measure is used in measuring solids, or bodies, which have length, breadth, and thickness or depth ; as boxes, earth, wood, stone, etc.

The unit of cubic measure is a cube, each of whose edges is a unit of some known length. Thus, a cubic inch is a cube, each of whose edges is one inch ; a cubic foot is a cube, each of whose edges is one foot ; etc.

TABLE.

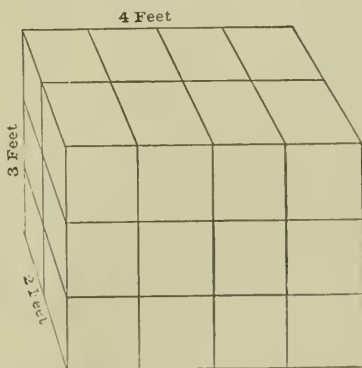
1728 Cubic Inches (<i>cu. in.</i>)	= 1 Cubic Foot . . .	<i>cu. ft.</i>
27 Cubic Feet	= 1 Cubic Yard . . .	<i>cu. yd.</i>

NOTES.—1. 128 cubic feet = 1 cord of wood, or bark. Tanners, in measuring bark, use a measure in which the foot is divided into tenths.

2. The U. S. measurement ton for freight contains 40 cubic feet.

159. The Volume or Solid Contents of a solid is an expression for that solid in terms of cubic or solid units.

The diagram represents a solid 4 feet long, 3 feet broad, and 2 feet thick. Each small cube is a cubic foot. Since the end of the solid contains (3×2) 6 square feet of surface, it is evident, if a section 1 foot thick be cut off from this end, it can be divided into 6 cubes, with edges 1 foot in length, and therefore the section will contain 6 cubic feet ; and since the whole solid is 4 feet long, and contains 4 like sections, it must contain 4 times 6 cubic feet, or twenty-four cubic feet. Hence the volume of a rectangular solid may be found by multiplying together the numbers expressing its length, breadth, and thickness, in the same denomination ; or, more briefly,



To find the volume of a rectangular solid, multiply together its length, breadth, and thickness.

160. Lumber is measured by *board measure*. The board foot is 1 *ft.* long, 1 *ft.* wide, and 1 *in.* thick ; hence it is $\frac{1}{12}$ of a cubic foot.

Boards, plank, scantling, joists, and sawed timber generally are usually measured by *board measure*; hewn and round timber by *cubic measure*.

161. When lumber is not more than one inch thick, to find the number of feet board measure : *Multiply the length in feet by the width in inches, and divide the product by 12.*

When more than 1 inch thick : *Multiply the length in feet by the width and thickness in inches, and divide the product by 12.*

LIQUID MEASURE.

162. Liquid Measure is used for measuring liquids.

The unit of this measure is the wine gallon, which contains 231 cubic inches.

TABLE.

	<i>gal.</i>	<i>qt.</i>	<i>pt.</i>	<i>gi.</i>
4 Gills (<i>gi.</i>) = 1 Pint. . . <i>pt.</i>	1	= 4	= 8	= 32
2 Pints = 1 Quart . . <i>qt.</i>		1	= 2	= 8
4 Quarts = 1 Gallon . . <i>gal.</i>			1	= 4

NOTES.—1. In estimating the capacity of tanks, cisterns, reservoirs, etc., 1 barrel = $31\frac{1}{2}$ gallons; 1 hogshead = 2 barrels = 63 gallons.

2. In commerce, the barrel and hogshead are not fixed measures, but their capacity is found by gauging, or actual measurement.

3. The imperial gallon of England contains 277.274 cubic inches, and is equivalent to 1.2 U. S. wine gallons.

4. The beer gallon contains 282 cubic inches. It is no longer used in the United States.

APOTHECARIES' FLUID MEASURE.

163. Apothecaries' Fluid Measure is used in prescribing and compounding liquid medicines.

The gallon and pint of this measure are the wine gallon and pint.

TABLE.

50 Minims (℥) = 1 Fluidrachm	$f\frac{3}{4}$.	Cong.	O.	$f\frac{3}{4}$.	\mathfrak{M} .
8 Fluidrachms = 1 Fluidounce	$f\frac{3}{4}$.	1=8=128=1024=61440			
16 Fluidounces = 1 Pint	O.	1=16=128=7680			
8 Pints = 1 Gallon	Cong.	1=8=480			
		1=60			

NOTES.—1. *Cong.* is for the Latin *congius*, gallon; *O.*, for the Latin *octarius*, one-eighth.

2. The symbols precede the numbers to which they refer; thus, *O.* } $f\frac{3}{4}$ 10, is 6 pints 10 fluidounces.

DRY MEASURE.

164. Dry Measure is used in measuring dry articles; as salt, grain, fruits, etc.

The unit of this measure is the Winchester bushel, which contains 2150.42 cubic inches.

TABLE.

2 Pints (<i>pt.</i>) = 1 Quart	<i>qt.</i>	<i>bu.</i>	<i>pk.</i>	<i>qt.</i>	<i>pt.</i>
8 Quarts = 1 Peck	<i>pk.</i>	1 = 4 = 32 = 64			
4 Pecks = 1 Bushel	<i>bu.</i>	1 = 8 = 16			
		1 = 2			

NOTES.—1. The half-peck or gallon of this measure contains 268.8 cubic inches.

2. The imperial bushel of England contains 2218.19 cubic inches, and is equal to 1.03 Winchester bushels.

3. Grain, seeds, etc., are usually sold by weight. For table of equivalents see Art. 168.

TROY WEIGHT.

165. Troy Weight is used in weighing gold, silver, coins, and jewels; also in philosophical experiments.

The unit of weight is the Troy pound, which contains 5760 grains. A cubic inch of distilled water weighs 252.458 of these grains, when the height of the barometer is 30 inches, and the temperature of the air and water 62° Fahrenheit.

TABLE.

		<i>lb.</i>	<i>oz.</i>	<i>pwt.</i>	<i>gr.</i>
24 Grains (<i>gr.</i>)	= 1 Pennyweight	<i>pwt.</i>	1=12=240=5760		
20 Pennyweights	= 1 Ounce	. . <i>oz.</i>	1= 20= 480		
12 Ounces	= 1 Pound	. . <i>lb.</i>		1= 24	

NOTE.—The carat, used in weighing diamonds, equals 3.2 Troy grains.

The term carat is also used to denote the fineness of gold, and means $\frac{1}{24}$ part. Thus, gold 18 carats fine contains 18 parts pure gold and 6 parts alloy.

APOTHECARIES' WEIGHT.

166. Apothecaries' Weight is used in prescribing and compounding medicines not liquid.

The pound, ounce, and grain of this weight are the same as those of Troy weight, the division of the ounce being different.

TABLE.

		<i>lb.</i>	$\frac{\text{℥}}{3}$	$\frac{\text{ʒ}}{3}$	$\frac{\text{℥}}{3}$	<i>gr.</i>
20 Grains (<i>gr.</i>)	= 1 Scruple	. . <i>sc.</i> or $\frac{\text{℥}}{3}$	1=12=96=288=5760			
3 Scruples	= 1 Dram	. . <i>dr.</i> or $\frac{\text{ʒ}}{3}$	1= 8= 24= 480			
8 Drams	= 1 Ounce	. . <i>oz.</i> or $\frac{\text{℥}}{3}$		1= 3= 60		
12 Ounces	= 1 Pound	. . <i>lb.</i> or <i>lb.</i>			1= 20	

NOTES.—1. The symbols precede the numbers to which they refer: thus, $\frac{\text{℥}}{3}$ 6 $\frac{\text{ʒ}}{3}$ 4, is 6 ounces 4 drams.

2. Drugs and medicines are sold in large quantities by Avoirdupois weight.

AVOIRDUPOIS WEIGHT.

167. Avoirdupois Weight is used in weighing all articles, excepting gold, silver, precious stones, and medicines in small quantities.

The Avoirdupois pound contains 7000 Troy grains.

TABLE.

16 Ounces (oz.)	=	1 Pound	lb.	T. cwt.	lb.	oz.
100 Pounds	=	{ 1 Hundred-weight, . cwt.	1=20=2000=32000			
		{ or 1 Cental	C.	1=	100=	1600
20 Hundred-weight	=	1 Ton	T.		1=	16

NOTES.—1. The ounce is divided into halves and quarters.

2. The dram, $\frac{1}{16}$ of an ounce, is now little used, except by silk manufacturers.

3. The Long or Gross ton, formerly used, contained 2240 pounds; the hundred-weight, 112 pounds; and the quarter, 28 pounds.

These weights are still used at the U. S. Custom Houses, in ocean freights, and in freighting and wholesaling coal from the mines.

168. In buying and selling grain, seeds, and other produce, the bushel is regarded as a certain number of pounds. The Boards of Trade of several of our leading cities, and the people generally, use the following equivalents: *

TABLE OF AVOIRDUPOIS POUNDS IN A BUSHEL.

Commodities.	Lbs.	Commodities.	Lbs.	Commodities.	Lbs.
Barley.....	48	Corn, shelled..	56	Peas....	60
Beans	60	Corn in the ear.	70	Rye.....	56
Buckwheat ...	48	Malt.....	34	Timothy Seed..	45
Clover Seed ...	60	Oats	32	Wheat....	60

In the Liverpool, San Francisco, and some other markets, produce is bought and sold by the *cental* of 100 pounds. Freight tariffs in the United States on grain, provisions, etc., are reckoned per *cwt.* or cental.

* These weights are the same as prescribed by the laws of most States, but the laws are not uniform. In inter-state commerce it is necessary to have common units, although they may differ from the units established by law. The laws are generally disregarded where the units prescribed by them differ from those prescribed by custom, or the laws of most of the States. As an instance of this irregularity, the State of New York prescribes 58 pounds as a bushel of corn, but the Boards of Trade and custom generally adopt 56 pounds as a bushel of corn. There can be no doubt but that an appeal to the courts of any one of the States would lead to a decision in accordance with the laws of that State in fixing the weight of a bushel of grain. It is further evident that decisions in State courts of last appeal might be as discordant upon this subject as the laws themselves.

169. The following units are used in commerce :

1 Quintal of Fish	=	100 lbs.
1 Barrel of Flour	=	196 lbs.*
1 Barrel of Pork	=	200 lbs.
1 Gallon Refined Petroleum	=	$6\frac{1}{2}$ lbs.
1 Gallon Crude Petroleum	=	$6\frac{1}{2}$ lbs.
1 Keg of Nails	=	100 lbs.

CIRCULAR MEASURE.

170. Circular or Angular Measure is used in measuring angles and arcs of circles. It is employed principally by surveyors in determining directions, by navigators in determining latitude and longitude of places, and by astronomers in making observations.

The unit of this measure is the *degree*, which is $\frac{1}{360}$ of the circumference of any circle.

TABLE.

60 Seconds (")	=	1 Minute '.
60 Minutes	=	1 Degree °.
360 Degrees	=	1 Circle C.

NOTES.—1. A quadrant is one-fourth of a circle, or 90°.

2. A sextant is one-sixth of a circle, or 60°.

3. 1 minute of the circumference of the earth is called a nautical, or geographic mile, and is about 1.15 statute or common miles.

COUNTING.

171. The following table is used in counting certain articles :

		<i>g. gr.</i>	<i>gr.</i>	<i>doz.</i>	<i>units.</i>
12 Units	=	1 Dozen . . .	<i>doz.</i>	1 = 12	144 = 1728
12 Dozen	=	Gross . . .	<i>gr.</i>	1 = 12	144
12 Gross	=	1 Great Gross .	<i>g. gr.</i>	1 =	12

* It is recommended by the leading Boards of Trade that all barrel flour contain 200 pounds, and all sack flour 50, 100, 150, or 200 pounds.

Flour is frequently exported from the United States to Great Britain in sacks of 140 pounds each. The sack of Great Britain usually contains 280 pounds.

PAPER.

172. The following table is used in the paper trade :

24 Sheets	= 1 Quire	. . .	qr.				
20 Quires	= 1 Ream	. . .	rm.		rm.	qr.	sheets.
2 Reams	= 1 Bundle.				1 =	20 =	480
5 Bundles	= 1 Bale.					1 =	24

UNITED STATES MONEY.

173. United States Money is the legal currency of the United States. It consists of gold coins, silver coins, treasury notes, and national bank notes.

174. Legal Tender.—The term legal tender is applied to money which may be legally offered in the payment of debts.

175. The unit of value is the gold dollar of 25.8 grains.

TABLE.

10 Mills	= 1 Cent	. . .	c., ct.
10 Cents	= 1 Dime	. . .	d.
10 Dimes or 100 Cents	= 1 Dollar	. . .	§.
10 Dollars	= 1 Eagle	. . .	E.

NOTES.—1. In business operations, dollars and cents are principally used. Eagles and dimes are used only as the names of coins.

2. In writing U. S. money, the decimal notation is used. *Dollars* are written at the left of the decimal point and form the integral part. *Cents* are written as *hundredths* of a dollar, and occupy the first two places at the right of the decimal point. *Mills* are written as *thousandths* of a dollar, and occupy the third decimal place.

Usually, in the final results of business operations, if the *mills* are more than *five*, they are regarded as an additional cent; if less than *five*, they are rejected.

3. In checks, notes, drafts, etc., *cents* are usually written as hundredths of a dollar in the form of a *fraction*. Thus, *six dollars and twenty five cents* may be written, \$6 $\frac{25}{100}$.

176. The legal coins of the United States are as follows:

GOLD.		SILVER.	
	Weight in grains.		Weight.
1 dollar piece,	25.8	Standard dollar,	412½ grains.
2½ dollar piece, or {	64.5	Half dollar, or {	12½ grams, or
Quarter-eagle, }		50 cent piece, {	192.9 grains.
3 dollar piece,	77.4	Quarter Dollar, or {	6½ grams, or
5 dollar piece, or {	129.	25 cent piece, {	96.45 grains.
Half-eagle, }		Dime, or {	2½ grams, or
10 dollar piece, or {	258.	10 cent piece, {	38.58 grains.
Eagle, }		COPPER AND NICKEL.	
20 dollar piece, or {	516.	5 cent piece,	5 grams, or 77.16 grains.
Double-eagle, }		3 cent piece,	30 grains.
		1 cent piece,	48 grains.

The Mill is not coined.

177. The Trade Dollar contains 420 grains of standard silver (.900 fine). It is not now coined, and is not a *legal tender*. It was originally coined for the purposes of trade in China and Japan.

178. The gold and silver coins of the United States contain 9 parts by weight of pure metal and 1 part alloy. The alloy of silver coins is copper; and the alloy of gold coins, copper, or copper and silver. (The silver in no case exceeds $\frac{1}{10}$ of the whole alloy.)

179. Gold Coins are a *legal tender* in all payments at their nominal value when not below the standard weight and limit of tolerance* provided by law; and, when reduced in weight, below said standard and tolerance, are a legal tender at valuation in proportion to their actual weight.

* "Any gold coin of the United States, if reduced in weight by natural abrasion not more than one-half of one per centum below the standard weight prescribed by law, after a circulation of twenty years, as shown by its date of coinage, and at a ratable proportion for any period less than twenty years, is received at its nominal value by the United States treasury and its offices." The "Coinage Act of 1873" allows a deviation from the standard weight of $\frac{1}{10}$ of a grain, or less, in the manufacture of the dollar piece.

180. Standard Silver Dollars are a *legal tender* at their nominal value for all debts and dues, public and private, except where otherwise expressly stipulated in the contract. The Secretary of the Treasury is authorized and directed to purchase silver bullion, not less than \$2,000,000 worth per month, nor more than \$4,000,000 worth per month, and cause the same to be coined into such dollars.

181. Silver Certificates.—Any holder of *standard silver dollars* may deposit the same with the Treasurer, or any Assistant Treasurer of the United States, in sums not less than \$10, and receive therefor certificates of not less than \$10, each corresponding with the denominations of U. S. notes (184). These certificates are receivable for customs and taxes, and when so received may be reissued.

182. Subsidiary Coins.—The present (1882) silver coins of the United States of smaller denominations than \$1 are a legal tender in all sums not exceeding \$10, in full payment of all dues, public and private.

“The holder of any of the silver coins of the United States of smaller denominations than \$1 may, on presentation of the same in sums of \$20, or any multiple thereof, at the office of the Treasurer or any Assistant Treasurer of the United States, receive therefor lawful money of the United States.”

183. Minor Coins.—The 5 and 3 cent pieces contain $\frac{3}{4}$ copper and $\frac{1}{4}$ nickel. The 1 cent piece contains 95% copper and 5% tin and zinc. These coins are a legal tender for any amount not exceeding twenty-five cents.

184. U. S. Notes (“Greenbacks”) are a legal tender for all debts, except duties on imports and interest on the public debt. Since Jan. 1, 1879, they have been redeemable in coin at the office of the Assistant Treasurer of the United States in New York, in sums of not less than \$50.

They represent the values of \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$500, \$1000, \$5000, and \$10,000. The Act of May 31, 1878, fixed their value at \$346,681,016, and forbade their further contraction.

185. National Bank Notes* are not a *legal tender*; but, since they are secured by bonds of the United States deposited with the U. S. Treasurer at Washington, and are redeemed in lawful money by the national banks and the Treasurer of the United States, they are usually accepted in the payment of debts in any part of the United States. They are receivable in all parts of the United States in payment of all taxes and excises and all other dues to the United States except duties on imports, and also for salaries and other debts and demands owing by the United States to individuals, corporations, and associations within the United States except interest on the public debt.

They represent the values of \$1, \$2, \$5, \$10, \$20, \$50, \$100, \$500 and \$1000. Since Jan. 1, 1879, no notes of the denomination of \$1 and \$2 have been issued to national banks. (R. S. 5175.) Since the act of Jan. 14, 1875, the volume of national bank notes has been unlimited. July 1, 1882, their total circulation, including gold banks, was \$358,742,034.

ENGLISH MONEY.

186. English or Sterling Money is the legal currency of Great Britain.

TABLE.

			Value in U. S. money.
4 Farthings	=	1 Penny . . . d. . . .	\$.02 +
12 Pence	=	1 Shilling . . . s.243 +
20 Shillings	=	{ 1 Pound, or . . . £. / . . . 1 Sovereign . . .	4.8665

NOTES.—1. 1 Crown = 5 shillings, or $\frac{1}{4}$ of a pound (\$1.216 +).

2. 1 Guinea = 21 shillings (\$5.11). It is not now coined.

3. The gold coins of Great Britain are 22 carats ($\frac{1}{2}$), or .916 $\frac{2}{3}$ fine. (The old carat system (165, note) is generally abandoned except for jewelry. 1 carat = .041 $\frac{2}{3}$.)

4. The silver coins of Great Britain are .925 ($\frac{7}{8}$) fine.

* For a more complete exposition of the National Banking system, see Art. 462.

187. FOREIGN MONETYS OF ACCOUNT AND THEIR VALUES IN UNITED STATES MONEY.

Country.	Monetary Unit.	Standard.	Value in U. S. Money.
Austria	Florin of 100 kreutzers	Silver.....	.40,6
Belgium.....	Franc of 100 centimes.	G. and S...	.19,3
Bolivia.....	Boliviano, 100 centavos	Silver.....	.82,3
Brazil.....	Milreis of 1000 reis...	Gold.54,6
British America..	Dollar of 100 cents....	Gold.....	\$1.00
Chili.	Peso of 100 centavos..	G. and S...	.91,2
Cuba.....	Peso of 100 centavos..	G. and S...	.93,2
Denmark.....	Crown of 100 öre.....	Gold.....	.26,8
Ecuador	Peso of 100 centavos..	Silver.....	.82,3
Egypt... ..	Piaster of 40 paras....	Gold.....	.04,9
France.....	Franc of 100 centimes	G. and S...	.19,3
Great Britain....	Pound sterling.....	Gold.....	4.86.6½
Greece.....	Drachma of 100 lepta..	G. and S...	.19,3
German Empire..	Mark of 100 pfennige..	Gold.....	.23,8
Hayti.....	Gourde.....	G. and S...	.96,5
India.....	Rupee of 16 annas ^a ...	Silver.....	.29
Italy.....	Lira of 100 centesimi.	G. and S...	.19,3
Japan.....	Yen of 100 sen.....	Silver.....	.88,7
Liberia.....	Dollar of 100 cents....	Gold....	1.00
Mexico.....	Dollar of 100 centavos.	Silver.....	.89,4
Netherlands....	Florin of 100 cents....	G. and S...	.40,2
Norway.....	Crown of 100 öre.....	Gold.....	.26,8
Peru.....	Sol of 100 centavos...	Silver.....	.82,3
Portugal	Milreis of 1000 reis...	Gold.....	1.08
Russia.....	Rouble of 100 copecks.	Silver.....	.65,8
Sandwich Islands	Dollar of 100 cents....	Gold.....	1.00
Spain.....	Peseta of 100 centimes.	G. and S...	.19,3
Sweden.....	Crown of 100 öre.....	Gold.....	.26,8
Switzerland	Franc of 100 centimes.	G. and S...	.19,3
Tripoli.....	Mahbub of 20 piasters.	Silver.....	.74,3
Turkey	Piaster of 40 paras....	Gold.....	.04,4
U. S. of Colombia	Peso of 100 centavos..	Silver.....	.82,3
Venezuela.....	Bolivar.	G. and S...	.19,3

The above rates, proclaimed by the Secretary of the Treasury, Jan. 1, 1882, are used in estimating, for Custom House purposes, the values of all foreign merchandise made out in any of said currencies.

(^a) The *anna* contains 12 *pies*.

REDUCTION.

188. Reduction of Denominate Numbers is the changing their denomination without changing their value.

189. To reduce denominate numbers from higher to lower denominations.

Ex. How many pence in £8 16s. 7d.?

OPERATION.

£	s.	d.
8	16	7
<hr/>		
	20	
160s.		
	16s.	
<hr/>		
	176s.	
	12	
<hr/>		
	2112d.	
	7d.	
<hr/>		
	2119d.	

ANALYSIS.—Since there are twenty shillings in 1 pound, in 8 pounds there are 8 times 20 shillings, or 160 shillings. (For convenience multiply by 20 as an abstract number.) 160 shillings plus 16 shillings equal 176 shillings. Since there are 12 pence in 1 shilling, in 176 shillings there are 176 times 12 pence, or 2112 pence. 2112 pence plus 7 pence equal 2119 pence. When possible, add mentally the number of the lower denomination to the product.

190. RULE.—Multiply the number of the highest denomination given by the number of the next lower denomination required to make 1 of this higher, and to the product add the given number, if any, of such lower denomination.

Treat this result, and the successive results obtained, in like manner until the number is reduced to the required denomination.

EXAMPLES.

191. Reduce :

1. £9 13s. 10d. to pence.
2. 6 gal. 3 qt. 1 pt. to gills.
3. £112 18s. 5d. to farthings.
4. 6 T. 12 cwt. 65 lb. to pounds.
5. The year 1896 to hours.
6. The year 1881 to minutes.
7. £245 15s. 3 far. to farthings.
8. 48 bu. 3 pk. 6 qt. to quarts.
9. The year 1900 to hours.
10. 18 lb. 8 oz. to pennyweights.

11. 5 *mi.* 36 *rd.* 11 *ft.* to feet. 16. 29 *sq. rd.* to square feet.
 12. 456 miles to feet. 17. 97 *sq. rd.* to square yards.
 13. 16½ hands to inches. 18. 5 *sq. mi.* to acres.
 14. 3 *mi.* 46 *ch.* 75 *l.* to links. 19. 5 miles square to acres.
 15. 7 *mi.* 55 *ch.* to rods. 20. 16 cords 112 *cu. ft.* to *cu. ft.*
 21. How many pounds in 16 *T.* 7 *cwt.* 22 *lb.*?
 22. How many pounds in 16 *T.* 3 *qr.* 18 *lb.* (Long Ton Table)?
 23. How many quarts in 3 *bb.* 24 *gal.* cider?
 24. How many pounds in 2375 bushels corn?
 25. At 1 cent each, what is the value of 20 great gross pens?
 26. How many days from Jan. 1, 1888, to Jan. 1, 1906?
 27. How many pounds in 2387 bushels wheat?
 28. Find the value in U. S. money of £375 Of 2345 francs.

192. To reduce denominate numbers from lower to higher denominations.

Ex. Reduce 2119 pence to higher denominations.

OPERATION.	ANALYSIS.—
12) 2119 <i>d.</i>	Since there are 12 pence in
20) 176 <i>s.</i> + 7 <i>d.</i>	1 shilling, in 2119 pence there are as many
£8 + 16 <i>s.</i>	shillings as 12 pence are contained times in
2119 <i>d.</i> = £8 16 <i>s.</i> 7 <i>d.</i>	2119 pence, or 176 shillings, and 7 pence
	remaining. Since there are 20 shillings in
	1 pound, in 176 shillings there as many
	pounds as 20 shillings are contained times
	in 176 shillings, or 8 pounds, and 16 shillings remaining. Therefore,
	2119 <i>d.</i> = £8 16 <i>s.</i> 7 <i>d.</i>

193. RULE.—*Divide the given number by the number of that denomination required to make 1 of the next higher, reserving the remainder, if any, as part of the answer.*

Treat the quotient, and the successive quotients obtained, in like manner, until the number is reduced to the required denomination. The last quotient and the several remainders will form the answer.

EXAMPLES.

194. Reduce

- | | |
|--|--|
| 1. 8475 <i>d.</i> to pounds. | 11. 13387 <i>d.</i> to pounds. |
| 2. 9683 <i>cu. ft.</i> to cords. | 12. 10224 <i>ft.</i> to fathoms. |
| 3. 7534 <i>pts.</i> to bushels. | 13. 60427 <i>l.</i> to chains. |
| 4. 9817 <i>pts.</i> to barrels. | 14. 16338 <i>ft.</i> to chains. |
| 5. 5280 <i>ft.</i> to miles. | 15. 5384 rods to chains. |
| 6. 7633 <i>sq. yds.</i> to <i>sq. rds.</i> | 16. 6375 <i>l.</i> to rods. |
| 7. 8437 days to <i>com. yrs.</i> | 17. 5316 <i>sq. rds.</i> to acres. |
| 8. 6375 <i>hrs.</i> to weeks. | 18. 4675 <i>d.</i> to pounds. |
| 9. 9537 <i>sec.</i> to hours. | 19. 38425 <i>sq. ch.</i> to <i>sq. mi.</i> |
| 10. 6239 <i>in.</i> to rods. | 20. 7685 poles to acres. |

21. What is the cost of 465 yards of cloth at $9\frac{1}{2}$ pence per yard?

22. What is the value of 49375 pounds of corn at \$0.64 per bushel?

23. What is the value of 27425 pounds of corn at \$0.95 per cental?

24. Required the value of 18643 pounds of oats at 75 *cts.* per bushel.

25. Find the cost of 17387 pounds of oats at \$1.88 per cental.

26. What cost 21370 pounds of straw at \$8 per ton?

27. Find the cost of 875 pounds of feed at \$1.15 per *cwt.*

28. In 327 days, how many months of 30 days each?

REDUCTION OF DENOMINATE FRACTIONS.

195. A Denominate Fraction is a fraction whose integral unit is a denominate number.

The principles, analyses, and rules of denominate fractions are essentially the same as those of denominate integers; therefore, no special rules are necessary for their reduction.

A sufficient number of illustrative examples are given to fully explain the different cases that may arise.

196. To reduce denominate fractions from higher to lower denominations.

Ex. Reduce $\frac{7}{16}$ of a £ to pence.

OPERATIONS.

$$\frac{7}{16} \times \frac{20}{1} = \frac{35}{4} s.$$

$$\frac{35}{4} \times \frac{12}{1} = 105d.$$

$$\text{Or, } \frac{7}{16} \times \frac{20}{1} \times \frac{12}{1} = 105d.$$

ANALYSIS.—Since there are 20 shillings in £1. in $\frac{7}{16}$ (.4375) of a £ there are $\frac{7}{16}$ (.4375) of 20 shillings, or $\frac{35}{4}$ (8.75) shillings. Since there are 12 pence in 1 shilling, in $\frac{35}{4}$ (8.75) shillings there are $\frac{35}{4}$ (8.75) times 12 pence, or 105 pence. Or, multiply the given fraction by the numbers of the scale required to reduce its denomination to the required denomination.

Ex. Reduce .4375 of a £ to pence.

OPERATIONS.

Or, £.4375

$$\begin{array}{r} .4375 \times 20 = 8.75s. \\ 8.75 \times 12 = 105d. \\ \hline 105.0000d. \end{array}$$

ANALYSIS.—As in previous example.

Ex. Reduce $\frac{7}{16}$ of a £ to integers of lower denominations, *i. e.* to shillings and pence.

OPERATION.

$$\frac{7}{16} \times \frac{20}{1} = \frac{35}{4} = 8\frac{3}{4} s.$$

$$\frac{3}{4} \times \frac{12}{1} = 9d.$$

$$\text{£} \frac{7}{16} = 8s. 9d.$$

ANALYSIS.—Multiplying by 20, $\text{£} \frac{7}{16} = 8\frac{3}{4}$ shillings. Reserve the integral part of the result, and reduce the fractional part to pence. Multiplying by 12, $\frac{3}{4}$ shilling = 9 pence. Hence, $\text{£} \frac{7}{16} = 8s. 9d.$

Ex. Reduce £.4375 to integers of lower denominations.

OPERATION.

$$\begin{array}{r} \text{£.4375} \\ \times 20 \\ \hline 8s. 7500 \\ \times 12 \\ \hline d. 9.0000 \end{array}$$

ANALYSIS.—Multiplying by 20, £.4375 = 8.75 shillings. Reserve the integral part of the result, and reduce the decimal part to pence. Multiplying by 12, .75 shilling = 9 pence. Hence, £.4375 = 8s. 9d.

EXAMPLES.

- 197.** 1. Reduce .625 of a £ to pence.
 2. Reduce .875 of a £ to shillings and pence.
 3. Reduce $\frac{3}{16}$ of a £ to pence.
 4. Reduce $\frac{9}{16}$ of a £ to integers of lower denominations.
 5. Change $2.333\frac{1}{3}$ yrs. to lower denominations.
 6. Change £16.467 to lower denominations.
 7. If 1 pound sterling can be bought for \$4.87, how many pounds can be bought for \$10000?
 8. Reduce 2.417 yr. to lower denominations.
 9. A cistern is 16.25 ft. long, 9.6 ft. wide, and 6.25 ft. deep; what is its capacity in cu. yd., etc.?
 10. A certain sum at a certain rate will in 1 yr. produce \$60 interest; in what time will the same sum at the same rate produce \$15.50 interest?
 11. Multiply £425 by $.03\frac{1}{2}$, and reduce the result to lower denominations.

198. To reduce denominate numbers to fractions (or decimals) of higher denominations.

Ex. Reduce $\frac{3}{5}$ of a penny to the fraction of a £.

OPERATIONS.

$$\frac{3}{5} \div 12 = \frac{1}{20}^s.$$

$$\frac{1}{20} \div 20 = \frac{1}{400}^{\text{d.}}$$

$$\text{Or, } \frac{3}{5} \times \frac{1}{12} \times \frac{1}{20} = \frac{1}{400}^{\text{d.}}$$

ANALYSIS.—Divide the given fraction by the numbers of the scale required to reduce pence to pounds.

If the answer is required in the form of a decimal, reduce the resulting fraction to a decimal by Art. 122.
 $\frac{1}{400} = \text{£}0.0025.$

Ex. Change 9 pence to the fraction of a £.

OPERATIONS.

$$\frac{9}{1} \times \frac{1}{12} \times \frac{1}{20} = \frac{3}{80}^{\text{d.}}$$

$$\text{Or, } \text{£}\frac{9}{240} = \text{£}\frac{3}{80}.$$

ANALYSIS.—For first operation, as in previous example.

Or, since there are 240 pence in £1, 1 penny equals $\frac{1}{240}$ of a £, and 9 pence equal $\frac{9}{240}$, or $\frac{3}{80}$ of a £.

Ex. Reduce 9 pence to the decimal of a £.

OPERATIONS.

$$12 \overline{) 9.} \quad d.$$

$$20 \overline{) .75} \quad s.$$

$$.0375 \quad £.$$

ANALYSIS.—As in previous example.

$$\text{Or, } £\frac{9}{240} = £\frac{3}{80} = £.0375.$$

Ex. Reduce 12s. 9d. to the fraction of a £.

OPERATION.

$$12s. 9d. = 153d.$$

$$£1 = 240d.$$

$$\frac{153}{240} = \frac{51}{80} £.$$

ANALYSIS.—12 shillings 9 pence = 153 pence.

Since £1 = 240 pence, 1 penny equals $\frac{1}{240}$ of a £, and 153 pence equal $\frac{153}{240}$, or $\frac{51}{80}$ of a £.

Ex. Reduce £18 12s. 9d. to the decimal of a £.

OPERATION.

$$12 \overline{) 9.} \quad d.$$

$$20 \overline{) 12.75} \quad s.$$

$$£18.6375$$

ANALYSIS.—Write the denominations given in a vertical column, the lowest denomination at the top.

Since there are 12 pence in 1 shilling, 9 pence are equal to .75 shilling; to which annexing the 12 shillings given, we have 12.75 shillings. Since there are 20 shillings in £1, 12.75 shillings are equal to £.6375,

to which annexing the £18, we have £18.6375. Hence, £18 12s. 9d. = £18.6375.

EXAMPLES.

199. 1. Reduce $\frac{5}{8}d.$ to the fraction of a pound.
2. Reduce .875 of a shilling to pounds.
3. Change 12 *cwt.* to the decimal of a ton.
4. Reduce 420 grains to the fraction of an ounce Troy.
5. Reduce $\frac{3}{4}$ of a penny to the decimal of a pound.
6. What decimal of a £ are 18s. 6d.?

NOTE.—The following method for reducing shillings, pence, and farthings to the decimal of a pound is sufficiently accurate for most business purposes: Write one-half of the greatest even number of shillings as tenths, and if there be an odd shilling write 5 hundredths; reduce the pence and farthings to farthings, and write their number as thousandths. If the number of farthings is between 12 and 36, add 1 to the thousandths; if between 36 and 48, add 2 to the thousandths. Thus, £8 17s. 8d. = £8 + £.85 + £.033 = £8.883.

7. Reduce 9s. 9d. to the decimal of a pound.
8. Reduce 116 cu. ft. to the decimal of a cord.
9. Reduce £247 14s. 9d. to pounds.
10. What decimal of an acre are 108 sq. rds.?
11. Reduce 75 feet to the fraction of a mile.
12. Reduce £27 10s. 6d. to pounds.
13. What is the cost of 22480 pounds of coal at \$4.25 per ton (2240 pounds)?
14. What is the cost of 16 tons 12 cwt. (16.6 T.) of "Nut" coal at \$6.80 per ton, and 8 tons 16 cwt. of "Chestnut" coal at \$6.10 per ton?
15. What is the cost of 8364 pounds of oats at \$1.65 per cental?
16. What is the cost of 8375 pounds of oats at \$0.56 per bushel?
17. If 1 pound is equivalent to $\$4.87\frac{3}{4}$, what is the value of £1234 16s. 9d. in U. S. money?
18. Reduce £25 12s. 6d. to the decimal of a £, and multiply the result by .05.

ADDITION.

200. Denominate numbers are *added*, *subtracted*, *multiplied*, and *divided* by the same general methods as are employed for like operations in abstract numbers. The only difference arises from the use of a *carrying* scale instead of the uniform scale of 10.

Ex. Add £5 11s. 4d., £7 14s. 9d., £6 16s. 8d., and £7 5s. 9d.

OPERATION.			ANALYSIS.—Write the numbers so that like denominations stand in the same column, and begin to add at the right. The sum of the pence is 30d. = 2s. 6d. Write the 6d. under the column of pence, and add the 2s. to the column of shillings, obtaining for the sum 48s. = £2 8s. Write the 8s. under the column of shillings, and add the £2 to the column of pounds, obtaining for the sum £27; which write under the column of pounds, producing the entire sum, £27 8s. 6d.
£	s.	d.	
5	11	4	
7	14	9	
6	16	8	
7	5	9	
27	8	6	

EXAMPLES.

201. 1. Add £16 5s. 4d., £12 8s. 9d., £13 14s. 8d., £42 0s. 7d., and 18s. 6d.

2. Add 3 *T.* 19 *cwt.* 2 *qr.* 16 *lb.*, 4 *T.* 13 *cwt.* 3 *qr.* 14 *lb.*, 18 *T.* 13 *cwt.* 24 *lb.*, and 42 *T.* 8 *cwt.* 1 *qr.* 22 *lb.* (Long Ton Table).

3. Add £163 16s. 11d., £52 8s. 6d., £3 14s. 2d., £84 12s. 11d., £106 1s. 4d., and £49 13s. 8d.

4. Add 1 *yr.* 6 *mo.* 10 *da.*, 3 *yr.* 8 *mo.* 24 *da.*, 4 *yr.* 11 *mo.* 16 *da.*, 3 *mo.* 18 *da.*, 1 *yr.* 8 *mo.* 8 *da.*

5. Add 8 *cu.* 106 *cu. ft.*, 3 *cu.* 85 *cu. ft.*, 2 *cu.* 113 *cu. ft.*, and 5 *cu.* 114 *cu. ft.*

6. Add 16 *hr.* 43 *min.* 48 *sec.*, 3 *hr.* 12 *min.* 40 *sec.*, 1 *hr.* 49 *min.* 13 *sec.*, and 5 *hr.* 19 *sec.*

7. Add $116^{\circ} 32' 44''$, $8^{\circ} 28' 53''$, $10^{\circ} 44' 12''$, and $16^{\circ} 18' 13''$.

8. Add 12 *ch.* 13 *l.*, 16 *ch.* 92 *l.*, 83 *ch.* 5 *l.*, and 5.05 *ch.*

9. Add 1 *lb.* 11 *oz.* 18 *pwt.* 14 *gr.*, 2 *lb.* 8 *oz.* 10 *pwt.*, 4 *lb.* 5 *oz.* 18 *gr.*, and 10 *oz.* 13 *pwt.* 12 *gr.*

10. Add 16 *gal.* 3 *qt.* 1 *pt.*, 45 *gal.* 2 *qt.*, 1½ *gal.* 1 *qt.* 1 *pt.*, 4 *gal.* 3 *qt.*, 15 *gal.* 1 *pt.*, and 24 *gal.* 3 *qt.* 1 *pt.*

11. Add £17 16s. 8d., £37 13s. 5d., £46 7d., £11 5s. 10d., £8 4s., £38 19s. 3d., and £45 12s. 8d.

12. Add £175 14s. 9d., £37 9s. 3d., £5 10s. 9d., 17s. 3d., £55 17s., £3 6s. 9d., £44 18s. 5d., £218 15s. 6d., and £3 11s. 11d.

SUBTRACTION.

202. Ex. From £10 6s. 4d. take £8 15s. 3d.

OPERATION.			ANALYSIS.—Write the numbers so that like denominations stand in the same column, and begin to subtract at the right. 3d. from 4d. leaves 1d., which write under the column of pence. Since 15s. cannot be subtracted from 6s., take £1 = 20s. from £10, leaving £9, and add it to the 6s., making 26s. 15s. from 26s. leaves 11s., which write under the column of shillings. £8 from £9 leaves £1, which write under the column of pounds. Hence the difference required is £1 11s. 1d.
£	s.	d.	
10	6	4	
8	15	3	
1	11	1	

EXAMPLES.

- 203.** 1. From £175 16s. 8d. take £87 12s. 6d.
 2. From £84 10s. 2d. take £63 5s. 10d.
 3. From £16 6s. 11d. take £12 12s. 8d.
 4. From £48 10s. 8d. take £24 16s. 10d.
 5. From 16 yr. 8 mo. 10 da. subtract 12 yr. 5 mo. 8 da.
 6. From 1880 yr. 10 mo. 16 da. take 1876 yr. 5 mo. 24 da.
 7. From 1881 yr. 4 mo. 25 da. take 1880 yr. 10 mo. 15 da.
 8. From 1882 yr. 3 mo. 20 da. take 1879 yr. 8 mo. 26 da.
 9. From 8 hr. 16 min. 44 sec. subtract 6 hr. 18 min. 40 sec.
 10. From $105^{\circ} 43' 12''$ subtract $87^{\circ} 49' 16''$.

204. To find the interval of time between two dates.

205. There are two methods in common use for finding the time between two dates: 1, by compound subtraction, in which the result is given in years, months, and days, and in which 12 months are considered a year, and 30 days a month; 2, the result is given in days, or in years and days, and the true number of days is taken for each month.

Ex. Find the time in months and days from Apr. 24 to Nov. 10.

OPERATION.	ANALYSIS.
mo. da.	—Represent the months and days by their numbers and find their difference by compound subtraction, writing the later date as the minuend and the earlier as the subtrahend.
11 10	
4 24	
6 16	

In many examples the interval may be found mentally as follows: From Apr. 24 to Oct. 24 are 6 mo.; in Oct. there are 6 more days after the 24th (regarding each month as 30 days), and in November to Nov. 10th inclusive, there are 10 days. Hence the total time between the given dates is 6 mo. 16 da.

The above methods may be used for finding the exact interval in days by making the necessary corrections. 6 mo. 16 da. = 196 da. From Apr. 24 to Nov. 10, there are 4 months containing 31 da. each; hence the true answer is 196 da. + 4 da., or 200 da.

NOTE.—When the month of February is included, subtract 2 days in a common year, and 1 day in a leap year.

Ex. Find the time from May 18, 1876, to Mar. 2, 1882.

OPERATION.

<i>yr.</i>	<i>mo.</i>	<i>da.</i>
1882	3	2
1876	5	18
<hr/>		
5	9	14

ANALYSIS.—As in preceding example.

Ex. What is the exact number of days from July 20, 1880, to Nov. 10, 1881?

OPERATION.

365 from July 20, 1880, to July 20, 1881.
 11 remaining in July.
 31 in August.
 30 in September.
 31 in October.
10 in November.

478 from July 20, 1880, to Nov. 10, 1881.

ANALYSIS.—In finding the interval between two dates the last day is counted, and not the first. Since the time is more than one year, write down 365 days as the

number of days from the first date to the same date of the next year. Next write down the number of days in the month of July after the 20th, then the number of days in each of the full calendar months, and finally the number of days in November to Nov. 10 inclusive. The sum of these numbers will be the required time.

EXAMPLES.

206. Find the time by compound subtraction from

1. Jan. 10 to Aug. 28.
2. Mar. 16 to Dec. 4.
3. Feb. 5, 1880, to Oct. 16, 1881.
4. Jan. 27, 1881, to July 4, 1883.
5. May 16, 1882, to Mar. 24, 1884.
6. June 28, 1881, to Apr. 10, 1882.
7. July 30, 1882, to May 12, 1883.
8. Aug. 16, 1883, to Jan. 1, 1885.

Find also the exact number of days between the above dates.

MULTIPLICATION.

207. Ex. Multiply £7 16s. 8d. by 11.

OPERATION.			ANALYSIS.—11 times 8d. are 88d. = 7s. 4d. Write the 4d. under the pence, and add the 7s. to the product of shillings. 11 times 16s. are 176s., plus 7s. from the preceding product are 183s. = £9 3s. Write the 3s. under the shillings, and add the £9 to the product of pounds. 11 times £7 are £77, plus £9 from the preceding product are £86, which write under the pounds. Hence the entire product is £86 3s. 4d.
£	s.	d.	
7	16	8	
		11	
86	3	4	

Ex. Multiply £8 12s. 6d. by .05.

ANALYSIS—Reduce the multiplicand to the decimal of a pound by Art. 198, perform the required multiplication, and reduce the result to shillings and pence by Art. 196. £8 12s. 6d. = £8.625. £8.625 × .05 = £.43125. £.43125 = 8s. 7.5d.

EXAMPLES.

- 208.** 1. Multiply £17 10s. 8d. by 9; by 11; by 15.
 2. How many cords of wood in 12 loads, each load containing 2 *cd.* 108 *cu. ft.*?
 3. Find the cost of 25 *yd.* of silk, at £1 2s. 6d. per *yd.*
 4. What is .05 of £127 16s. 6d.? Of £145 15s. 9d.?
 5. What is the weight of 24 silver spoons, each spoon weighing 1 oz. 13 *pwt.*?
 6. Multiply 1 *hr.* 38 *min.* 22 *sec.* by 15; by 12; by 18.
 7. If 15 men perform a certain piece of work in 3 *da.* 16 *hr.* 52 *min.*, how long would it take one man to perform it?
 8. Multiply £138 8s. 9d. by .02½; .06; by .07.
 9. What will 50 *gal.* of wine cost at 8s. 3d. per gallon?
 10. How much grain in 12 bins, each containing 13 *bu.* 3 *pk.* 6 *qt.*?
 11. If a man walk 4 *mi.* 3 *fur.* 32 *rd.* in one hour, how far will he walk in 10 hours? In 16 hours?

DIVISION.

209. Ex. If 6 *yds.* of cloth are worth £8 18*s.* 6*d.* what is 1 *yd.* worth ?

OPERATION.

£	s.	d.
6) 8	18	6
	1	9 9

ANALYSIS.—1 *yd.* is worth 1 *sixth* as much as 6 *yds.*
 $\frac{1}{6}$ of £8 is £1 and £2 remaining. Write the £1 in the quotient, and reduce the £2 to shillings. £2 = 40*s.*, plus 18*s.* in the dividend = 58*s.* $\frac{1}{6}$ of 58*s.* is 9*s.* and 4*s.* remaining. Write the 9*s.* in the quotient, and reduce the 4*s.* to pence. 4*s.* = 48*d.*, plus 6*d.* in the dividend = 54*d.* $\frac{1}{6}$ of 54*d.* is 9*d.*, which write in the quotient. £1 9*s.* 9*d.* is the quotient required.

NOTE.—When the divisor is a denominate number, as in Ex. 2, reduce both divisor and dividend to the same denomination, and divide as in simple numbers.

EXAMPLES.

210. 1. Divide £13 12*s.* 3*d.* by 11 ; by 9 ; by 33.

2. How many yards of muslin at 7*d.* per yard can be bought for £5 12*s.*? For £9 9*s.*? (See note.)

3. Divide 17*s.* 3*d.* by .05 ; by .09 ; by .15.

Reduce the dividend to the decimal of a pound, then divide in the usual manner, and reduce the quotient to pounds, shillings, and pence.

4. How many yards of silk at £1 19*s.* 2*d.* per yard can be purchased for £86 3*s.* 4*d.*? (See note.)

5. Divide 85° 18' 30" by 15 ; by 18 ; by 27.

6. If 48 shares of a certain stock are worth £2013 8*s.*, what is the value of 1 share?

When the divisor is large and a composite number, divide successively by its factors. Thus, in the above example, first divide by 6 and the resulting quotient by 8.

7. Divide 322 A. 90 *sq. rd.* by 10 ; by 13 ; by 16.

8. A pile of wood 4 *ft.* wide and 6 *ft.* high contains 18 *cd.* 72 *cu. ft.*; what is the length of the pile?

9. If 120 spoons weigh 32 *lb.* 9 *oz.* 15 *pwt.*, what does 1 weigh?

10. If 42 *yd.* of cloth cost £20 16*s.* 6*d.*, what is the price of 1 *yd.*? Of 12 *yd.*? Of 20 *yd.*?

LONGITUDE AND TIME.

211. The whole circle of the earth, or 360° , passes under the sun in 24 hours, and in 1 hour passes $\frac{1}{24}$ of 360° , or 15° ; in 1 minute, $\frac{1}{60}$ of 15° ($15 \times 60'$), or $15'$; and in 1 second, $\frac{1}{60}$ of $15'$ ($15 \times 60''$), or $15''$.

212. Comparison of Longitude and Time.

For a difference of	There is a difference of
15° in Longitude	1 <i>hr.</i> in Time.
$15'$ “ “	1 <i>min.</i> “ “
$15''$ “ “	1 <i>sec.</i> “ “
1° “ “	4 <i>min.</i> “ “
$1'$ “ “	4 <i>sec.</i> “ “
$1''$ “ “	$\frac{1}{4}$ <i>sec.</i> “ “

213. RULE.—1. *The difference in longitude of two places, expressed in degrees, minutes, and seconds, divided by 15 will produce their difference in time expressed in hours, minutes, and seconds.*

2. *The difference in time of two places, expressed in hours, minutes, and seconds, multiplied by 15 will produce their difference in longitude expressed in degrees, minutes, and seconds.*

214. TABLE OF LONGITUDES.

Albany.....	$73^\circ 44' 50''$ W.	New York.....	$74^\circ 0' 3''$ W.
Ann Arbor.....	$80^\circ 43'$ W.	New Orleans ...	$90^\circ 2' 30''$ W.
Boston.....	$71^\circ 3' 30''$ W.	Paris.....	$2^\circ 20' 23''$ E.
Berlin.....	$13^\circ 23' 45''$ E.	Philadelphia...	$75^\circ 10'$ W.
Calcutta....	$88^\circ 19' 2''$ E.	Rome.....	$12^\circ 27' 14''$ E.
Cincinnati	$84^\circ 29' 31''$ W.	Richmond, Va....	$77^\circ 25' 45''$ W.
Chicago.....	$87^\circ 37' 45''$ W.	San Francisco...	$122^\circ 23' 45''$ W.
Jefferson City, Mo.	$92^\circ 8'$ W.	St. Paul, Minn....	$95^\circ 4' 55''$ W.
London.....	$0^\circ 5' 38''$ W.	St. Louis, Mo....	$90^\circ 15' 15''$ W.
Mexico.....	$99^\circ 5'$ W.	Washington, D. C.	$77^\circ 0' 15''$ W.

EXAMPLES.

215. Find the difference in longitude between

- | | |
|-------------------------------|--------------------------------|
| 1. New York and London. | 4. St. Louis and Calcutta. |
| 2. Boston and Paris. | 5. Philadelphia and Berlin. |
| 3. Chicago and San Francisco. | 6. San Francisco and Calcutta. |

Find the difference in time between

- | | |
|----------------------------|----------------------------|
| 7. New York and Greenwich. | 10. Rome and London. |
| 8. Chicago and New York. | 11. Paris and Albany. |
| 9. Richmond and Calcutta. | 12. Calcutta and St. Paul. |

13. The difference in time between New York and Greenwich is 4 *hr.* 56 *min.* $\frac{1}{2}$ *sec.*; what is the difference in longitude?

14. A gentleman travels west until his watch is 50 minutes fast. How many degrees of longitude does he travel?

15. What change would a person be obliged to make in his watch in going from New York to Paris? (For longitudes, see table.)

16. A navigator finds that when it is noon at his place of observation, it is 16 *min.* 34 *sec.* past 10 P.M. by his chronometer, Greenwich time; what is his longitude?

17. When it is 6 o'clock P.M. at Richmond, Va., what is the time at St. Louis, Mo.?

18. If the difference of time between two places is 1 *hr.* 18 *min.* 4 *sec.*, what is the difference of longitude?

19. When it is 20 *min.* past 2 P.M. at Boston, Mass., what o'clock is it at San Francisco?

20. When it is 9 o'clock P.M. in San Francisco, it is 3 *min.* $3\frac{2}{5}$ *sec.* past 11 A.M. in Calcutta; what is the longitude of San Francisco, if the longitude of Calcutta is 88° 19' 2" E.?

21. When it is noon in Chicago, it is 5 *min.* 29 $\frac{1}{2}$ *sec.* of 1 P.M. in New York; what is the longitude of Chicago, the longitude of New York being 74° 3" W.?

THE METRIC SYSTEM.

216. In the **Metric System**, the *Meter* is the basis of all the weights and measures which it employs.

217. The **Meter** is the unit of length, and is equal to one ten-millionth part of the distance measured on a meridian of the earth from the equator to the pole, and equals about 39.37 inches.

The standard meter is a bar of platinum carefully preserved at Paris. Exact copies of the meter and the other units have been procured by the several nations, including the United States, that have legalized the system.

218. The names of the *higher* denominations, or *multi-
ples*, of the unit are formed by prefixing to the several *units* the Greek numerals, *deka* (10), *hecto* (100), *kilo* (1000), and *myria* (10000); as *dekameter*, 10 meters, *hectometer*, 100 meters, etc.

To assist the memory, observe that the initial letters of the multiples are in alphabetical order; thus, *D*, *H*, *K*, and *M*.

219. The names of the *lower* denominations, or *divisions*, of the unit are formed by prefixing to the several *units* the Latin numerals, *deci* ($\frac{1}{10}$), *centi* ($\frac{1}{100}$), *milli* ($\frac{1}{1000}$); as *decimeter*, $\frac{1}{10}$ meter, *centimeter*, $\frac{1}{100}$ meter, etc.

To assist the memory observe that the following words are derived from the same roots; *dime*, *decimal*, *decimate*, *decennial*, etc.; *cent*, *cental*, *century*, *centennial*, etc.; *mill*, *millennium*, etc.

* The use of the metric system is obligatory in Belgium, France, Germany, Greece, Netherlands, Italy, Portugal, Roumania, Spain, and Switzerland; in the Argentine Republic, Brazil, Peru, San Domingo, United States of Colombia, and Uruguay; while its use is partial or legalized in Austria, Azores, Madeira and Cape de Verde Islands, Central American States, Denmark, Japan, Sweden, Norway, Turkey, Spanish Possessions, Great Britain and the British Possessions, and our own country. For the year ending June 30, 1877, the value of our imports from countries where the metric system is obligatory amounted to \$177,897,469; partially in use, \$17,378,735; legalized, \$265,211,585; not legalized or in use, only \$23,801,142. Of the amount received from countries where its use is legalized, Great Britain and British Possessions furnish \$185,667,400. With these countries our present system is partly in harmony, but unfortunately the bulk of our trade with them is made up of articles measured by the bushel and gallon, neither of which standards corresponds to any bushel or gallon of this country.

LINEAR MEASURE.

220. TABLE.

1 Millimeter	($\frac{1}{1000}$ of a meter)	=	.03937 <i>in.</i>
10 <i>mm.</i> = 1 Centimeter	($\frac{1}{100}$ of a meter)	=	.3937 <i>in.</i>
10 <i>cm.</i> = 1 Decimeter	($\frac{1}{10}$ of a meter)	=	3.937 <i>in.</i>
10 <i>dm.</i> = 1 METER	(1 meter)	=	39.37 <i>in.</i>
10 <i>m.</i> = 1 Dekameter	(10 meters)	=	32.8 <i>ft.</i>
10 <i>Dm.</i> = 1 Hektometer	(100 meters)	=	328.09 <i>ft.</i>
10 <i>Hm.</i> = 1 Kilometer	(1000 meters)	=	.62137 <i>mi.</i>

NOTES—1. The meter, like the yard, is used in measuring cloths, ribbons, laces, short distances, etc.

2. The kilometer is used in measuring long distances, and is about $\frac{5}{8}$ of a mile.

3. The centimeter and millimeter are used by artisans and others in measuring minute lengths. The other denominations are rarely used.

EXAMPLES.

221. 1. Reduce 875275 meters to kilometers.

ANALYSIS.—Since 1 kilometer equals 1000 meters, in 875275 meters there are as many kilometers as 1000 is contained times in 875275, or 875.275. To divide by 1000 place the point three places to the left (**138**, 3).

2. Reduce 675.318 kilometers to meters.

ANALYSIS.—Since 1 kilometer equals 1000 meters, in 675.318 kilometers there are 675.318 times 1000, or 675318 meters. To multiply by 1000, place the point three places to the right (**135**, note).

3. Reduce 383.64 meters to centimeters; to kilometers.

4. Reduce 175.16 centimeters to kilometers; to meters.

5. Reduce to meters and find the sum of 876.2 decimeters, 30347 centimeters, 176.48 meters, 8.175 kilometers.

6. A ship sails 5712 kilometers in 48 days; how many kilometers does she sail per day?

7. What is the value of 56.4 meters of silk at \$1.75 per meter?

8. 16 pieces of cloth contain 38.5 meters each; 18 pieces contain 39 meters each; and 24 pieces contain 41.2 meters each; how many meters in all?

SQUARE MEASURE.

222. The unit of square measure is the *square meter*.

TABLE.

100 Square Centimeters, <i>sq. cm.</i>	= 1 Square Decimeter	= 15.5 + <i>sq. in.</i>
100 Square Decimeters, <i>sq. dm.</i>	= 1 SQUARE METER, <i>Sq. M.</i>	= 1.196 + <i>sq. yd.</i>

NOTES.—1. The square meter is used in measuring flooring, ceilings, etc.; the square decimeter and the square centimeter are used for minute surfaces.

2. Since units of square measure form a scale of hundreds, each denomination must have two places of figures.

223. The unit of **Land Measure** is the *are*, and is equal to a square dekameter (100 square meters), or 119.6 square yards.

TABLE.

1 Centare...	(1 square meter)	= 1550 <i>sq. in.</i>
100 Centares, <i>ca.</i>	= 1 Are.....	(100 square meters) = 119.6 <i>sq. yd.</i>
100 Ares, <i>A.</i>	= 1 Hectare...	(10000 square meters) = 2.471 acres.

NOTE.—The hectare is the ordinary unit for land.

EXAMPLES.

224. 1. Write 16 *sq. m.*, 8 *sq. dm.*, 24 *sq. cm.*, having the square meter as the unit. *Ans.* 16.0824.

2. Write 83 *sq. m.*, 9 *sq. dm.*, having the *sq. m.* as the unit.

3. In 47 ares how many square meters?

4. In 60.25 hectares how many centares?

5. How many square meters in a building lot 8 *m.* by 32 *m.*?

6. How many building lots, each containing 225 *sq. m.*, can be formed from a field containing 9 hectares?

7. How many hectares in a farm 1.024 *Km.* in width and 1.625 *Km.* in length?

8. What is the cost of a mirror 2.25 *m.* by 1.44 *m.*, at \$3.84 per *sq. m.*?

9. How many lots 25 *m.* wide by 60 *m.* deep, or having an equivalent area, can be laid out from 6 hectares?

10. A man bought a piece of land for \$6950.50, and sold it for \$7603.30, by which transaction he made \$6.80 a hectare; how many hectares were there?

11. What will be the cost of 4 plate-glass windows, each 3.2 *m.* by 2.5 *m.*, at \$2.75 per square meter?

12. What will be the cost of plastering a room 6.4 *m.* long, 4.5 *m.* wide, and 2.8 *m.* high, at 75 cents a square meter, allowing 30 *sq. m.* for doors, windows, etc.?

13. A rectangular farm containing 64 hectares is 625 meters wide; what is its length?

CUBIC MEASURE.

225. The unit for measuring ordinary solids is the *cubic meter*.

TABLE.

1000 Cu. Millimeters, <i>cu. mm.</i>	= 1 Cu. Centimeter	= .061 <i>cu. in.</i>
1000 Cu. Centimeters, <i>cu. cm.</i>	= 1 Cu. Decimeter	= 61.027 <i>cu. in.</i>
1000 Cu. Decimeters, <i>cu. dm.</i>	= 1 CU. METER	= { 35.317 <i>cu. ft.</i> 1.308 <i>cu. yd.</i>

NOTES—1. The cubic meter is used in measuring embankments, excavations, etc.; cubic centimeters and cubic millimeters for minute bodies.

2. Since units of cubic measure form a scale of thousands, each denomination must have three places of figures.

226. The unit of Wood Measure is the *ster*, and is equal to a cubic meter, or 35.317 cubic feet.

TABLE.

10 Decisters, <i>ds.</i>	= 1 Ster. (1 Cubic Meter)	= { .2759 cord. 35.317 <i>cu. ft.</i>
10 Sters, <i>s.</i>	= 1 Dekaster, <i>Ds.</i> (10 Cubic Meters)	= 2.759 cords.

EXAMPLES.

227. 1. Write 29 *cu. m.*, 75 *cu. dm.*, having the cubic meter as the unit. *Ans.* 29.075 *cu. m.*

2. Write 17 *cu. m.*, 218 *cu. dm.*, 27 *cu. cm.*, having the cubic meter as the unit.

3. How many cubic meters in a box 3.5 *m.* by 3.2 *m.* by 2.5 *m.*?

4. Bought 12 sters of wood; having sold 8.7 cubic meters, how much remained?

5. There are 13 blocks of marble, each containing 370.16 *cu. dm.*; how many cubic meters in all?

6. How many cubic meters in an excavation 13.2 *m.* by 18.5 *m.* by 8.4 *m.*?

7. At \$1.25 a cubic meter, what will it cost to dig a cellar 6.5 *m.* long, 5.4 *m.* wide, and 2.5 *m.* deep?

8. How many sters of wood in a pile 2.5 *m.* high, 2 *m.* wide, and 16.5 *m.* long? What is the length of a pile of the same height and width containing 216 sters?

9. Find the value of a pile of wood 12.3 *m.* long, 3.2 *m.* wide, and 2.5 *m.* high, at \$2.25 per ster.

10. Find the volume of a stick of timber 12.5 *m.* long, the end of which is .32 *m.* square.

DRY AND LIQUID MEASURE.

228. The unit of Dry and Liquid Measure is the *liter*, which is equal to a cubic decimeter, 1.0567 wine quarts, or .908 dry quart.

TABLE.

	Dry Measure.	Liquid Measure.
1 Milliliter. . . ($\frac{1}{1000}$ of a liter) =	.06103 <i>cu. in.</i> , or	.0338 <i>fl. oz.</i>
10 <i>ml.</i> = 1 Centiliter . . ($\frac{1}{100}$ of a liter) =	.6103 <i>cu. in.</i> , or	.338 <i>fl. oz.</i>
10 <i>cl.</i> = 1 Deciliter . . ($\frac{1}{10}$ of a liter) =	6.1027 <i>cu. in.</i> , or	.845 <i>gi</i>
10 <i>dl.</i> = 1 LITER . . . (1 liter) =	.908 <i>qt.</i> , or	1.0567 <i>qt.</i>
10 <i>l.</i> = 1 Dekaliter . . (10 liters) =	9.08 <i>qt.</i> , or	2.6418 <i>gal.</i>
10 <i>hl.</i> = 1 Hectoliter . . (100 liters) =	28.375 <i>bu.</i> , or	26.418 <i>gal.</i>
10 <i>kl.</i> = 1 Kiloliter . . (1000 liters) =	28.375 <i>bu.</i> , or	264.18 <i>gal.</i>

NOTES—1. The liter is commonly used in measuring wine, milk, etc., in moderate quantities. For minute quantities the centiliter and milliliter are employed; and for large quantities the dekaliter.

2. For measuring grain, etc., the hectoliter (=28375 bushels) is commonly used.

3. Instead of the kiloliter and milliliter, it is customary to use their equals, the cubic meter and cubic centimeter.

EXAMPLES.

229. 1. How many liters in a vessel whose capacity is 1 cubic meter?

2. How many hectoliters in 1 cubic meter?

3. What is the cost of sixteen liters of milk at 8 cents a liter?

4. How many hectoliters of wheat can be bought for \$396 at \$5.50 per hectoliter?

5. How many hectoliters of grain can be put in a rectangular bin, 4 *m.* long, 3.5 *m.* wide, and 1.2 *m.* high?

6. How many liters in 63.5 dekaliters? In 83.75 hectoliters?

7. At \$1.75 a liter, what is the cost of 85.6 dekaliters of wine?

8. How many hectoliters in 16 cubic meters?

9. How many bags, each holding 1 hectoliter, can be filled from a bin, 1.5 *m.* high, 2.4 *m.* wide, and 5 *m.* long?

10. A cistern 3.5 *m.* by 3.2 *m.*, and 9 *m.* deep, will hold how many dekaliters?

11. A merchant bought 4 hectoliters of nuts at \$8.50 per hectoliter, and retailed them at 12 cents a liter; what was his profit?

12. What must be the length of a bin, whose width is 4.8 *m.* and depth 2.4 *m.*, to hold 720 hectoliters of grain?

13. How many liters of wine, at 75 cents per liter, must be given in exchange for 480 hectoliters of wheat at \$5.25 per hectoliter?

14. How many liters will a cistern hold that is 4.8 *m.* long, 3.6 *m.* wide, and 3.2 *m.* deep?

WEIGHT.

230. The unit of weight is the *gram*, which is equal to the weight of a cubic centimeter of distilled water in a vacuum, at its greatest density (39.2° F.), or 15.432 grains.

TABLE.

	1 Milligram....($\frac{1}{1000}$ of a gram)	=	.01543 <i>gr.</i> Tr.
10 <i>mg.</i>	= 1 Centigram....($\frac{1}{100}$ of a gram)	=	.1543 <i>gr.</i> Tr.
10 <i>cg.</i>	= 1 Decigram....($\frac{1}{10}$ of a gram)	=	1.543 <i>gr.</i> Tr.
10 <i>dg.</i>	= 1 GRAM..... (1 gram)	=	15.432 <i>gr.</i> Tr.
10 <i>g.</i>	= 1 Dekagram....(10 grams)	=	.3527 <i>oz.</i> Av.
10 <i>Dg.</i>	= 1 Hectogram....(100 grams)	=	3.5274 <i>oz.</i> Av.
10 <i>Hg.</i>	= { 1 Kilogram, { ..(1000 grams)	= {	2.6792 <i>lb.</i> Tr.
	or Kilo		2.2046 <i>lb.</i> Av.
10 <i>Kg.</i>	= 1 Myriagram....(10000 grams)	=	22.046 <i>lb.</i> Av.
10 <i>Mg.</i> , or }	= 1 Quintal.....(100000 grams)	=	220.46 <i>lb.</i> Av.
100 Kilos }			
10 <i>Q.</i> , or }	{ 1 Tonneau, { ...(1000000 grams)	=	{ 2204.6 <i>lb.</i> Av.
1000 Kilos }			
	or TON		1.1023 <i>T.</i>

NOTES.—1. The above table is used in computing the weights of all objects from the smallest atom to the largest known body. The gram, kilogram (or kilo), and ton are principally used.

2. The gram is used in weighing letters, gold, silver, precious stones, and medicines.

3. The kilogram, or kilo, like the pound, is used in weighing groceries and coarse articles. It is approximately $2\frac{1}{5}$ pounds Av.

4. The ton is the weight of a cubic meter of water, and is used in weighing very heavy articles, as coal, iron, etc.

5. The pound of Germany, Austria, and Denmark is equal to $\frac{1}{2}$ of a kilogram; the centner (100 German pounds) to $\frac{1}{2}$ of a quintal.

EXAMPLES.

231. 1. What is the weight in grams of a cubic meter of water? Of a *cu. dm.* of water?

2. A farmer sells to A 3.716 *T.* of hay, to B 4.325 *T.*, to C 8775 kilos; how many tons does he sell?

3. The U. S. 50-cent piece weighs 12.5 grams; how many can be coined from a kilogram of standard silver?

4. The U. S. 5-cent piece (copper and nickel) weighs 5 grams; how many 5-cent pieces are equivalent in weight to 12 50-cent pieces?

5. How much alloy must be used in making 1200 U. S. twenty-five-cent pieces? (See Art. 176.)

6. What is the cost of 75.6 kilos of sugar at 18 cents a kilo?

7. How many kilos of lard can be bought for 1248 francs at 1.04 francs per kilo?

8. What is the weight of 10 *cu. m.* of ice, it being .93 as heavy as water?

232. TABLE OF EQUIVALENTS.

1 inch = 2.54 centimeters.....	1 centimeter = 0.3937 inch.
1 foot = 3.048 decimeters.....	1 decimeter = 0.328 foot.
1 yard = 0.9144 meter.....	1 meter = 1.0936 yards = 39.37 in.
1 rod = 0.5029 dekameter.....	1 dekameter = 1.9884 rods.
1 mile = 1.6093 kilometers.....	1 kilometer = 0.62137 mile.
1 sq. inch = 6.452 sq. centimeters.	1 sq. centimeter = 0.155 sq. inch.
1 sq. foot = 9.2903 sq. decimeters..	1 sq. decimeter = 0.1076 sq. foot.
1 sq. yard = 0.8361 sq. meter.....	1 sq. meter = 1.196 sq. yards.
1 sq. rod = 25.293 sq. meters.....	1 are = 3.954 sq. rods = 119.6 sq. yds
1 acre = 0.4047 hectare.....	1 hectare = 2.471 acres.
1 sq. mile = 2.59 sq. kilometers..	1 sq. kilometer = 0.3861 sq. mile.
1 cu. inch = 16.387 cu. centimeters.	1 cu. centimeter = 0.061 cu. inch.
1 cu. foot = 28.317 cu. decimeters..	1 cu. decimeter = 0.0353 cu. foot.
1 cu. yard = 0.7645 cu. meter.....	1 cu. meter = 1.308 cu. yards.
1 cord = 3.624 sters.....	1 ster = 0.2759 cord.
1 liquid quart = 0.9463 liter.....	1 liter = 1.0567 liquid quarts.
1 gallon = 0.3785 dekaliter.....	1 dekaliter = 2.6417 gallons.
1 dry quart = 1.101 liters.....	1 liter = 0.908 dry quart.
1 bushel = 3.524 dekaliters.....	1 hektoliter = 2.8375 bushels.
1 ounce av. = 28.35 grams.....	1 gram = 0.03527 ounce av.
1 pound av. = 0.4536 kilogram..	1 kilogram = 2.2046 pounds av.
1 pound av. = 0.9072 Ger. pounds..	1 Ger. pound = 1.1023 pounds av
1 ton (2000 lbs.) = 0.9072 met. ton..	1 met. ton = 1.1023 tons.
1 grain Troy = 0.0648 gram.....	1 gram = 15.432 grains Troy
1 ounce Troy = 31.1035 grams...	1 gram = 0.03215 ounce Troy
1 pound Troy = 0.3732 kilogram..	1 kilogram = 2.679 pounds Troy

EXAMPLES.

233. 1. In 225 meters how many yards? How many inches?

2. Reduce 6 miles to kilometers; to meters.

3. Reduce 640 acres to hectares; to ares.

4. In 10 kilometers, how many feet? How many miles?

5. In 315.6 kilos, how many pounds?

6. How many German pounds in 225 English or U. S. pounds?

7. What is the weight of the U. S. standard silver dollar in grams? Of the trade dollar?

8. In 5000 U. S. bushels, how many hectoliters? How many dekaliters?

9. In 875 *cu. yd.* how many *cu. m.*?

10. In 1000 *cu. m.* how many *cu. yd.*?

11. Reduce 1728 *gal.* wine to liters; to dekaliters.

12. In 244 *sq. m.* how many *sq. yd.*? How many *sq. ft.*?

13. Reduce 220 *oz. Av.* to grams; to kilograms.

14. Reduce 4387 meters to miles.

234. APPROXIMATE VALUES.*

When no great accuracy is required, we may, for all practical purposes, consider—

1 decimeter	= 4 inches.	1 cu. met. or ster	= $1\frac{1}{3}$ cu. yd., or $\frac{1}{4}$ cord.
1 meter	= 39 inches.	1 liter	= 1 quart.
5 meters	= 1 rod.	1 hectoliter	= $2\frac{1}{2}$ bushels.
1 kilometer	= $\frac{5}{8}$ mile.	1 gram	= $15\frac{1}{2}$ grains.
1 square meter	= $10\frac{3}{4}$ sq. feet.	1 kilogram	= $2\frac{1}{2}$ pounds.
1 hectare	= $2\frac{1}{2}$ acres.	1 ton	= 2200 pounds.

* For short approximate rules for reducing the metric system to the common system of weights and measures, and *vice versa*, see the **PACKARD COMMERCIAL ARITHMETIC**, pages 85-87.

For other foreign weights and measures, see the **PACKARD COMMERCIAL ARITHMETIC**, pages 88-90.

REVIEW EXAMPLES.

235. 1. How many days from Mar. 16 to Oct. 4?
From June 30 to Dec. 25?

2. Find the time by compound subtraction from Aug. 23, 1882, to Jan. 15, 1884.

3. How many leap years from 1881 to 1897? From 1795 to 1845? From 1889 to 1909?

4. Reduce 2.375 years to years, months, and days.

5. Suppose a person's income to be \$1000 per day, how much is that per minute?

6. In 4376 feet how many chains? How many inches?

7. In 396 rods, how many chains? How many feet?

8. In 37.56 chains, how many feet? How many rods?

9. Children's size 1 of shoemakers' measure is $4\frac{1}{8}$ inches long; what is the length of boys' size 8, youths' size 1, and men's size 10? (Size 1 of the second series is one size longer than size 13 of the first series. See Art. 152.)

10. How many square feet in a lot 25 feet front and 100 feet deep?

11. How many square feet in a rectangular lot, whose breadth is $25\frac{3}{4}$ feet and whose length is $116\frac{1}{2}$ feet?

12. How many acres in a rectangular field, 28.50 chains by 46.38 chains?

13. How many square yards in a floor, 16 *ft.* 6 *in.* by 12 *ft.* 9 *in.*?

14. How much will it cost to carpet a floor 16 *ft.* by 18 *ft.*, with carpeting $\frac{3}{4}$ *yd.* wide, at \$1.60 per yard?

15. What is the value of a field 320 *rd.* long and 160 *rd.* wide at \$22.50 an acre?

16. A rectangular lot contains 24 acres; what is its width, its length being 1056 feet?

17. How much will it cost to dig a cellar 36 *ft.* long, 30 *ft.* wide, and 6 *ft.* deep, at 30 cents per cubic yard?

18. If a pile of bark is 40 *ft.* long and 4 *ft.* wide, how high must it be to contain 10 cords?

19. How many feet, board measure, in 16 boards each 18 *ft.* long, 10 *in.* wide, and 1 *in.* thick?

20. How many feet, board measure, in 12 planks, each 10 *ft.* long, 12 *in.* wide, and 2 *in.* thick?

21. How many board feet in 225 cubic feet?

22. Paid \$222 75 for boards at \$13.50 per M.; how many feet were purchased?

23. What is the value of 27315 *ft.* of lumber at \$12 per M.?

24. How many pills, each containing 5 grains, can be made from 1 *lb.* av. of quinine?

25. In 70 *oz.* Tr., how many *oz.* av.?

26. In 70 *lb.* Tr., how many *lb.* av.?

27. What is the cost of 11 *T.* 12 *cwt.* of "Nut" coal at \$6.95 per ton, and 9 *T.* 16 *cwt.* of "Chestnut" coal at \$6.25 per ton?

28. What is the freight of 16 *T.* 17 *cwt.* 25 *lb.* at \$5 per ton (2240 *lb.*)?

29. What is the cost of 15669 pounds meal at \$1.10 per *cwt.*?

30. What cost 16450 pounds of hay at \$15.50 per ton?

31. In 27318 pounds of corn, how many bushels? What is the value of the same at $48\frac{3}{4}$ cents per bushel?

32. What is the value of 27318 pounds of corn, at 87.1 cents per cental?

NOTE.—Examples 35 and 36 illustrate the present and the cental systems of buying and selling produce, and show the calculations saved by using the latter.

33. In 7346 pounds of oats, how many bushels?

34. What is the cost of $273\frac{1}{2}$ *bu.* oats, at 58 cents per bushel?

35. Find the value of 281 *lb.* peas at \$1.05 per bushel?

36. What is the value of 291 *lb.* of peas at \$1.75 per cental?

37. What is the value of 186 *lb.* of beans at \$2.25 per bushel?

38. Find the cost of 192 *lb.* beans at \$3.75 per cental.

39. At what price per bushel is rye at \$1.227 per cental ?
Oats at \$1.66 per cental ? Barley at \$2.126 per cental ?

40. How many bushels in 27316 pounds of wheat ? in
24375 pounds of corn ? In 16218 pounds of oats ? In
21412 pounds of barley ? In 17387 pounds of malt ?

41. How many bushels in 54 centals of wheat ? In 87
centals of corn ? In 46 centals of oats ? In 53 centals of
barley ?

42. How much per cental, is wheat at \$1.85½ per bushel ?
Corn at 76¼ cents per bushel ? Oats at 48¾ cents per bushel ?
Barley at 87 cents per bushel ?

43. How much per bushel is wheat at \$1.27 per cental ?
Corn at \$1.323 per cental ?

44. How much per cental is timothy seed at \$1.75 per
bushel ? Clover seed at \$8.55 per bushel ?

45. What is the cost of 561²³ bushels oats at 43 cents per
bushel ? Of 411¹⁴ bushels corn at 46 cents per bushel ?

NOTE.—The small figures represent pounds.

46. A quartermaster purchased 75000 pounds of corn, at
31¼ cents per bushel ; 32113 pounds of oats, at 32½ cents
per bushel ; and 79500 pounds of hay, at \$22.37½ per ton
(2000 pounds). What was the total cost of the purchase ?

47. A farmer sold 18360 pounds of corn, at 64 cents per
cental ; 22450 pounds of oats, at 94 cents per cental ; and
36650 pounds of hay, at \$1.31 per cental. How much was
realized from the sale ?

48. How many sheets of paper in 5 reams ?

49. How many grains of gold and alloy respectively are
required for the coinage of 6983 gold dollars ?

50. How many Troy ounces of pure silver would be re-
quired in the coinage of 2,000,000 standard silver dollars ?
How much copper ?

51. Find the value of an *oz.* Tr. of standard gold, making
no allowance for the alloy and coinage. Of an *oz.* av.

52. The coinage at the mints of the United States during the fiscal year ending June 30, 1879, was as follows :

GOLD—Double-eagles, \$57,234,340 ; eagles, \$1,031,440 ; half-eagles, \$1,442,130 ; three-dollars, \$109,182 ; quarter-eagles, \$1,166,800 ; dollars, \$3,020 ; total gold, \$———.

SILVER—Dollars, \$27,227,050 ; half-dollars, \$225 ; quarter-dollars, \$112.50 ; dimes, \$45 ; total silver, \$———.

MINOR COINAGE—5-cents, \$1,175 ; 3-cents, \$984 ; cents, \$95,639 ; total minor coinage, \$———.

How many pieces were coined and what was the total value of the coinage?

53. Add £27 16s. 10d., £6 10s. 8d., £47 15s. 11d., £25 7s. 6d., £3 14s. 8d., and £23 16s. 3d.

54. In 47 guineas, how many shillings and pounds?

55. What is the value of 45000 tons of steel rails at 97s. 6d. per ton? What is the value per ton in U. S. money? Of total in U. S. money?

56. How many yards of cloth at 3s. 6d. per yard can be bought for £7?

57. Reduce £19 16s. 9d. to the decimal of a pound.

58. If £1 sterling is worth \$4.87, what is the value of £225 18s. 6d.?

59. What is the value of 20 yd. silk at 10s. 6d. per yard?

60. If 1 franc is worth \$.193, what is the value of \$1 in francs?

61. What is the value in U. S. money of 875 Napoleons? (1 Napoleon = 20 francs.)

62. What is the cost of 50 meters silk at 8.25 francs (8 francs 25 centimes) per meter?

63. What is the value in U. S. money of 24000 marks?

64. What is the value in U. S. money of 5,528,364 Brazilian reis? Of 7387 Portuguese milreis? (See Art. 187.)

65. In 8375 pies (money of India), how many annas and rupees? (See Art. 187.)

66. The difference in the local time of two places is 3 hr. 43 min. 12 sec.; what is the difference in longitude?

67. What is the value of a pile of wood, 28 *ft.* long, 6 *ft.* high, and 8 *ft.* broad, at \$5.65 per cord?

68. Find the cost of papering a room 22 *ft.* 6 *in.* long, 13 *ft.* 6 *in.* wide, and 9 *ft.* high, the paper being worth 45¢ per roll, and each roll containing 8 *yds.*, 18 *in.* wide.

69. When it is 4 *hr.* 40 *min.* A. M. at Chicago, what is the time at Calcutta?

70. How many bushels will a box 10 *ft.* long, 5 *ft.* wide, and 4 *ft.* high contain?

NOTE.—Since a bushel is about $1\frac{1}{4}$ cubic feet, the following approximate rules may be used for all practical purposes:

To reduce cubic feet to bushels: *Deduct one-fifth.*

The result will be too small by about $4\frac{1}{2}$ bushels for every 1000 bushels of the result.

To reduce bushels to cubic feet: *Add one-fourth.*

The result will be too great by about $4\frac{1}{2}$ cubic feet for every 1000 cubic feet of the result.

Solve the above example, both exactly and approximately, and compare the results.

71. How many hectoliters of grain will a box 4 meters long, 3.2 meters wide, and 2.5 meters high contain?

72. How many gallons of water will a cistern hold which is 8 *ft.* long, 7 *ft.* wide, and 10 *ft.* deep?

73. What is the capacity in liters of a cistern 25 meters long, 2.2 meters wide, and 3 meters deep?

74. In 52 meters cassimere, how many yards?

75. The specific duty on Brussels carpet is 44 cents per square yard; what is the duty per square meter?

76. In a pane of glass 24 *in.* by 30 *in.*, how many square decimeters?

77. The duty on pig-iron is \$7 per ton (2240 *lb.*); what is the duty per metric ton or millier?

78. The U. S. custom duty on alcohol is \$2 per gallon; what is the duty per liter?

79. The duty on tallow candles is $2\frac{1}{2}$ cents per pound; what is the duty per kilogram?

PERCENTAGE.

236. **Percentage** is a term applied to all operations in which 100 is used as the basis of computation.

It is also the name given to any number of hundredths of a number.

237. **Per Cent.** (%) is an abbreviation of the Latin *per centum*, meaning *on* or *by the hundred*.

Thus, 5% means 5 of every hundred, or 5 hundredths ($\frac{5}{100}$, or .05).

238. Any *per cent.* may be expressed in the form of a *decimal* or *fraction*.

Thus, 5 *per cent.* = 5% = 5 hundredths = .05 = $\frac{5}{100}$ = $\frac{1}{20}$. The first two forms are used in the statements of questions; the others in the operations.

239. In percentage, three elements are considered, viz.: the *Base*, the *Rate*, and the *Percentage*. Any two being given, the other can be found.

240. The **Percentage** is the result obtained by taking a certain number of hundredths of a number.

241. The **Base** is the number of which a certain number of hundredths are taken.

242. The **Rate** is the number of hundredths, or the number per cent.

Thus, in the statement, 6% of 300 is 18, the **Percentage** is 18, the **Base** 300, and 6 per cent. (.06) is the **Rate**.

243. **Applications of Percentage.**—The principle of percentage are applied to many of the most common business transactions. Among the most important of these are Trade Discounts, Commission, Insurance, Profit and Loss, Duties, Interest, and Exchange.

244. Ex. What is 5 per cent. of 300?

OPERATION.	ANALYSIS.—
300 Base.	5% is equivalent to 5 hundredths ($\frac{5}{100}$, or .05). 5 hundredths of a number may be found by multiplying it by 5 hundredths. For convenience, the multiplication is performed by expressing the 5 hundredths in the form of a decimal. $.05 \times 300 = 15$, the percentage required.
.05 Rate.	
15.00 Percentage.	

Therefore, the Percentage is the product of two factors, the Base and the Rate.

Or, 1% of 300 is 3, and 5% is 5 times 3, or 15.

Ex. 15 is 5 per cent. of what number?

OPERATION.	ANALYSIS.—
Rate. Percentage.	In this example there is given the Percentage and Rate, to find the Base. Since the Percentage = the Base \times the Rate, the Base = the Percentage \div the Rate
.05) 15.00	
Base. 300.	

Or, if 15 is 5% of a certain number, 1% is $\frac{1}{5}$ of 15, or 3; and the number, or 100%, is 100 times 3, or 300.

Ex. 15 is what per cent. of 300?

OPERATION.	ANALYSIS.—
Base. Percentage. Rate.	The Base and Percentage are given, to find the Rate. Since the Percentage = the Base \times the Rate, the Rate = the Percentage \div the Base. $15 \div 300 = .05$ (5%), the required per cent.
300) 15.00 (.05	Or, 15 is $\frac{15}{300}$ or $\frac{1}{20}$ of 300. $\frac{1}{20} = \frac{5}{100}$, or 5%.

Ex. What is 4% of £247 13s. 6d.?

OPERATION.	ANALYSIS.—
£ s. d.	Multiply the number of each denomination by .04, as in the margin, and then reduce the decimal parts to integers of lower denominations (196).
247 13 6	
.04	
£ 9.88 .52 .24	Or, reduce shillings and pence to the decimal of a pound (See note, Ex. 6. Art. 199), take the required per cent. and reduce the decimal result to lower denominations. Thus,
20	
s. 18.12	
12	
d. 1.68	

$$\begin{aligned} \text{£}247 \text{ } 13\text{s. } 6\text{d.} &= \text{£}247.675. \\ \text{£}247.675 \times .04 &= \text{£}9.907 = \text{£}9 \text{ } 18\text{s. } 1.68\text{d.} \end{aligned}$$

245. These principles may be expressed by the following formulae:

$$P = B \times R; \quad B = P \div R; \quad R = P \div B.$$

246. RULES.—1. To find the percentage, multiply the base by the rate expressed decimally.

2. To find the base, divide the percentage by the rate expressed decimally.

3. To find the rate, divide the percentage by the base.

NOTE.—In finding the rate, to produce a quotient of hundredths, make the decimal places of the dividend exceed those of the divisor by 2.

247. When the rate is an aliquot part of 100, it is generally more convenient to use the equivalent fraction. Thus,

$$\begin{array}{lll} 50\% = .50 = \frac{1}{2}. & 16\frac{2}{3}\% = .16\frac{2}{3} = \frac{1}{6}. & 6\frac{1}{4}\% = .06\frac{1}{4} = \frac{1}{16}. \\ 33\frac{1}{3}\% = .33\frac{1}{3} = \frac{1}{3}. & 12\frac{1}{2}\% = .12\frac{1}{2} = \frac{1}{8}. & 5\% = .05 = \frac{1}{20}. \\ 25\% = .25 = \frac{1}{4}. & 10\% = .10 = \frac{1}{10}. & 3\frac{1}{3}\% = .03\frac{1}{3} = \frac{1}{30}. \\ 20\% = .20 = \frac{1}{5}. & 8\frac{1}{2}\% = .08\frac{1}{2} = \frac{1}{12}. & 2\frac{1}{2}\% = .02\frac{1}{2} = \frac{1}{40}. \end{array}$$

EXAMPLES.

248. What is

Find

1. $\frac{1}{4}$ of 1728?

6. 16% of \$375.

2. $\frac{25}{100}$ of 2456?

7. 8% of \$24.25.

3. .25 of 5280?

8. $2\frac{1}{2}\%$ ($\frac{1}{4}$ of 10%) of 876.

4. 25% of 8424?

9. $7\frac{1}{2}\%$ ($10\% - \frac{1}{4}$ of 10%) of \$1678.

5. $\frac{1}{8}\%$ of 1000?

10. $\frac{3}{4}\%$ ($1\% - \frac{1}{4}\%$) of \$21275.

11. What is the difference between $2\frac{1}{2}\%$ of \$16000 and 5% of \$8475?

12. A bought goods amounting to \$375.60, and sold them so as to gain 30% of the cost; how much did he gain?

13. A lawyer collected \$2875, and charged 5% for his services; how much did he retain for his services, and how much did he pay over?

14. What is the duty on twelve watches valued at \$75 each, at 25% of the value?

15. Jan. 10, a merchant buys a bill of goods amounting to \$876.40 on the following terms: 4 months, or less 5% if paid in 10 days. How much would settle the bill Jan. 18?

16. The product of two factors is 75 ; if one of the factors is .03, what is the other factor ?

17. The percentage is 60, and the rate $2\frac{1}{2}\%$; what is the base ?

18. \$18.08 are 5% of what ? 22. 165 *fl.* are $33\frac{1}{3}\%$ of what ?

19. \$324 are 3% of what ? 23. £240 are $3\frac{1}{4}\%$ of what ?

20. \$37.56 are $2\frac{1}{2}\%$ of what ? 24. \$12.25 are $6\frac{1}{8}\%$ of what ?

21. \$17.28 are 24% of what ? 25. 96 francs are $\frac{4}{5}\%$ of what ?

26. An agent sells a house and lot for \$16450, and receives 5% for his services ; what does he pay to the owner of the property ?

27. Bought 375 barrels of flour at \$4.88 per barrel, and sold it at a gain of $12\frac{1}{2}\%$. What was the selling price ?

28. If a man fails to pay his tax until he is charged 8% additional, how much will he lose if his tax is \$36.75 ?

29. If the rate is 20% and the percentage 440, what is the base ?

30. A has 35% of his property invested in stocks, 10% in horses and cattle, 18% in grain, and the remainder, which is \$24235, in real estate. What is the total value of his property ?

31. A merchant, failing in business, pays 43% of his indebtedness ; he owes A \$3750, and B \$6280 ; how much does he pay each ?

32. The product of two numbers is 375 ; if one of the numbers is 30000, what is the other number ? Express answer in hundredths.

33. The assets of a bankrupt are \$27387, and his liabilities \$82161 ; what % of his indebtedness can he pay ?

What per cent. of

34. 375 is 75 ?

38. \$1000 is \$12.50 ?

35. \$1728 is \$144 ?

39. \$3720 is \$232.50 ?

36. \$3456 is \$72 ?

40. \$2416 is \$60.40 ?

37. 5280 *fl.* is 165 *fl.*

41. \$1484 is \$21.20 ?

42. A merchant paid for goods \$345 and sold them for \$258.75; the loss is what % of the cost?

43. If a paymaster receives \$150000 from the treasury, and fails to account for \$225 thereof, what is the percentage of loss to the government?

44. If a premium on fire insurance is $\frac{3}{4}\%$, what sum has to be paid on a risk of \$8800?

45. \$640 being increased by a certain % of itself equals \$720; required the rate %.

46. A commission merchant sold 450 barrels of flour at \$5.30 per barrel; how much should he send to the miller, if he charges $2\frac{1}{2}$ per cent. for making the sale?

47. A horse was sold for \$658, which was $16\frac{2}{3}\%$ more than it cost; what was the cost?

NOTE.—The cost of the horse was $\frac{100}{116\frac{2}{3}}\%$, or 100% of itself; since the gain was $16\frac{2}{3}\%$ of the cost, the selling price (the cost plus the gain) was $116\frac{2}{3}\%$ of the cost. \$658 is $116\frac{2}{3}\%$ of what number?

What number increased by What number decreased by

48. 25% of itself is 500? 51. 5% of itself is \$307.80?

49. 8% of itself is \$1004.40? 52. 40% of itself is 3726?

50. 125% of itself is 999? 53. 25% of itself is \$342.60?

54. When the premium on gold was $17\frac{3}{8}\%$, what amount of gold was it necessary to sell to pay a note of \$3000 in currency?

55. 144 is 120% of what number?

56. 275 is 100% of what number?

57. A merchant buys a bill of goods amounting to \$1000 on a credit of four months, or 6% off for cash. He pays \$500 cash. For what amount should his account be credited?

58. What is 116% of 1200? 63. Find $2\frac{1}{2}\%$ of £64 16s.

59. 375 is what % of 300? 64. Find 4% of £75 12s. 6d.

60. Find 95% of \$1260. 65. Find 10% of £37 8s. 9d.

61. \$187.50 are $2\frac{1}{2}\%$ of what? 66. 16s. is $2\frac{1}{2}\%$ of what?

62. Find 5% of £375. 67. £1 8s. 4d. is 4% of what?

DISCOUNTS.

249. It is customary in many branches of business for manufacturers and dealers to have fixed price-lists of certain kinds of merchandise; and when the value changes, instead of changing a long price-list, the rate of discount is changed. The fixed price is called the *List-Price*, and the discount allowed the *Trade Discount*.

Books are usually sold by publishers and jobbers at certain discounts from the retail prices.

250. Many kinds of merchandise are sold at "time" prices, subject to certain rates of discount if paid at an earlier period.

1. Thus, the following or similar announcements are usually found upon the bill-heads of wholesale dealers: "Terms, 4 months, or 30 days, less 5%"; or, "Terms 60 days, or 1% discount in 30 days, or 2% discount in 10 days."

2. When no rate of discount has been offered, merchants are generally willing, when bills are paid before maturity, to deduct the interest on the amount of the bill for the remainder of the time at the legal rate per annum.

Ex. The list price of a scale is \$80; what is the net price if a discount of 25% and 10% is allowed?

OPERATION.

\$80 List-price.

20 25%, or $\frac{1}{4}$.

60

6 10%, or $\frac{1}{10}$.

54 Net-price.

ANALYSIS—The first rate of discount is reckoned upon, and deducted from the list-price, and the others are deducted from the successive remainders.

The result is not affected by the order in which the discounts are taken. A discount of 25% and 10% is the same as a discount of 10% and 25%.

EXAMPLES.

251. 1. The gross amount of a bill of shoes is \$82.68. What is the net amount, the rate of discount being 5%?

2. A stove is sold for \$45 less 30%; required the net price?

NOTE.—If the discount is not required, multiply by 70 ($100\% - 30\%$); the product will be the net price.

3. The gross amount of a bill of mdse. is \$106.36; what is the net amount, the rates of discount being 20% and 10%?

4. The gross amount of a bill of notions is \$49.75; what is the net amount, the rates of discount being 10% and 10%?

5. What is the value of 12 pair shoes @ \$1.60 per pair, less 5%?

6. What direct discount is equivalent to a discount of 15% and 10%? 45% and 10%? 20% and 12½%? 60% and 10%?

7. What is the net value of one case prints containing 2273 yd., @ 43 cts., less 5%, cooperage 25 cts.?

8. A bill of merchandise amounting to \$442.38 was bought Aug. 18, 1879, on the following time: "4 months or 5% off 30 days." How much would settle the bill Sept. 16, 1879?

9. What is the net value of a bill of iron amounting to \$1103.75, at a discount of 45, 10, and 2 per cent.?

10. What is the net value of 1 case prints containing 30392 yd. @ 5 cts. per yd., less a discount of 3%; cooperage \$25?

11. The net amount of a bill of files was \$36.75; what was the gross amount, the rate of discount being 10%?

12. What is the difference on a bill of \$875 between a discount of 40% and a discount of 30% and 10%?

13. A bill of tinware is sold at the following discounts: \$74.20 at 20% and 10%; \$43.75 at 40% and 5%; and \$69 at 33⅓% and 10%. What is the net amount of the bill?

14. A bill of dry goods amounting to \$914.37 is sold, Aug. 19, on the following terms: "60 days, or less 1% if paid in 30 days, or less 2% if paid in 10 days." How much would settle the bill Sept. 18? How much Aug. 27?

15. Of a bill of hardware, \$61.51 are sold at a discount of 60 and 5%; \$18.75 at a discount of 10%; \$16.86 at a discount of 12½%; \$44.25 at a discount of 40 and 5%; \$29.60 at a discount of 40, 12½, and 10%; \$28.04 at a discount of 55%; \$18.70 at a discount of 50%; \$19.75 at a discount of 20%; \$18.50 at a discount of 15%; \$307.55 at a discount of 75 and 12½%; \$36.61 at a discount of 60 and 10%; and \$218.25 net. What is the net amount of the bill?

BILLS AND INVOICES.

252. A Bill is a detailed statement of merchandise sold, or of services rendered. Bills of merchandise state the place and date of the sale, the names of the buyer and seller, the terms of the sale, the quantity, price, and distinguishing marks and numbers of the merchandise, and other details.

The terms *Bill* and *Invoice* are used by many interchangeably. The term Invoice is applied more particularly to statements rendered by consignees to commission merchants, showing marks, numbers, values, and accrued charges of goods shipped; to bills rendered to jobbers; and to bills received from foreign countries.

EXAMPLES.

253. Copy and extend the following bills:

(1. Canned Goods.)

Folio 316. WILMINGTON, DEL., Nov. 16, 1876
Messrs. WM. DOLTON & Co.,
Bought of JAMES MORROW & SON.

<i>Cases.</i>	<i>Doz.</i>						
2	4	3	lb. Peaches	2 ²⁵ ^a	9	00	
1	2	2	" Saco Corn	1 ⁸⁵	*	**	
1	2	2 ¹ ₂	" Salmon	3 ⁸⁵	*	**	
2	4	3	" Tomatoes	1 ⁸⁰	*	**	
1	2	2 ¹ ₂	" Col. Pears	4 ⁰⁰	*	**	\$*** **
			^a Price per dozen.				

(2. Flour.)

BUFFALO, N. Y., Dec. 6, 1880.
Messrs. DANIEL CROUSE & SONS.
Bought of St HOELLKOPF & MATTHEWS.
Interest charged on all accounts after 30 days.

	20	Bbbs. Flour "Eric" Sacks .	\$7 05	***	**		
	25	" " " Bbbs. .	7.25	***	**		
	25	" " "Victor" Sacks .	6.05	***	**		
	25	" " " Bbbs. .	6.25	***	**		
	15	" " "Dakota" Sacks .	5.30	**	**		
20 bags		2177 lb. Sifted Meal . . .	1.20 ^a	**	**		
70 "		264 ⁹ ₃₂ bu. Onts56 ^b	***		***	**

^a \$1.20 per hundredweight.

^b 56 cents per bushel

(3. Fish.)

GLOUCESTER, MASS., Sept. 28, 1876

Messrs. DANIEL WIEDMAN & Co.,

Bought of CLARK & SOMES.

Subject to sight draft without notice after thirty days.

2	Qtl. New Geo. Cod	5.75	**	**		
1	Bbl. Ex. #1 Mackerel . . .	20.00	**	**		
10	Kits 15 lbs Ex. #1 Mackerel	1.80*	**	**		
10	" 20 lbs Bay #1 "	1.80	**	**		
2	Bbls. #2 Shore "	12.00	**	**		
10	Kits 20 lbs. #2 Shore .	1.50	**	**		
5	Halves New Labrador Herring	3.82	**	**		
3	" Round Shore "	2.95	*	**		
	Box #3, ctg. in Boston #90		*	**	\$***	**
* \$1.80 per kit.						

(4. Groceries.)

Order Book, 410-22.
Day Book, 115-797.

NEW YORK, Feb. 1, 1880

Messrs. EDWARDS & Co.,

Bought of H. K. & F. B. THURBER & Co.Terms Cash 30 days.
Shipped per National Line.When you desire to order goods, same as had before, give
date of purchase, and the Order and Day Book pages.

#4385	1	Cask Old Prunes 1544-134=**** lbs..	4 $\frac{3}{4}$	**	**	
	3	Boxes Old Muscatel Raisins	1 $\frac{1}{2}$	*	**	
	3	" New "	2 10	*	**	
	4	" " Layer "	1 15	*	**	
	1	" Cream Tartar, $\frac{1}{4}$ foil. . . .	20 lbs. .39	*	**	
	2	" Yeast-Cakes, 3 doz. ea., . . .	6 doz. .65	*	**	
	25	lbs. Whole Pepper16	*		
	10	" Nutmegs #1	1 10	**		
		Bag				15
	1	Box O. K. Mustard, $\frac{1}{4}$'s	12 lbs. .25	*		
	1	" " " $\frac{1}{2}$'s	12 " .25	*		
		Cartage on all. . . .		1		
				***	**	

NOTE.—1st item.—#4385 is distinguishing number upon the cask; 1544 lbs gross wt.; 134 lbs. tare or weight of cask. 5th item.— $\frac{1}{4}$ foil, put up in $\frac{1}{4}$ lb. packages and wrapped in tin-foil. 10th item.— $\frac{1}{4}$'s, quarter pound packages. 11th item.— $\frac{1}{2}$'s, half pound packages.

(5. Groceries.)

NEW YORK, Aug. 13, 1876.

Messrs. HORTON, CRARY & Co.,

Bought of AUSTIN & NICHOLS.

Terms.—Syrup, 60 days; balance, 30 days.

W. B.	1	Bag ²⁰ Rio Coffee	132	23	30	56
A 599	1	" ²⁰ "	131	21 $\frac{1}{2}$	**	**
	1	Bbl. ²⁵ Roa Java Coffee	121 21	100	25 $\frac{1}{2}$	**
	2	" ⁵⁰ " Rio "	112—22 109—20	221 42	***	24
H. R.	1	Case Conc Lye	*		5	50
Union.	2	Boxes Yeast Cakes, ea. 3 doz.	*	65	*	**
	25	lbs. Spice, Bag 20¢		15 $\frac{1}{2}$	*	**
	5	Mats Cassia		21 $\frac{1}{2}$	*	**
A. & N.	1	Keg Gr. Mustard		50	35	**
	10	lbs. White Glue		40	*	**
			257—20 269—20	**** **		
A. & N.	5	Bbls. X. C. Sugar	256—21 253—18 253—20	11 $\frac{3}{4}$	***	**
2134	1	" W. D. Syrup	47 $\frac{1}{2}$	**	60	**
2114	1	" C. D. "	45 $\frac{1}{2}$	**	50	**
		Ctg.			1	50
					\$***	**

NOTE.—The letters and numbers on the margin correspond with the distinguishing marks and numbers on the bags, cases, boxes, barrels, kegs, etc. The small figures at the right of the words "bag" and "bbl." are the prices of the same. 3rd item.—121 lbs. gross wt., 21 lbs. tare, 100 lbs. net wt. 4th item.—112 and 109, gross weights; 22 and 20, tare; 221, total gross weight; 42, total tare. 11th item.—1st column, gross weight; 2nd column, tare. 12th item.— $\frac{1}{2}$, $\frac{1}{2}$ gallon allowance for leakage.

(6. Provisions.)

CLEVELAND, O., Oct. 9, 1876.

Messrs. L. C. MAGAW & SON,

Bought of J. P. ROBISON & Co.

Terms Cash.—No goods sold on 30 days.

10	Bbls S. M. Pork	17 ⁰⁰	***		
5	" Mess Beef	10 ⁷⁵	**	**	
5	" Hams 90 ^a 1376 ^b —98 ^c	***d	14 ^c	***	**
3	" Should's 58 744—57	**	9 ^c	**	**
1	" Dr. Beef 33 241—22	**	14 ^c	**	**
1	Tc. Lard 406—63	**	11 ^c	**	**

^a No. of pieces. ^b Gross weight. ^c Tare, or weight of barrel or tierce. ^d Net wt.

(7. Books.)

CHICAGO, ILL., May 7, 1878

Mr. JOHN BERWOLD,

Bought of HADLEY BROS.

Terms Cash.

12	Randall's Arithmetics, Part 160	7	20		
18	" " " 250	*			
24	Smith's Primers (paper)06	*	**		
36	" Spellers22	*	**		
18	" 2d Readers45	*	**		
12	" 3d "70	*	**		
6	" 4th " . . .	1.15	*	**		
6	" 5th " . . .	1.35	*	**		
6	Doz. Brown's Copy Books . . .	1.80	**	**		
			**	**		
	Less 33 $\frac{1}{3}$ % . . .		**	**	**	**
6	Jones' Geographies #135	2	10		
6	" " 263	*	**		
6	" " 3 . . .	1.10	*	**		
6	" " 4 . . .	2.00	**			
			**	**		
	Less 25% . . .		*	**	**	**
3	Boxes Chalk Crayons . . .	18				**
3	Doz. Blank Copy Books50			*	**
					\$**	**

(8. Hosiery.)

Claims for Damages or Errors must
be made on receipt of Goods.

NEW YORK, June 28, 1880.

Messrs. JOHN FORD, SONS & Co.,

Bought of JAMES TALCOTT.

Net 30 Days. Note to your own order payable at a Bank in New York City.

1789	35	Doz. 458 ^b Mixed $\frac{1}{2}$ Hose80	28		
	25	" 132 Fancy " "80	*		
	12	" 853 Col'd " " . . .	1.00	**		
	18	" 759 Fancy " "75	**	**	
	16	" 716 " " " . . .	1.10	**	**	
	6	" 436 Fch mx $\frac{1}{2}$ " "90	*	**	
	22	" Job Misses " "75	**	**	\$**** **

^a Number on box or case.^b Manufacturer's distinguishing numbers on goods (stock numbers).

(9. Dry Goods.)

NEW YORK, *March 20, 1879.*

Messrs. FIELD, LEITER & Co.,

Bought of H. B. CLAFLIN & Co.

Terms Cash in 30 days less 5%, or 4 months' note delivered within 30 days, and payable at Bank in New York exchange.

287	1	Bale Boott M. Brown	800 ^a	06 ^{3b}	54	
803	1	" Continental C. do.	800	07 ¹	**	
336	1	" Pequot A. 36 in.	967	07 ²	**	**
129	1	" Great Falls E.	1111	07 ¹	**	**
159	1	" Atlantic H.	1038	07 ³	**	**
217	1	" Pepperell 600 Drill	622	07 ³	**	**
250	1	Case Blackstone A. A.	1649	07 ¹	***	**
651	1	" Dwight Anchor	1139	09	***	**
298	1	" Great Falls Q.	1492	08	***	**
165	1	" Pearl River Ticking	708	15 ²	***	**
		Cooperage				75
					***	**

^a Number of yards in each bale or case.^b Price per yard.

(10. Dry Goods.)

NEW YORK, *March 23, 1878*

Messrs. DAVIDGE, LANDFIELD & Co.,

Bought of TEFFT, WELLER & Co

2	Naumkeag Bleached Jean	48 ³ 47 . .	95	09	8	55
4	Roll Cambric	46 46 40 46	***	05 ²	*	**
1	Lowell ¹⁰ / ₄ Brown	45 ³ 45 45 ¹ 58 ¹	38	14 ²	*	**
5	New Market N.	46 ²	***	06 ¹	**	**
2	Champion Cheviot	48 ¹ 50 ¹ . .	***	09	*	**
2	Otis B. B. Dark Stripe	57 ³ 57 ¹ . .	***	10	**	**
1	Hamilton 30 in. Tick	58 ³	48 ³	11 ²	*	**
2	Thorndike C.	61 . .	***	08 ²	**	**
2	Wamsutta C. Bleached	58 ¹ 68 ¹ . .	***	12	**	**
8	Andros L.	52 52 49 51 ¹ 51 ² 51 ³ 51 52 ²	***	07 ³	**	**
1	Pepperell ¹⁰ / ₄		36 ³	22	*	**
	Cooperage				1	25
					***	**

^a Number of yards in the several pieces.

(11. Dry Goods.)

NEW YORK, *March 20, 1878.*

Messrs. JORDAN, MARSH & CO.,

Bought of E. S. JAFFRAY & CO.

Job,	8	Cases Gordon Fancy							
J. U.		\$4561	2810						
S. B. R.		4157	2902 ¹						
H. Z.		3473	2787 ²						
S. J. L.		4224	2880 ²						
G. Q.		2777	2821 ¹						
J. B.		3504	2842 ²						
J. Z.		3970	2883 ¹						
J. H.		4198	2863 ¹	*****	.05	****	**		
			Less 5%			**	**	****	**

(12. Dry Goods.)

*Book 174, Page 148.*NEW YORK, *March 30, 1878.*MR. JAMES MORGAN, *Milwaukee, Wis.,**Bought of H. B. CLAFLIN & CO.*

Terms: Net 60 Days, or 1% discount in 30 days, or 2% discount in 10 days, N. Y. Funds. No Exchange allowed.

246	53	Pc's Gordon Prints (Job)							
		21 ² 48 ² 38	41 ¹ 48 ² 48 ³ 37 ² 48	48					
		44	49 ² 44 ³ 48 ² 49 ² 49 ³ 49 ² 42	56					
		48 ² 49 ¹ 28 ² 49 ¹ 49	48 ³ 49 ¹ 28	48 ³					
		37	33 ² 49 ² 52	33 ³ 40	48	49 ¹ 49 ¹			
		24	48 ² 48 ² 52	48 ³ 49	47 ² 48 ¹ 48 ²				
		49 ¹ 49 ² 48 ³ 48 ² 48 ² 43 ² 49 ¹ 49 ²				*****			
226	54	Pc's Do.							
		48 ³ 48	49	42	22 ¹ 49 ¹ 49	48 ² 53 ²			
		48 ² 47 ³ 48 ³ 48 ² 49	44	49	49 ² 48 ²				
		49 ² 49	49	48 ² 47 ³ 47	48 ² 49 ¹ 56				
		50 ² 49 ¹ 41 ¹ 48 ¹ 50	27 ¹ 49	48 ² 48 ³					
		21 ³ 29 ¹ 51 ³ 46 ³ 48 ² 48 ² 28 ² 48 ² 49 ¹							
		49 ² 45 ² 47	48 ² 40 ² 50 ¹ 39 ² 48 ² 46 ¹			*****			
247	54	Pc's Do.							
		30 ² 49 ² 42	49 ² 32	48	46	48 ² 46 ²			
		42 ³ 47 ² 22 ¹ 33	46	48	49 ² 48 ² 48				
		42	42	48	28	48 ¹ 49 ² 48 ² 49	49		
		49 ² 48 ² 28 ² 49 ² 43	49 ¹ 48 ² 49 ² 48						
		38 ² 29	25	26 ³ 49 ¹ 49 ³ 49 ¹ 49	48 ²				
		34 ³ 48 ³ 45	49	49 ¹ 49 ² 48 ¹ 36	48	****			
						****	.04 ²	****	**

(13. Tinware.)

ROCHESTER, N. Y., Oct. 16, 1880.

Messrs. MCCARTHY & REDFIELD,

Bought of JOHN H. HILL.

Terms 60 days. If paid in 10 days 2 per cent. discount.

2	Doz. #21 Pieced Dish Pans . .	8.25	**	**		
1½	" 9 in. Wash Boilers . .	36.00	**			
6	" Pieced Bread Pans 3 × 9 × 3	2.00	**			
1	" #5 Pieced Covered Pails		2	50		
3	" #13 " Cups90	*	**		
1	" #15 " Dippers . .		1	25		
2	" #25 " . . .	1.75	*	**		
6	Nests #26 Flaring Pails . .	1.14	*	**		
			**	**		
	Less 20 & 12½% .				**	**
1	Doz. Champion Nutmeg Graters				1	75
1	" Nests #4 Fancy Cov'd Pails		6	00		
1	" #4 Burnished Tea Pots .		6	75		
			**	**		
	25 & 12½% .				*	**
1	" #9 Pudding Pans . .		3	50		
2	" #10 " " . . .	4.25	*	**		
1½	" #20 Pressed Kettles . .	5.50	*	**		
1	" #22 " " . . .		7			
			**	**		
	37½% .				**	**
6	2 qt. En'd Sauce Pans63	*	**		
6	3 " " " "73	*	**		
			*	**		
	40% .				*	**
	.75 .90					
6	Enameled Kettles Ea. 4—5 qt.		*	**		
	1.10 1.30					
12	" " Ea. 6—8 qt.		**	**		
			**	**		
	60% .				**	**
7	W. H. Tea Kettles95	*	**		
	45% .				*	**
					**	**

NOTE.—19th item.—4 qt. size, 75 cents each; 5 qt. size, 90 cents each.

COMMISSION AND BROKERAGE.

254. **Commission** or **Brokerage** is an allowance made to an agent for transacting business for another ; as, the sale or purchase of property, the collection or investment of money, etc.

An additional percentage is usually charged by commission merchants for guaranteeing the payment of sales made on credit.

255. The party who transacts the business is called a **Commission Merchant**, or **Broker** ; and the one for whom he acts is called a **Principal**.

NOTES.—1. Commission Merchants usually have possession of the merchandise to be sold, and make sales and purchases in their own name.

2. Brokers do not have possession of the merchandise bought or sold, and generally make contracts in the names of those who employ them, and not in their own. They simply effect bargains and contracts.

256. A **Consignment** is a quantity of merchandise sent by one party to another. The party who sends it is called the **Consignor** ; and the party to whom it is sent, the **Consignee**.

257. The **Net Proceeds** of a consignment is the balance due the consignor after all charges have been deducted.

The whole amount realized from a sale is called the *gross proceeds*. The commission is usually a certain per cent. of this amount.

258. An **Account Sales** is a detailed statement rendered by the Commission Merchant to the Consignor, showing the sales of certain goods, the charges or expenses attending the same, and the difference or net proceeds.

The charges embrace freight, cartage, inspection, advertising, storage, insurance, commission and guarantee, etc.

259. An **Account Purchase** is a detailed statement rendered by the Commission Merchant to his Principal, showing the cost of certain goods, and the charges or expenses attending the purchase.

260. Commission or brokerage is usually computed at a certain per cent. of the amount realized or invested, or of the amount involved in the transaction. In such cases the general principles of percentage are applied.

NOTES.—1. In buying and selling stocks, bonds, etc., the par value, and not the actual value, is taken as the base.

2. The commission for buying and selling some kinds of merchandise is usually computed at a certain price per unit of weight or measurement ; as, grain per bushel, cotton per bale, etc.

EXAMPLES.

261. 1. A commission merchant sold goods to the amount of \$864; what was his commission at $2\frac{1}{2}\%$?

2. A salesman sells goods at a commission of $2\frac{1}{2}\%$; what must be his sales, that he may have a yearly income of \$5000?

3. What is the brokerage for selling 850 bales of cotton at the rate of \$25 per 100 bales?

4. A lawyer collected a note of \$2375; how much did he pay to the owner of the note, his commission being 5%?

5. My agent in Chicago purchases for me 600 barrels of flour at \$3.75 per barrel; how much do I owe him, his commission for purchasing being 2%?

6. An officer collected \$17850, and deposited \$17493 in the Treasury, retaining the remainder as his commission. What was the rate per cent. of the commission?

7. Sent to a commission merchant in Toledo \$2080.80 to invest in flour, his commission being 2% on the amount expended; how many barrels of flour would be purchased at \$4.25 per barrel?

8. A commission merchant sells merchandise amounting to \$3325; how much is paid to the consignor of the merchandise, the charges being, for transportation \$117.50, for advertising \$10, for storage \$15, for commission $2\frac{1}{2}\%$?

9. My agent buys for me 1187.76 cents wheat at \$2.123 per cental. What is his commission at $\frac{1}{2}$ per cent.?

10. A commission merchant purchased for me 928 (28 *lbs.*, see Art. 168) bushels of clover seed at \$8.55 per bushel. How much should I send to him in settlement, if his commission for purchasing is 1 per cent.?

11. A broker buys 8375 *lbs.* of leather at 26¢ per pound. What is his brokerage at $\frac{3}{4}\%$, and what is the net amount received by the seller, the brokerage being paid by him?

12. A freight broker procures transportation for 375 tons of merchandise at \$3.50 per ton; what is his brokerage at 5%?

13. A collector deposits \$28117, retaining 3% on the whole amount collected. What amount did he collect and what was his commission?

14. A lawyer, collecting a note at a commission of 5% thereon, received \$6.25; what was the face of the note?

15. An agent sold 6 mowing-machines at \$120 each, and 12 at \$140 each. He paid for transportation \$72, and, after deducting his commission, remitted \$2208 to the manufacturer. What was the % of his commission?

16. A merchant instructs his agent in Cincinnati to buy pork to the amount of \$5000. The charges on the pork being \$16, and the agent's commission $1\frac{1}{2}\%$, how much must be remitted to settle the bill?

17. What are the net proceeds of the sale of 12372 pounds of leather at 22 cents per pound, the charges being \$31. and a commission of $2\frac{1}{2}\%$ being paid for selling and $2\frac{1}{2}\%$ for guaranteeing payment?

18. A real estate agent, who charged $2\frac{1}{2}\%$ for making the sale, paid to the owner of a house and lot \$42412.50; what was the value of the property?

19. A commission merchant sells 240 *bb.* of potatoes at \$3.75 per *bb.*, and 260 *bb.* at \$3.60 per *bb.* How much is due the consignor, the commission being $12\frac{1}{2}$ cents per barrel?

20. What were the proceeds in currency of \$2611.06 gold, @ 1.06 $\frac{1}{8}$, commission for selling $\frac{1}{8}\%$?

21. A, having a claim against the government of \$10970, agreed to pay an agent 8 per cent. of the amount collected. But the amount collected was 22 per cent. less than the amount of the claim. How much was received by A?

22. A commission merchant sold 300 bales of cotton, averaging 462 lb. to the bale, at 15.7¢, his commission being 25¢ per bale, and the charges \$161. He purchased for the consignor dry goods amounting to \$2576.37, charging a commission of 1½%. How much was still due the consignor?

Copy and extend the following accounts:

(23. Account Sales.)

NEW YORK, Oct. 19, 1880.

Sold for account of A. W. RANDOLPH & Co.,

By DAVID DOWS & Co.

1880.									
Sept.	12	100 Bbls. "Sunshine"	- - -	5.75	***				
"	18	125 " "Idaho"	- - -	6.25	***	**			
"	30	150 " "Sunshine"	- - -	6.	***				
Oct.	14	125 " "Idaho"	- - -	6.50	***	**	****	**	
<i>Charges.</i>									
Sept	10	Transportation 500 Bbls. @ 27¢	- - -		***				
"	10	Cartage 400 " @ 5¢	- - -		**				
Oct.	19	Storage 400 " @ 3¢	- - -		**				
"	19	Commission and Guarantee 5%	- - -		***	**	****	**	
Net proceeds				- - - - -			****	**	

(24. Account Purchase.)

TOLEDO, O., Mar. 6, 1887

Purchased by A. L. BACKUS & SONS.

For account and risk of L. A. & W. B. SHAW.

9	Bags "Montank"	- - - - -	.21	*	**		
227	Bu. Mammoth Clover Seed	- - - - -	900	**	**		
928	" Clover Seed	- - - - -	855	**	**		
931	" Timothy Seed	- - - - -	175	**	**	****	25
<i>Charges.</i>							
	Cartage	- - - - -			25		
	Commission 1%	- - - - -		*	**	*	*c
	Charge your ac	- - - - -				****	**

PROFIT AND LOSS.

262. Profit and Loss treats of the gains (profits) and losses which arise in business transactions.

The profit or loss is always estimated on the cost price, or the amount invested. Discounts are reckoned on the market or asking price. (See Art. 249.)

263. The difference between the cost of goods and the price at which they are sold is a **profit** or a **loss**,—profit if the selling price is the greater, loss if the cost is the greater.

EXAMPLES.

264. 1. A man purchased a horse for \$250, and sold it at a gain of 16%. What was the gain? (Gain = $.16 \times \text{cost}$.)

2. Sold goods that cost \$325 at an advance of 12%; what was the selling price? (Gain = $.12 \times \text{cost}$, and selling price = cost + gain; or, selling price = $1.12 \times \text{cost}$.)

3. Bought a farm for \$3600, and sold it at an advance of 25%; what was the gain?

NOTE.—If, as in the above example, the rate per cent. is an aliquot part of 100, it is more convenient to use the equivalent fraction (247) Thus, $25\% = .25 = \frac{1}{4}$; gain = $\frac{1}{4}$ of cost.

4. Cloth is bought at \$6 per yard, and sold at a loss of 20%. Find the selling price. (Selling price = $\frac{4}{5}$ of cost.)

5. Bought a house for \$3475; at what price must it be sold to gain 36%?

6. Purchased flour at \$6.25 per barrel; at what price must it be sold to gain 20%?

7. If I buy hats at \$27 per dozen, at what price must they be sold apiece to gain $33\frac{1}{3}\%$?

8. A factory which cost \$8775 was sold at a gain of 16%. What was received for it?

9. If silk costs \$1.68 per yard, and is sold at an advance of $12\frac{1}{2}\%$, what is the profit per yard?

10. A merchant purchased goods to the amount of \$8735, and sold them at a loss of 12% ; what was his loss?

11. Bought 125 barrels of flour for \$600. If sold at an advance of 15% , what was the profit per barrel?

12. A lot of dry goods was sold at an advance of 18% . If the gain was \$436.50, what was the cost? (Gain = $.18 \times \text{cost}$; hence, $\text{gain} \div .18 = \text{cost}$.)

13. A farm was bought for \$7200, and sold at a gain of \$900; what was the gain per cent.? (Gain = $\text{gain} \% \times \text{cost}$; hence, $\text{gain} \% = \text{gain} \div \text{cost}$.)

14. A man paid for merchandise \$875, and sold it for \$1015; what per cent. did he gain?

15. A man paid for merchandise \$1015, and sold it for \$875; what per cent. did he lose?

16. Find the rate $\%$ of profit on goods bought for \$324 and sold for \$364.50.

17. A painting was sold for \$2343, at a gain of 32% ; what was the cost? [Selling price = $1.32 (100\% + 32\%) \times \text{cost}$; hence, $\text{cost} = \text{selling price} \div 1.32$.]

18. Find the cost of goods sold at an advance of $12\frac{1}{2}\%$, being a profit of \$76.

19. How much was paid for a farm sold for \$9878, at 12% below cost?

20. What is the profit on iron sold for \$4520, at an advance of 13% on cost?

21. What is the selling price of tea which cost 32 cents per pound and is sold at a profit of $37\frac{1}{2}\%$?

22. Sold drugs for \$168, at an advance of 15% ; what was the profit?

23. A merchant sold for \$2576 a lot of dry goods for which he paid \$3360. What was the per cent. of loss?

24. A speculator sold two building lots for \$4800 each. On one he gained 20% , and on the other he lost 20% . Did he gain or lose, and how much?

25. If, by selling tea at $47\frac{1}{2}$ cents per pound, I lose 5%, at what price must I sell it to gain 15%?

26. By selling goods for \$126, I lose 16%; what per cent. would I gain or lose if I sell them for \$168?

27. A merchant's price is 25% above cost price. If he allows a customer a discount of 12% on his bill, what per cent. profit does he make?

28. If cloth, when sold at a loss of 25%, brings \$5 per yard, what would be the gain or loss per cent. if sold at \$6.40 per yard?

29. Goods that cost \$168 are sold at an advance of 25%; what is the selling price?

30. What must be the asking price of goods costing \$32, that I may deduct 20% from it, and still gain 25% on the cost?

31. Sold a horse at a gain of $33\frac{1}{3}\%$, and with the proceeds purchased another horse, which I sold for \$120 at a loss of 20%. What was the gain or loss?

32. What must ribbon be sold per yard so as to gain 20%, if $22\frac{1}{2}$ yards cost \$6.75?

33. Books are purchased at a discount of 30% from the list price (249). What is the gain per cent. by selling at the list price?

34. A mixture is made of 1 gallon of wine at 50 cents a gallon, 3 at 90 cents, 4 at \$1.20, and 12 at 40 cents. What per cent. would be gained by selling the mixture at \$1.60 a gallon?

35. A merchant purchases goods at a discount of 25% from the list price. What per cent. is gained by selling at the list price? What per cent. if goods are purchased at a discount of $33\frac{1}{3}\%$? 35%? 25% and 5%? 20% and $12\frac{1}{2}\%$?

36. A merchant's retail price for boots is \$4.75 per pair, by which he makes a profit of $33\frac{1}{3}\%$. He sells to a wholesale customer at a discount of 20% from the retail price. What per cent. does he gain or lose, and what does he receive per pair?

INTEREST.

DEFINITIONS.

265. **Interest** is a sum charged for the use of money, or its equivalent; or more strictly speaking, it is *the use of money*, or the service rendered in its use.

266. The **Principal** is the sum for the use of which interest is charged.

267. The **Rate** is the per cent., or number of hundredths, of the principal, charged for its use for a certain time, usually for one year (per annum). When no time is mentioned with the rate in the contract, a year is understood.

268. The **Amount** is the sum of the principal and interest.

If \$1000 is loaned for one year at 6% per annum, \$60 would be the interest, \$1000 the principal, and \$1060 the amount.

269. **Simple Interest** is interest on the principal only for the full time.

270. **Compound Interest** is interest not only on the principal, but on the interest also after it becomes due.

If \$1000 is loaned Jan. 1, 1881, for 2 years, the amount due Jan. 1, 1883, at 6% simple interest, would be \$1000 (Principal) plus \$120 (Simple Interest), or \$1120. At compound interest the amount due Jan. 1, 1882, would be \$1060 (\$1000 + \$60); the amount due Jan. 1, 1883, would be \$1060 plus \$63.60 (6% of \$1060), or \$1123.60. The simple interest for 2 years would be \$120; the compound interest for the same time, \$123.60. When the word interest is used alone, simple interest is understood.

271. Legal Interest is the interest according to the rate per cent. fixed by law for cases in which the rate per cent. is not specified. By special agreement between parties in certain States, interest may be received at a rate higher than the legal rate

272. Usury is the taking of a higher rate of interest than that allowed by law. A person taking usury is liable to certain penalties differing in the several States.

273. The following table shows in the first column the legal rate of interest when no rate is specified in the contract, and in the second column the maximum rate allowed by law.

State or Territory.	Rate.		State or Territory.	Rate.	
Alabama.....	8%	8%	Mississippi.....	6%	10%
Alaska (Ter.)	Missouri.	6%	10%
Arkansas.....	6%	10%	Montana (Ter.)...	10%	Any
Arizona (Ter.) . . .	10%	Any	Nebraska.....	7%	10%
California....	10%	Any	Nevada	10%	Any
Colorado.....	10%	Any	New Hampshire	6%	6%
Connecticut.....	6%	6%	New Jersey.....	6%	6%
Dakota (Ter.) . . .	7%	12%	New Mexico (Ter.)	Any
Delaware.....	6%	6%	New York	6%	6%
Florida	8%	Any	North Carolina...	6%	8%
Georgia.....	7%	8%	Ohio.	6%	8%
Idaho (Ter.).....	10%	18%	Oregon	10%	12%
Illinois	6%	8%	Pennsylvania.....	6%	6%
Indian (Ter.)	6%	Any	Rhode Island....	6%	Any
Indiana.....	6%	8%	South Carolina...	7%	7%
Iowa	6%	10%	Tennessee.....	6%	6%
Kansas	7%	12%	Texas.....	8%	12%
Kentucky.....	6%	6%	Utah (Ter.).....	Any
Louisiana.....	5%	8%	Vermont.....	6%	6%
Maine.....	6%	Any	Virginia.....	6%	6%
Maryland.....	6%	6%	Washington (Ter.)	10%	Any
Massachusetts . . .	6%	Any	West Virginia...	6%	8%
Michigan	7%	10%	Wisconsin	7%	10%
Minnesota.....	7%	10%	Wyoming (Ter.)...	12%	Any

274. Interest for Parts of a Year.—Although many of the States have rigid laws in regard to the rate per cent. to be charged per annum, few of them specify on what basis interest should be reckoned for a period of time less than a year. The following methods are in common use:

1. Finding the time in months and days (Compound Subtraction, Art. 205, 1), and regarding the months as twelfths of a year, and the days as thirtieths of a month or 360ths of a year. This method, although implied by the general interest laws* of the State of New York, is not uniform, since it allows the same interest for February with its 28 days as for March with its 31 days. Its results are sometimes greater and sometimes less than those of accurate interest.

2. Finding the exact time in days (205, 2) and regarding the days as 360ths of a year. Since a day is $\frac{1}{360}$ of a year, this method produces too great a result. It is however used by merchants, brokers, and bankers generally, and by many banks † in discounting notes. 6% by this method is equivalent to $6\frac{1}{12}\%$ accurate interest.

3. **Accurate Interest.**—Finding the exact time in days (205, 2) and regarding the days as 365ths of a year. This method is used by the United States government, and by some merchants and banks; but, on account of its inconvenience when interest tables are not used, it is not generally adopted.

NOTES.—1. By the first method, the time from July 10 to Sept. 10, would be 2 months, and the interest would be $\frac{2}{12}$ or $\frac{1}{6}$ of the interest for one year. On \$10000 at 6% for 2 months, the interest would be \$100 ($\frac{1}{6}$ of .06 of \$10000).

* "For the purpose of calculating interest, a month shall be considered the twelfth part of a year, and as consisting of thirty days; and interest for any number of days less than a month shall be estimated by the proportion which such number of days shall bear to thirty." (R. S., page 1165.)

† According to the banking laws of the State of New York, banks are authorized in discounting notes to charge interest in advance for the exact number of days which the note has to run (Ch. XVII, Title 2, § 300).

2. By the second method, the interval between the same dates would be 62 days, and the interest would be $\frac{62}{360}$ of the interest for one year. On \$10000 at 6% for $\frac{62}{360}$ of a year, the interest would be \$103.33 ($\frac{62}{360}$ of .06 of \$10000).

3. By the third method, the interval between the same dates would be 62 days as in the second method, and the interest would be $\frac{62}{365}$ of the interest for one year. On \$10000 at 6% for $\frac{62}{365}$ of a year, the interest would be \$101.92 ($\frac{62}{365}$ of .06 of \$10000).

4. The difference between ordinary interest and accurate interest for the same number of days is $\frac{1}{3}$ of the former, or $\frac{1}{2}$ of the latter (292). Thus in the above example, the difference between the results, \$1.41 (\$103.33-\$101.92), is $\frac{1}{3}$ of \$103.33, or $\frac{1}{2}$ of \$101.92.

5. Unless the words "Accurate Interest" are used, all computations in this book are made on the basis of 360 days to the year.

275. Interest is an application of percentage, the element of time being introduced. Therefore the four elements or parts in interest are the Principal (the Base), the Rate, the Interest (the Percentage), and the Time; any three of which being given, the other may be found.

276. To find the interest for any number of years and months.

Ex. What is the interest and amount of \$324, for 2 yr. 3 mo., at 8%?

OPERATIONS.			
\$324	Principal.	Or	\$324
.08			.18
<hr/>			<hr/>
25.92	Interest for 1 yr.		25.92
<hr/>			<hr/>
24			324
<hr/>			<hr/>
648			58.32
<hr/>			<hr/>
5184			324.
<hr/>			<hr/>
58.32	Interest for 24 yr.		\$382.32
<hr/>			
324	Principal.		
<hr/>			
\$382.32	Amount for 24 yr.		

ANALYSIS.—At 8%, the interest of \$324 for 1 year is .08 of \$324 (the Principal), or \$25.92. If the interest of \$324 for 1 year at 8% is \$25.92, for 2 yr. 3 mo. (24 yr.), it is 24 times \$25.92, or \$58.32. The amount is \$324 plus \$58.32, or \$382.32.

277. RULE.—*To find the interest, multiply the principal by the rate per cent. expressed decimally, and that product by the number of years, and the months as a fraction of a year.*

To find the amount, add the principal to the interest.

NOTES.—1. When the rate per month is given, apply the same rule, *i. e.*, multiply the principal by the rate per month expressed decimally, and that product by the number of months.

2. Instead of multiplying by the rate and time separately, the process may be shortened by multiplying the principal by the product of the rate and time. In the above example, multiply \$324 by .18 ($2\frac{1}{2} \times .08$).

EXAMPLES.

278. Find the interest of

1. \$875 for 2 y., at 7%. 6. \$816.40 for 5 y. 3 m., at 5%.
2. \$642.50 for 3 y., at 6%. 7. \$1275 for 7 y. 4 m., at 6%.
3. \$1010 for 6 y., at 8%. 8. \$2789.40 for 3 y. 2 m., at $4\frac{1}{2}\%$.
4. \$3010.75 for 3 y., at 7%. 9. \$456.75 for 4 y. 6 m., at 5%.
5. \$3745.80 for 4 y., at 6%. 10. \$10180 for 3 y. 1 m., at 10%.

NOTE.—In the following examples find the time by Compound Subtraction.

11. What is the interest of \$6488 from May 3, 1879, to Sept. 3, 1881, at 7%?

12. What is the amount of \$396.60 from Aug. 16, 1880, to Dec. 16, 1882, at 8%?

13. Find the interest of \$864.30 from Jan. 1, 1881, to June 1, 1883, at 4%.

14. Compute the interest of \$250.75 from Nov. 20, 1882, to July 20, 1884, at $4\frac{1}{2}\%$.

15. Loaned on interest, New York, Dec. 16, 1880, \$1739.75 (no rate specified); what amount should I receive, June 16, 1881?

16. In settling with a merchant Oct. 3, 1882, I gave my note for \$254.60, at 7%; what must be paid Aug. 3, 1883?

279. To find the ordinary interest (360 days to the year) for any rate and time.

280. 60-day Method at 6%.—6% for 12 months or 1 year, is equivalent to 1% for 2 months (60 days), or $\frac{1}{6}$ of one year. 1% of any amount is readily ascertained by placing the point two places to the left. Hence the interest of any sum at 6% per annum for 2 months, or 60 days, may be found by placing the point two places to the left.

NOTE.—It will be found advantageous to use a perpendicular line as a separatrix in solving examples by this method. All necessity for pointing off will then be dispensed with, and confusion prevented.

Ex. What is the interest of \$864 for 1 yr. 10 mo. 15 da. at 6%?

OPERATION.		ANALYSIS.—The interest of \$864
\$8	64 = int. for 60 da.	at 6% for 2 mo. is \$8.64. For 1 yr.
	11	10 mo. (22 mo.), it will be 11 times
95	04 = int. for 22 mo.	\$8.64, or \$95.04. If the interest for
2	16 = " " 15 da.	60 da. is \$8.64, for 15 da. ($\frac{1}{4}$ of 60),
		it will be $\frac{1}{4}$ of \$8.64, or \$2.16. Hence
\$97	20 = required int.	the interest for the given time will
		be \$95.04 plus \$2.16, or \$97.20.

Ex. What is the interest of \$1732.80 for 2 yr. 9 mo. 23 da., at 7%?

OPERATION.	
\$17	32.80 = int. for 2 mo. or 60 da.
	16 $\frac{1}{2}$
8	6640
103	9680
173	280
5	776 = int. for 20 da.
	866 = " " 3 "
6) 292	554 = " " given time at 6%.
48	759 = " " " " 1%.
\$341	313 = " " " " 7%.

ANALYSIS.—The interest for 2 *mo.*, forming the basis, is \$17.328. Multiply this by $16\frac{1}{2}$, to find the interest for 33 *mo.* (2 *yr.* 9 *mo.*). As 23 is not an aliquot part of 60, take 20, which is $\frac{1}{3}$ of 60, and 3, which is $\frac{1}{20}$ of 60. Divide the basis, which is the interest for 60 *da.*, by 3, to find the interest for 20 *da.* (\$5.776); and the same sum by 20, to find the interest for 3 *da.* (\$.0866). By adding these various sums, we have the interest for the given time at 6% (\$292.554). To this result add $\frac{1}{4}$ of itself, which is the interest for the given time at 1%, and the required interest is obtained (\$341.31).

Ex. What is the interest of \$1236 for 80 *da.*, at 6%?

OPERATION.		ANALYSIS.
\$12	36 = int. for 60 <i>da.</i>	at 6% for 60 <i>da.</i> is \$12.36. If the interest for 60 <i>da.</i> is \$12.36, for 20 <i>da.</i> ($\frac{1}{3}$ of 60), it will be $\frac{1}{3}$ of \$12.36, or \$4.12. Hence for 80 <i>da.</i> it will be \$12.36 plus \$4.12, or \$16.48.
4	12 = " " 20 <i>da.</i>	
\$16	48 = " " 80 <i>da.</i>	

281. Aliquot Parts of 60.— $1 = \frac{1}{60}$; $2 = \frac{1}{30}$; $3 = \frac{1}{20}$; $4 = \frac{1}{15}$; $5 = \frac{1}{12}$; $6 = \frac{1}{10}$; $10 = \frac{1}{6}$; $12 = \frac{1}{5}$; $15 = \frac{1}{4}$; $20 = \frac{1}{3}$; $30 = \frac{1}{2}$.

NOTES.—1. To divide by 10, place the figures of the basis one place to the right.

2. To divide by 20, 30, or 60, divide by the first figure and write the figures of the quotient one place to the right.

282. If the number of days given is other than any of the above, which are aliquot parts of 60, it will need to be so separated that the component parts will be aliquot parts of 60.

Numbers not aliquot parts of 60, with best divisions: $7 = 6 + 1$; $8 = 6 + 2$; $9 = 6 + 3$; $11 = 6 + 5$, or $10 + 1$; $13 = 10 + 3$; $14 = 12 + 2$; $16 = 10 + 6$; $17 = 12 + 5$, or $15 + 2$; $18 = 12 + 6$. (The interest for 18 days may be found by multiplying the basis by 3, and placing the figures of the product one place to the right); $19 = 15 + 4$, or $10 + 6 + 3$; $21 = 15 + 6$; $22 = 20 + 2$ ($2 = \frac{1}{10}$ of 20); $23 = 20 + 3$; $24 = 12 + 12$ (Or multiply by 4 and place the figures of the product one place to the right); $25 = 20 + 5$ ($5 = \frac{1}{4}$ of 20); $26 = 20 + 6$; $27 = 12 + 15$; $28 = 12 + 12 + 4$ ($4 = \frac{1}{3}$ of 12), or $20 + 6 + 2$; $29 = 12 + 12 + 5$, or $20 + 6 + 3$.

283. RULE.—Draw a perpendicular line two places to the left of the decimal point; the result will be the interest at 6% for 2 months, or 60 days, the dollars being on the left, and the cents on the right of this line. Multiply this result by one-half the total number of months. To this product, add that proportion of the interest for 60 days, which the given number of days is of 60.

284. The interest for any other rate may be found from the interest at 6% as follows: At 1%, divide by 6; at $1\frac{1}{2}\%$, divide by 4; at 2%, divide by 3; at 3%, divide by 2; at 4%, subtract $\frac{1}{3}$; at $4\frac{1}{2}\%$, subtract $\frac{1}{4}$; at 5%, subtract $\frac{1}{6}$; at 7%, add $\frac{1}{6}$; at 8%, add $\frac{1}{3}$; at 9%, add $\frac{1}{2}$; at 10%, divide by 6, and multiply by 10 by placing the point to the right one place; at 12%, multiply by 2. At any per cent., divide by 6 and multiply by the rate.

285. Six Per Cent. Method.—At 6%, the interest for one year is .06 of the principal. For one month, $\frac{1}{12}$ of a year, it will be $\frac{1}{12}$ of .06, or $.00\frac{1}{2}$ (.005). For one day, $\frac{1}{30}$ of a month, it will be $\frac{1}{30}$ of .005, or $.000\frac{1}{6}$.

Ex. What is the interest of \$864, at 6%, for 2 yr. 7 mo. 20 da.?

OPERATION.

$$\begin{aligned} 2 \times .06 &= .12 \\ 7 \times .00\frac{1}{2} &= .035 \\ 20 \times .000\frac{1}{6} &= .003\frac{1}{3} \\ &\quad .158\frac{1}{3} \end{aligned}$$

$$\begin{array}{r} \$864 \\ .158\frac{1}{3} \\ \hline 288 \\ 6912 \\ 4320 \\ 864 \\ \hline \$136.800 \end{array}$$

ANALYSIS.—If the interest for 1 yr. is .06 of the principal, for 2 yr. it will be twice .06, or .12. If the interest for 1 mo. is $.00\frac{1}{2}$ of the principal, for 7 mo. it will be 7 times $.00\frac{1}{2}$, or .035. If the interest for 1 day is $.000\frac{1}{6}$ of the principal, for 20 da. it will be 20 times $.000\frac{1}{6}$, or $.003\frac{1}{3}$. Hence the interest for the given time will be $.158\frac{1}{3}$ of the principal (\$864), or \$136.80.

286. RULE.—*Multiply the given principal by the decimal obtained by taking for every year six hundredths, one-half as many hundredths as there are months, and one-sixth as many thousandths as there are days. The product will be the interest at 6%.*

NOTES.—1. In using this method, to multiply by $\frac{2}{3}$, write $\frac{1}{3}$ twice; to multiply by $\frac{5}{6}$, take $\frac{1}{2}$ and $\frac{1}{3}$.

2. The interest at any other per cent. may be found as in Art. 284.

3. The decimal obtained by the above rule, if regarded as cents and mills, expresses the interest of \$1 for the given time at 6%. The interest of \$1 at 6% for 1 year is \$.06; for 1 month, \$.00 $\frac{1}{2}$, or \$.005; for 1 day, \$.000 $\frac{1}{6}$.

287. 6% Method for Days.—This is a modification of the preceding method, and may be applied to any example if the time is reduced to days.

Ex. What is the interest of \$1735 for 173 days at 6%?

OPERATION.

$$\begin{array}{r}
 \$1735 \\
 173 \\
 \hline
 5205 \\
 12145 \\
 1735 \\
 \hline
 6) 300155 \\
 \hline
 \$50.025 +
 \end{array}$$

ANALYSIS.—The interest of \$1735 for 173 days is equivalent to the interest of 173 times \$1735, or \$300155 for 1 day. Since the interest of \$1 for 1 day is $\frac{1}{6}$ of a mill, or .000 $\frac{1}{6}$ of the principal, the interest of \$300155 for 1 day is as many mills as 6 is contained times in 300155, or 50025 mills, or \$50.03.

288. RULE.—*Multiply the principal by the number of days, divide the product by 6, and place the point 3 places to the left. The result will be the interest at 6%.*

NOTES.—1. The interest at any other per cent. may be found as in Art. 284. To find the interest at 3%, divide by 12 instead of 6; at 4%, by 9; at 9%, by 4.

2. If the principal is a multiple of the divisor, time can be saved by performing the division first. Thus, to find the interest of \$1200 for 113 days, divide 1200 by 6 and multiply the quotient 200 by 113, producing 22600. By pointing off three places, the required interest is \$22.60.

EXAMPLES.

289. What is the interest of

1. \$375.60 for 8 mo. 20 da., at 6%?
2. \$1727 for 7 mo. 15 da., at 6%?
3. \$449.38 for 1 yr. 4 mo. 12 da., at 6%? At 7%?
4. \$285 for 1 yr. 5 mo. 10 da., at 6%? At 5%?
5. \$432.65 for 2 yr. 2 mo. 6 da., at 6%? At 8%?
6. \$1235 for 2 yr. 5 mo. 5 da., at 6%? At 4%?
7. \$445.25 for 5 mo. 4 da., at 6%? At 9%?
8. \$1000 for 93 days, at 6%? At 7%?
9. \$2416.60 for 72 days, at 6%? At 7%?
10. \$3210 for 62 days, at 6%? At 8%?
11. \$735 for 75 days, at 6%? At 5%?
12. \$812.45 for 121 days, at 6%? At 4%?
13. \$2440.50 for 97 days, at 6%? At 7%?
14. \$3125 for 38 days, at 6%? At 7%?
15. \$247.50 for 69 days, at 6%? At 5%?
16. \$512.45 for 5 mo. 11 da., at 6%? At 7%?
17. \$1478 for 1 yr. 2 mo. 13 da., at 6%? At 8%?
18. \$2810.60 for 9 mo. 24 da., at 6%? At 5%?
19. \$944.50 for 1 yr. 10 mo. 22 da., at 6%? At 4½%?
20. \$575 for 2 yr. 8 mo. 16 da., at 6%? At 9%?
21. \$1112 for 3 mo. 14 da., at 6%? At 4½%?
22. \$5285 for 1 yr. 6 mo. 21 da., at 6%? At 3%?
23. \$7218 for 11 mo. 18 da., at 6%? At 12%?
24. \$416.75 for 8 mo. 17 da., at 6%? At 7%?
25. \$1235 for 2 yr. 1 mo. 19 da., at 6%? At 8%?
26. \$575.60 for 1 yr. 4 mo. 23 da., at 6%? At 5%?
27. \$2214 for 4 mo. 25 da., at 6%? At 4%?
28. \$6315 for 5 mo. 29 da., at 6%? At 9%?
29. \$4312 for 4 mo. 26 da., at 6%? At 4½%?
30. \$384.30 for 2 mo. 28 da., at 6%? At 3%?
31. \$1296 for 1 yr. 11 mo. 27 da., at 6%? At 12%?
32. \$4375 for 2 yr. 8 mo. 24 da., at 6%? At 5½%?
33. \$1234 for 6 mo. 24 da., at 5%? At 4¾%?

NOTE.—Find the time in the following examples both in months and days, and in exact days (205).

34. \$444.40 from Jan. 13 to Nov. 2, at 4%? At 5½%?

35. \$575.20 from June 5, 1882, to Feb. 4, 1883, at 7%?
At 5%?

36. \$2375 from July 17, 1884, to Nov. 27, 1885, at 6%?
At 3½%?

37. \$3212 from Aug. 24, 1881, to Jan. 20, 1884, at 4%?
At 4½%?

38. \$475.80 from May 12, 1882, to Feb. 1, 1884, at 7%?
At 10%?

39. Find the interest of \$180 for 253 days, at 6%. At 8%.

NOTE.—In many examples, labor can be saved by having the time and principal exchange places. In the above example, the interest of \$180 for 253 days is the same as \$253 for 180 days ($\253×3).

40. Find the interest of \$600 for 173 days at 9%.
At 4%.

41. Find the interest of \$3000 for 111 days at 12%.
At 3%.

42. Find the interest of \$1800 from Jan. 17 to Oct. 2, at 6%. At 4¾%.

43. Find the interest of \$540 from May 11 to Dec. 18, at 5%. At 4½%.

44. If \$9200 is loaned Sept. 18, 1882, at 6%, what is due May 9, 1885? (Time by Compound Subtraction.)

45. What is a banker's gain in 1 year on \$10000 deposited at 6%, and loaned 11 times at 1½% a month?

46. A note for \$1421, with interest after 4 months, at 7%, was given Dec. 1, 1881, and paid Aug. 12, 1883. What was the amount due? (Compound Subtraction.)

47. Nov. 6, 1881, I bought a lot of grain for \$753.20; Dec. 16, I sold a part of it for \$375.60; and, Dec. 31, I sold the remainder for \$411.40. Money being worth 6%, how much did I gain by the transaction?

ACCURATE INTEREST.

290. To find the accurate interest (365 days to the year) for any rate and time. See Art. 274.

Ex. What is the accurate interest of \$865, at 4%, from June 21 to Dec. 13?

OPERATION.

$$\begin{array}{r}
 \$865 \text{ Principal.} \\
 .04 \\
 \hline
 34.60 \text{ Interest for 1 yr.} \\
 175 \\
 \hline
 365 \text{) } 6055.00 \text{ (} 16.59
 \end{array}$$

ANALYSIS.—From June 21 to Dec. 13, there are 175 days. The interest of \$865 for 1 yr., at 4%, is \$34.60. For 175 days, $\frac{175}{365}$ of 1 yr., it is $\frac{175}{365}$ of \$34.60 $\left(\frac{175 \times 34.60}{365} \right)$, or \$16.59.

291. RULE.—*Multiply the principal by the rate per cent. expressed decimally. The result will be the interest for 1 year.*

Multiply the interest for 1 year by the number of days, and divide the product by 365.

NOTES.—1. When the number of days is a multiple of 5, multiply the interest for 1 year by $\frac{1}{5}$ the number of days, and divide the product by 73. In the above example, $\$865 \times .04 \times 35 \div 73 = \16.59 .

2. To find the interest at 5%, multiply the principal by the number of days, divide the product by 73, and point off 2 places. From this result to find the interest at 6%, add $\frac{1}{5}$; $4\frac{1}{2}\%$, subtract $\frac{1}{10}$; 4%, subtract $\frac{1}{5}$.

292. Accurate Interest from Ordinary Interest.—The difference between ordinary interest and accurate interest for 1 day equals the difference between $\frac{1}{360}$ and $\frac{1}{365}$ of a year's interest.

$$\begin{aligned}
 \frac{1}{360} - \frac{1}{365} &= \frac{365 - 360}{365 \times 360} = \frac{5}{365 \times 360} = \frac{5}{365} \text{ of } \frac{1}{360} = \frac{1}{72} \\
 \text{of } \frac{1}{360} \cdot \frac{5}{365 \times 360} &= \frac{5}{360} \text{ of } \frac{1}{365} = \frac{1}{72} \text{ of } \frac{1}{365}.
 \end{aligned}$$

The difference between the two methods is $\frac{1}{3}$ of ordinary interest, or $\frac{1}{72}$ of accurate interest (274, Note 4). Therefore, from ordinary interest to find accurate interest subtract $\frac{1}{72}$.

In reckoning accurate interest, on account of the many short methods of ordinary interest, many accountants prefer to calculate ordinary interest first, and then make the necessary deduction.

EXAMPLES.

293. What is the accurate interest of

1. \$435, at 6%, for 25 days? 5. \$292, at $3\frac{1}{2}\%$, for 140 days?
2. \$6030, at 5%, for 141 days? 6. \$438, at 6%, for 210 days?
3. \$780, at 6%, for 90 days? 7. \$350, at 4%, for 150 days?
4. \$437, at 7%, for 63 days? 8. \$500, at $4\frac{1}{2}\%$, for 100 days?
9. \$3110.45, at $5\frac{1}{2}\%$, for 90 days?
10. \$373.70, at 7%, from June 4 to Dec. 28?
11. \$500, at 6%, from July 24 to Sept. 16?
12. \$365, at 6%, from June 30 to Dec. 21?
13. \$1080, at 5%, from May 9, 1878, to Jan. 30, 1879?
14. \$1728, at 7%, from Jan. 6, 1878, to Jan. 21, 1880?
15. Required the exact interest on three U. S. bonds of \$5000 each, at $3\frac{1}{2}\%$, from July 1 to Aug. 11.
16. What is the interest on three U. S. bonds of \$1000 each, at $4\frac{1}{2}\%$, from Sept. 1 to Nov. 15?
17. What is the interest on a \$5000 U. S. bond, at 4%, from Oct. 1 to Dec. 16?
18. What is the interest on a U. S. bond of \$1000, bearing $3\frac{1}{2}\%$ interest, from May 1 to July 19?
19. What is the interest on a \$500 U. S. bond, at 4%, from Apr. 1 to May 10?
20. What is the interest on a \$5000 U. S. bond from Nov. 1, 1881, to Jan. 3, 1882, at $3\frac{1}{2}\%$?
21. What is the difference between the ordinary and the accurate interest of \$10000 for 219 days at 6%?
22. The ordinary interest on a certain sum for a certain time at a certain rate is \$8.76. What is the accurate interest?

PROBLEMS IN INTEREST.

294. To find the rate, the principal, interest or amount, and time, being given.

Ex. At what rate will \$720 in 1 yr. 4 mo. 10 da., produce \$44.10 interest?

OPERATION.	
\$7	20
—	—
57	60
1	20
6) 58	80
\$9	80) \$44.10 (4½

ANALYSIS.—The interest on a given principal for a given time is in proportion to the rate per cent. At one per cent., \$720 will in 1 yr. 4 mo. 10 da., produce \$9.80 interest. To produce \$44.10 interest, the required rate must be as many times 1%, as \$9.80 are contained times in \$44.10, or 4½ times. Hence the answer is 4½%.

295. RULE.—*Divide the given interest by the interest of the given principal, for the given time, at 1%.*

NOTE.—When the amount is given, find the interest by subtracting the principal from the amount.

EXAMPLES.

296. At what rate will

1. \$864 in 8 mo. 10 da. produce \$42 interest?
2. \$1000 in 9 mo. 9 da. produce \$54.25 interest?
3. \$852 in 1 yr. 7 mo. 16 da. amount to \$935.21?
4. \$1926 in 2 yr. 8 mo. 24 da. amount to \$2189.22?
5. \$375.60 in 1 yr. 10 mo. 22 da. amount to \$425.41?
6. \$1872 in 7 mo. 17 da. produce \$41.31 interest?
7. \$435.60 in 1 yr. 2 mo. 18 da. amount to \$478?
8. \$1338.72 in 6 mo. 27 da. produce \$34.64 interest?
9. \$1728 in 8 mo. 21 da. amount to \$1778.11?
10. \$3456 in 5 mo. 8 da. produce \$91.01 interest?
11. \$5280 in 11 mo. 11 da. amount to \$5720.12?
12. \$1234 in 8 mo. 22 da. produce \$80.83 interest?
13. \$6975 in 3 mo. 28 da. amount to \$7215.06?
14. \$525 in 1 yr. 11 mo. 18 da. amount to \$834.75?

297. To find the time, the principal, interest or amount, and rate, being given.

Ex. In what time will \$426, at 6%, produce \$59.427 interest ?

OPERATIONS.

\$426	Or	\$426
.06		.06
\$25.56) \$59.427 (yr. 2.325		\$25.56) \$59.427 (2 yr.
51 12	12	51 12
8 307	mo. 3.900	8.307
7 668	30	12
6390	da. 27.000	\$25.56) 99.684 (3 mo.
5112		76 68
12780		23.004
12780		30
0		\$25.56) 690.120 (27 da.

ANALYSIS.—The interest on a given principal at a given rate %, is in proportion to the time. In one year, \$426, at 6%, will produce \$25.56 interest. To produce \$59.427 interest, it will require as many years as \$25.56 is contained times in \$59.427, or 2.325 yr. 2.325 yr. equal 2 yr. 3 mo. 27 da. (196.)

298. RULE.—*Divide the given interest by the interest of the given principal, at the given rate, for 1 year. The integral part of the quotient will be years. Reduce the decimal, if any, to months and days (196).*

EXAMPLES.

299. In what time will

1. \$3000, at 7%, produce \$108.50 interest ?
2. \$1728, at 6%, amount to \$1872 ?
3. \$3932, at 7%, produce \$597.88 interest ?
4. \$735, at 5%, amount to \$742.66 ?
5. \$1222.25, at 6%, produce \$39.52 interest ?

In what time will

6. \$375.60, at 7%, amount to \$425.41?
7. \$1461.75, at 6%, produce \$420.25 interest?
8. \$1200, at $3\frac{3}{4}\%$, amount to \$1413?
9. \$4500, at 5%, produce \$181.25 interest?
10. \$276.50, at 10%, amount to \$303.46?
11. \$1020, at 6%, produce \$89.25 interest?
12. \$6495, at 7%, amount to \$7161.81?
13. \$100, at 6%, produce \$100 interest?

300. To find the principal, the interest, time, and rate, being given.

Ex. What principal will produce \$152.64 interest, in 1 yr. 5 mo. 20 da., at 6%?

OPERATION.	
\$\$.088\frac{1}{3}) \$152.64
3	3
<hr/>	
.265) 457.920 (1728

duce \$152.64, the principal must be as many times \$1 as \$.088\frac{1}{3} is contained times in \$152.64, or \$1728.

ANALYSIS.—The interest on any principal is as many times greater than the interest of \$1, as that principal is greater than \$1. One dollar, in 1 yr. 5 mo. 20 da., at 6% (**285**), will produce \$.088\frac{1}{3} interest. To pro-

301. RULE.—*Divide the given interest by the interest of \$1 for the given time, at the given rate.*

EXAMPLES.

302. What principal will produce

1. \$1235 interest, in 1 yr. 8 mo. 12 da., at 6%?
2. \$49.81, in 9 mo. 24 da., at 7%?
3. \$186.75, in 1 yr. 4 mo. 20 da., at 6%?
4. \$244.44, in 7 mo. 18 da., at 5%?
5. \$375.60, in 2 yr. 4 mo. 6 da., at 8%?
6. \$54.25, in 3 mo. 3 da., at 7%?
7. \$387.40, in 2 yr. 8 mo., at 4\frac{1}{2}\%?
8. \$456, in 93 da., at 6%?
9. \$375, in 63 da., at 7\frac{1}{4}\%?

10. \$1000, in 1 yr. 18 da., at 3%?
11. \$538.80, in 10 mo. 24 da., at 5%?
12. \$416.75, in 8 mo. 21 da., at 4%?
13. \$645.39, in 4 yr. 8 mo. 10 da., at 4%?

303. To find the principal, the amount, time, and rate, being given.

Ex. What principal will amount to \$1880.64, in 1 yr. 5 mo. 20 da., at 6%?

OPERATION.

$$\begin{array}{r} \$1.088\frac{1}{3}) \$1880.64 \\ \underline{3 3} \\ 3.265 \end{array}$$

$$3.265 5641.920 (1728$$

the principal must be as many times \$1 as \$1.088 $\frac{1}{3}$ are contained times in \$1880.64, or \$1728.

ANALYSIS.—The amounts of different principals for the same time and rate %, are to each other as the principals. One dollar, in 1 yr. 5 mo. 20 da., at 6%, will amount to \$1.088 $\frac{1}{3}$. To amount to \$1880.64,

304. RULE.—Divide the given amount by the amount of \$1 for the given time, at the given rate

EXAMPLES.

305. What principal will amount to

1. \$1272.254, in 6 mo. 6 da., at 6%?
2. \$5538.72, in 8 mo. 12 da., at 7%?
3. \$3695.04, in 1 yr. 4 mo. 18 da., at 5%?
4. \$442.71, in 2 yr. 2 mo. 24 da., at 8%?
5. \$14794.31, in 3 yr. 3 mo. 3 da., at 6%?
6. \$1793.38, in 7 mo. 17 da., at 6%?
7. \$1010.65, in 5 yr. 8 mo. 6 da., at 7%?
8. \$977.75, in 1 yr. 10 mo. 10 da., at 6%?
9. \$1716.75, in 3 yr. 4 mo. 21 da., at 4%?
10. \$2808.08, in 2 yr. 8 mo. 12 da., at 8%?
11. \$4312.22, in 1 yr. 2 mo. 11 da., at 12%?
12. \$6528.49, in 4 yr. 7 mo. 6 da., at 5%?
13. \$1763.02, in 1 yr. 2 mo. 21 da., at 6%?
14. \$2457.28, in 2 yr. 5 mo. 23 da., at 6%?

TRUE DISCOUNT.

306. The **Present Worth** of a debt due at some future time is its value now. Theoretically, it is such a sum that, if placed at interest to-day for the given time, would amount to the face of the debt.

307. The **True Discount** is the difference between the face of the debt and the present worth.

NOTES—1. This subject is an application of the principle illustrated in Art. **303**, the face of the debt being the amount, the present worth the principal, and the true discount the interest.

2. In actual business, true discount is little used, banks and merchants generally using bank discount (**327**). True discount is the interest on the present worth for the given time, while bank discount is the interest on the face of the debt. The difference is therefore equivalent to the interest on the true discount. For discount on bills, etc., when time does not enter in as an element, see Art. **249**.

Ex. Mr. B owes me \$212, payable one year from to-day without interest: what is the present worth of the debt, the current rate of interest being 6%?

ANALYSIS.—Since \$1 in one year, at 6%, amounts to \$1.06, it would require as many dollars to amount to \$212, as \$1.06 are contained times in \$212, or \$200. The true discount is \$212 — \$200, or \$12.

308. RULE.—*I. To find the present worth, divide the face of the debt by the amount of \$1 for the given time, at the given rate.*

II. To find the true discount, subtract the present worth from the face of the debt.

EXAMPLES.

309. The current rate of interest being 6%, what is the present worth and true discount of

1. \$1000, due 2 years hence? 3. \$600, due in 1 yr. 7 mo.?
2. \$500, due in 2 yr. 4 mo.? 4. \$800, due in 9 mo. 24 da.?
5. \$325, due in 2 yr. 5 mo. 12 da.?

6. Mr. C. desiring to pay a bill of \$1728 4 months before it was due, was allowed a discount equivalent to the interest on the face of the bill for the unexpired time at 6% per annum (bank discount). How much greater was this discount than the true discount?

7. Goods to the amount of \$3750 are sold on a credit of 4 months. For how much cash could the merchant afford to sell the same goods, money being worth 10% per annum?

8. If \$10000 will be due me May 28, and \$8000 May 16, what discount should I make on the two claims Apr. 1, money being worth 8%?

REVIEW EXAMPLES.

310. 1. What is the interest of \$375.60, for 1 yr. 10 mo. 16 da., at 6 %?

2. Find the amount of \$1765 for 7 mo. 20 da., at 7%.

3. At what rate will \$1234, in 2 yr. 2 mo. 26 da., produce \$138.14 interest?

4. In what time will \$585, at 6%, produce \$67.08 interest?

5. What principal will, in 1 yr. 8 mo. 14 da., at 6%, produce \$176.22 interest?

6. The semi-annual interest on a mortgage at 7% is \$350. What is the face of the mortgage?

7. I invest \$49500 in a business that pays me \$594 per month. What annual rate of interest do I receive?

8. Which is the better investment, and what per cent., one of \$8400, yielding \$336 semi-annually, or one of \$15000, producing \$1425 annually?

9. May 18th, a speculator bought 1600 bushels of wheat, at \$1.50 a bushel. He afterward sold the whole for \$2472 cash, his profit being equivalent to 8% per annum on the amount invested. What was the date of the sale?

10. The par value of Mr. A's bank stock is \$9000, and he receives a semi-annual dividend of \$315. What per cent. is the dividend per annum?

11. Mrs. C.'s son is now 16 *yr.* old ; how much must she invest for him at 6%, that, on arriving at age, he may have, with simple interest, \$25000 ?

12. What is the interest of \$10000 for 2 days, at 6% per annum, and a commission of $\frac{1}{8}\%$ per day ?

13. A gentleman loaned \$15000, at 6%. Jan. 1, 1880, interest and principal together equalled \$20000. When was the money loaned ?

14. Find the interest on \$3000, from Mar. 16 to Dec. 4, at 6%, by the following methods (274) : 1, ordinary interest and compound subtraction ; 2, ordinary interest and exact number of days ; 3, accurate interest.

15. Oct. 1, 1880, the loans and discounts of the National Banks of the United States amounted to \$1,041,000,000. At 6%, what would be the difference between the ordinary (360 days) and accurate (365 days) interest of this amount for 1 day ?

16. How much is paid for the use of \$1000 from Dec. 2 to Dec. 17, accurate interest at 6%, and a commission of $\frac{1}{32}\%$ per day being charged ?

17. 6% per annum accurate interest and a bonus of $\frac{1}{32}\%$ per day is equivalent to what rate per annum ?

18. A loaned B a sum of money, payable in 5 months, with interest at the rate of 6%, and at the end of that time received \$666.25 in return. How much did he loan ?

19. A speculator borrowed \$10925 at 6%, May 16, 1882, with which he purchased flour at \$6.25 per barrel. June 11, 1883, he sold the flour at \$7.50 per barrel, cash. What did he gain by the transaction ?

20. A bill of goods amounting to \$4316.75 is due May 27; how much would settle it May 1 at 6% ? How much July 3 ? (Exact days. See Art. 307, Note 2.)

21. A buys a bill of goods amounting to \$2776.40, on the following terms:—"4 months, or less 5% cash." He accepts the latter, and borrows the money at 6% to pay the bill. How much does he gain ?

22. B bought 225 A. 24 sq. rd. of land, Aug. 18, 1882, at \$4 an acre, borrowing the money to pay for it at 5%. He sold the land April 7, 1886, at an advance of \$299.40 on cost. If meanwhile he paid \$46.50 for taxes on the land, did he gain or lose, and how much?

23. C of New York sells for D of Atlanta, a quantity of cotton, amounting to \$1317.83, and charges a commission of $2\frac{1}{2}\%$. By instructions, he invests the proceeds in dry goods, after deducting a commission of $1\frac{1}{2}\%$ of the amount expended. What was the total commission?

24. A owes B £260, with interest at 5%, for 143 days. He pays 25% of the amount due; how much remains?

NOTE.—In England, interest is usually computed on the basis of 365 days to the year, when the time is given in days. The legal rate in England is 5%. To calculate interest on English money, reduce the shillings and pence to the decimal of a pound (See Art. 199, Ex. 6. Note), apply any of the methods under Art. 291, and reduce the resulting decimal to shillings and pence.

Find the accurate interest of

25. £425, from Aug. 4 to Dec. 28, at 5%.

26. £625 12s., from Jan. 12 to April 1, at 4%.

27. £717 16s. 10d., from Mar. 3 to June 16, at $4\frac{1}{2}\%$.*

28. £429 10s. 8d., from Sept. 16 to Nov. 30, at 3%.

29. £516 18s. 3d., from Aug. 1 to Oct. 18, at $3\frac{1}{2}\%$.

30. £612 6s. 11d., from July 1 to Nov. 3, at 5%.

31. A commission merchant sold 24160 pounds of leather at $26\frac{3}{4}$ cents a pound, paid transportation \$60.40, cartage \$20, his commission being $2\frac{1}{2}\%$, and his charge for inspection \$20. What were the net proceeds?

32. What per cent. profit is made by buying at a discount of 20, 10, and $12\frac{1}{2}\%$, and selling at the list price?

33. At what per cent. above cost must goods be marked, so that when sold at a discount of 5%, there would be a profit of 25%?

* When the time is less than 1 year, and the rate is 6% or less, reject the part of the principal, if less than 6; add 1 shilling, if more than 6. The result will be sufficiently accurate.

COMPOUND INTEREST.

311. Compound Interest is interest not only on the principal, but on the interest also after it becomes due (**270**).

1. Interest may be compounded annually, semi-annually, quarterly, etc.

2. Compound interest cannot be collected by law, that is, payment cannot be enforced; but such a payment is equitable, and the receiving of it, if the debtor is willing or can be induced to pay it, does not constitute usury in the legal sense of the word. In the State of Missouri, parties may contract in writing for the payment of interest upon interest, but it shall not be compounded oftener than once a year.

Ex. What is the compound interest of \$1000 for 3 years, at 6%?

OPERATIONS.

\$1000.00	Principal.	Or	\$1000
<u>60.00</u>	Interest for 1 <i>yr.</i>		<u>1.06</u>
1060	Amount for 1 <i>yr.</i> , or 2d principal.		1060
<u>63.60</u>	Interest of \$1060 for 1 <i>yr.</i>		<u>1.06</u>
1123.60	Amount for 2 <i>yr.</i> , or 3d principal.		1123.60
<u>67.416</u>	Interest of \$1123.60 for 1 <i>yr.</i>		<u>1.06</u>
1191.016	Amount for 3 <i>yr.</i>		1191.016
<u>1000</u>	Original principal.		<u>1000</u>
191.016	Compound interest for 3 <i>yr.</i>		191.016

312. RULE.—*Find the amount of the given principal for the first period of time, and make it the principal for the second. Find the amount of the second principal for the second period of time, and make it the principal for the third; and so continue for the whole time. The last amount is the amount required.*

The last amount, less the given principal, will be the compound interest.

313. Table showing the sum to which \$1 will increase, at compound interest, in any number of years not exceeding 20.

Yrs.	2%.	2½%.	3%.	3½%.	4%.	4½%.	5%.	6%.	Yrs.
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0600	1
2	1.0404	1.0506	1.0609	1.0712	1.0816	1.0920	1.1025	1.1236	2
3	1.0612	1.0769	1.0927	1.1087	1.1249	1.1412	1.1576	1.1910	3
4	1.0824	1.1038	1.1255	1.1475	1.1699	1.1925	1.2155	1.2625	4
5	1.1041	1.1311	1.1593	1.1877	1.2167	1.2462	1.2763	1.3382	5
6	1.1262	1.1597	1.1941	1.2293	1.2653	1.3023	1.3401	1.4185	6
7	1.1487	1.1887	1.2299	1.2723	1.3159	1.3609	1.4071	1.5036	7
8	1.1717	1.2184	1.2668	1.3168	1.3686	1.4221	1.4775	1.5938	8
9	1.1950	1.2489	1.3048	1.3629	1.4232	1.4861	1.5513	1.6895	9
10	1.2190	1.2801	1.3439	1.4106	1.4802	1.5530	1.6289	1.7908	10
11	1.2434	1.3121	1.3842	1.4600	1.5395	1.6229	1.7103	1.8983	11
12	1.2682	1.3449	1.4258	1.5111	1.6010	1.6959	1.7956	2.0122	12
13	1.2936	1.3785	1.4685	1.5640	1.6651	1.7722	1.8856	2.1329	13
14	1.3195	1.4130	1.5126	1.6187	1.7317	1.8519	1.9799	2.2609	14
15	1.3459	1.4483	1.5580	1.6753	1.8009	1.9353	2.0789	2.3966	15
16	1.3728	1.4845	1.6047	1.7340	1.8730	2.0224	2.1829	2.5404	16
17	1.4002	1.5216	1.6528	1.7947	1.9479	2.1134	2.2920	2.6928	17
18	1.4282	1.5597	1.7024	1.8575	2.0258	2.2085	2.4066	2.8543	18
19	1.4568	1.5987	1.7535	1.9225	2.1068	2.3079	2.5270	3.0256	19
20	1.4859	1.6386	1.8061	1.9898	2.1911	2.4117	2.6533	3.2071	20

To find the sum to which a given amount will increase, at compound interest, at any of the rates per cent., and number years expressed in the above Table: *Multiply the given amount by the sum to which one dollar will increase at the rate and for the number of years required.*

NOTES.—1. The amount for any number of years not given in the table may be computed by finding the product for any two numbers of years whose sum equals the given time.

2. The compound interest of \$1 is \$1 less than the amounts in the above table.

314. Table showing the sum to which \$1, paid at the beginning of each year, will increase at compound interest, in any number of years not exceeding 20.

Yrs.	3%.	3½%.	4%.	5%.	6%.	7%.	8%.	Yrs.
1	1.0300	1.0350	1.0400	1.0500	1.0600	1.0700	1.0800	1
2	2.0909	2.1662	2.2416	2.1525	2.1835	2.2149	2.2461	2
3	3.1836	3.2149	3.2435	3.3101	3.3746	3.4399	3.5061	3
4	4.3091	4.3625	4.4163	4.5256	4.6371	4.7507	4.8666	4
5	5.4681	5.5592	5.6390	5.8019	5.9753	6.1533	6.3359	5
6	6.6625	6.7791	6.8983	7.1420	7.3978	7.6540	7.9228	6
7	7.8923	8.0517	8.2142	8.5491	8.8975	9.2598	9.6366	7
8	9.1591	9.3685	9.5828	10.0266	10.4913	10.9780	11.4876	8
9	10.4639	10.7314	11.0061	11.5779	12.1838	12.8164	13.4866	9
10	11.8078	12.1420	12.4864	13.2068	13.9746	14.7836	15.6455	10
11	13.1920	13.6021	14.0258	14.9171	15.8699	16.8885	17.9771	11
12	14.6178	15.1130	15.6238	16.7130	17.8821	19.1406	20.4952	12
13	16.0863	16.6770	17.2919	18.5986	20.0151	21.5305	23.2149	13
14	17.5989	18.2957	19.0236	20.5786	22.2769	24.1290	26.1521	14
15	19.1563	19.9710	20.8245	22.6575	24.6705	26.8881	29.3243	15
16	20.7616	21.7050	22.6975	24.8401	27.2129	29.8402	32.7592	16
17	22.4114	23.4997	24.6454	27.1324	29.9757	32.9990	36.4502	17
18	24.1169	25.3573	26.6712	29.5330	32.7600	36.3790	40.4463	18
19	25.8704	27.2797	28.7781	32.0690	35.7856	39.9955	44.7620	19
20	27.6765	29.2695	30.9692	34.7193	38.9927	43.8652	49.4229	20

EXAMPLES.

315. 1. What will \$450 amount to at compound interest, in 4 years, compounded annually at 4%? At 3%?

2. Find the compound interest of \$360, for 2 years, interest compounded semi-annually at 6%. At 5%.

NOTE.—Find the compound interest for 4 periods of 6 months each, at one-half the annual rate.

3. What is the compound interest of \$800 for 1 yr. 3 mo. at 8%, interest compounded quarterly?

4. At compound interest, what is the amount of \$1728 for 3 yr. 4 mo. 16 da., interest compounded annually at 3%? At 6%?

NOTE.—First find the amount for 3 years, and use this amount as the principal for the remaining time.

5. B holds a mortgage against A's property dated Apr. 1, 1881, for \$20000, interest payable annually at 6%. The interest due Apr. 1, 1882, is not paid until May 26, 1882. How much is then due, A having consented to pay interest upon interest? (See Note 2, Art. 311.)

NOTE.—In solving the following examples, use the tables in Art. 313-314.

6. A gentleman deposits in a savings bank \$100 when his child is twelve years old. How much will this amount to when he is 21 years old, interest being compounded semi-annually at 4%? At 5%?

7. At the age of 20, and every year thereafter, a young man places \$200 at compound interest at 6%. How much will he have at the age of 30? At the age of 40? (See Art. 314.)

8. How much will a gentleman have at the end of three years, if he places at compound interest at 5%, \$300 at the beginning of each year?

9. Mr. B., whose life is insured for \$4000, pays an annual premium of \$114. How much would this amount to at 6% compound interest in 20 years?

10. What sum must be placed at compound interest, at 6%, to amount to \$1000 in 5 years ?

NOTE.—In compound interest, as in simple interest, the amounts are proportional to the principals ; hence the amount of any principal is as many times greater than the amount of \$1, as that principal is greater than \$1.

To find the principal, divide the given amount by the amount of \$1 for the given time and rate.

In simple interest, the interest on a given principal for a given time is in proportion to the rate per cent., and at a given rate, in proportion to the time ; but, in compound interest, such is not the case. If the rate or time be doubled, the interest is more than doubled.

11. How much should a gentleman invest at compound interest, at 6%, for his son who is now 6 years old, so that, when he becomes 21 years of age, he may have \$10000 ?

12. In the above example, how much should be invested at the beginning of each year to produce the same sum ?

13. A gentleman at his death left \$7850 for the benefit of his only son, 12 years old, the money to be paid to him when he should be 21 years of age. How much did he receive, interest at 6%, compounded semi-annually ?

COMMERCIAL PAPER.

316. Commercial Paper embraces notes, drafts, bills of exchange, etc.

317. A **Note** (also called a Promissory Note) is a written promise to pay a certain sum of money on demand or at a specified time.

318. The **Maker** of a note is the person who signs it, and thus becomes responsible for its payment. The **Payee** is the person to whom, or to whose order, it is made payable. The **Face** of a note is the sum promised.

In Note 1, Art. **324**, Peter Cooper is the maker ; George Peabody is the payee ; the face of the note is \$1000.

319. A **Negotiable Note** is a note which is made payable to bearer or to the order of some person.

1. A note is *non-negotiable* when it is payable only to the party named in the note.

2. A negotiable note made in New Jersey must contain the words "without defalcation or discount;" in Missouri, the words "negotiable and payable without defalcation or discount."

3. Negotiable notes payable to order may be sold or transferred by the payee writing his name upon the back of the note. He then becomes an indorser.

320. The **Indorser** of a note or draft is the person who writes his name on the back of it, and by so doing guarantees its payment.

If Mr. Erastus Corning desires to sell or transfer Note 3, Art. 324, it will be necessary for him to indorse it. If he writes his name only, it is called an indorsement *in blank*, and the note is then payable without further indorsement to any person lawfully holding the same. He may indorse it *in full* by making it payable to a particular person, thus—"Pay to the order of Henry R. Pierson. Erastus Corning." Before it can be again transferred, it will require the indorsement of Henry R. Pierson. For greater security, checks, notes, drafts, etc., are indorsed in full when sent by mail.

321. A **Draft**, or **Bill of Exchange** is an order or request addressed by one person to another directing the payment of a specified sum of money to a third person or to his order.

322. The **Drawer** of the draft is the person who signs it. The **Drawee** is the person on whom it is drawn. The **Payee** is the person to whom, or to whose order, it is made payable.

In Draft 5, Art. 324, C. P. Huntington is the drawer; Drexel, Morgan & Co. are the drawees; J. & W. Seligman & Co. are the payees.

1. After the bill is presented and accepted, the drawee is called the acceptor, and the draft, an acceptance. The draft then has the same legal significance as a promissory note.

2. A person accepts or promises to pay a draft by writing the word "Accepted" and the date over his name across its face.

3. To "honor" a draft is to accept it or pay it on being presented.

323. A **Protest** is a formal statement made by a Notary Public declaring that a draft or note has been presented for payment or acceptance, and was refused.

324. FORMS OF NOTES AND DRAFTS.

1. DEMAND NOTE.

\$1000.

NEW YORK, *August 19, 1881.*

On demand, I promise to pay GEORGE PEABODY, or bearer, One Thousand Dollars. Value received.

PETER COOPER.

The above note is payable on demand,—that is, whenever presented; is negotiable (payable to bearer); and bears interest from date at the legal rate of the State in which it is made. If the words “or bearer” were omitted the note would not be negotiable.

2. TIME NOTE—INTEREST-BEARING.

\$875 $\frac{44}{100}$.

CINCINNATI, OHIO, *July 16, 1882.*

Six months after date, I promise to pay GEO. C. MILLER, or order, Eight Hundred Seventy-five and $\frac{44}{100}$ Dollars, with interest at eight per cent. Value received.

ALEX. McDONALD.

The above note is payable 6 *mo.* 3 *da.* after its date, or Jan. 19, 1883; is negotiable (payable to order); and draws interest from its date at 8% per annum. If the rate of interest was omitted, it would bear interest at the legal rate of the State for such cases, 6%.

3. TIME NOTE—WITHOUT INTEREST.

\$6000.

ALBANY, N. Y., *December 4, 1881.*

Sixty days after date, I promise to pay to the order of ERASTUS CORNING, Six Thousand Dollars, at the Second National Bank. Value received.

E. C. KOONZ.

The above note is payable 63 days from Dec. 4, 1881, or Feb. 5, 1882. It is payable at the Second National Bank. No interest will be due at maturity (Feb. 5). If the note is not paid at maturity, it will bear interest from that date.

4. JOINT AND SEVERAL NOTE.

\$416 $\frac{32}{100}$.WORCESTER, MASS., *May 27, 1882.*

Four months after date, we jointly and severally promise to pay JOHN S. BALLARD, or order, Four Hundred Sixteen $\frac{32}{100}$ dollars, with interest from date, value received.

T. K. EARLE.

CHAS. W. SMITH.

If the above note were written "we jointly promise, etc.," it would be called a *joint note*. The makers of a joint note must be sued jointly, each being responsible for one-half of the amount of the note. The makers of a joint and several note may be sued separately, either being responsible for the full amount of the note.

5. SIGHT DRAFT.

\$8000.

SAN FRANCISCO, CAL., *May 1, 1882.*

At sight, pay to the order of J. & W. SELIGMAN & Co., Eight Thousand Dollars, value received.

C. P. HUNTINGTON.

To DREXEL, MORGAN & Co., New York.

6. TIME DRAFT.

\$5000.

BURLINGTON, IOWA, *June 18, 1881.*

At sixty days' sight, pay to the order of ADDISON BALLARD, Five Thousand Dollars, value received, and charge to account of

A. G. ADAMS.

To BARTON & JONES, Chicago, Ill.

Drafts are sometimes drawn a certain number of "days after date."

NOTES.—1. A note should contain the words "Value received," as a contract without a consideration is not legally binding.

2. Negotiable securities are good in the hands of one who purchases in good faith and before maturity, although the seller may have found or stolen them.

3. Where no place of payment is specified, a promissory note is payable at the maker's place of business, or if none is known, at the residence of the maker.

325. Days of Grace and Maturity.—The day of maturity is the day on which a note becomes legally due. According to the laws of most of the States, a note is not legally due until three days after the expiration of the time specified in the note, except the note contain the words “without grace.” These days are called *days of grace*, but they are of no advantage to the payer, since interest is charged for them as for any others.

1. California has abolished days of grace altogether. In Georgia, Alabama, and Kentucky, grace is allowed on promissory notes only in case they are made payable, or are discounted or left for collection at a bank or private banker's.

2. In the State of New York and most of the States, all bills and notes due on Sunday are payable on Saturday, and all due on a legal holiday are made payable on the business or secular day next preceding. Thus, if a holiday falls on Thursday, all notes, etc., must be paid on Wednesday; if a holiday falls on Monday, all notes due Sunday or Monday would be payable on Saturday; if a holiday falls on Saturday, notes due Saturday or Sunday would be payable on Friday.

3. The legal holidays in the State of New York are New Year's Day, Washington's Birthday (Feb. 22), Decoration Day (May 30), Independence Day (July 4), Election day (the first Tuesday after the first Monday in November), Thanksgiving Day (the day appointed by the President of the United States, usually the last Thursday of November), and Christmas (Dec. 25).

4. When a legal holiday falls on Sunday, Monday is, by the statute of New York, made a legal holiday, and notes, etc., maturing on Sunday or Monday, must be paid on the preceding Saturday.

5. When the time of a note is expressed in months, calendar months are used to determine the day of maturity; when in days, the exact number of days is used. Thus, a note dated July 16, and payable two months from date, would nominally mature Sept. 16, and, including the three days of grace, would legally mature Sept. 19. A note having the same date, and payable sixty days from date, would nominally mature Sept. 14, and, including the three days of grace, would legally mature Sept. 17.

6. A note due in one or more months from date, matures on the corresponding day of the month up to which it is reckoned, *if there are so many days in that month*; but if not so many, it then matures on the last day of said month, to which the usual grace must be added.

Thus, notes dated Jan 28, 29, 30, or 31, and payable one month from date, would become due Mar. 3 (Feb. 28 with 3 days' grace added).

7. When drafts are payable a certain time after *sight*, the date of acceptance and the time of the draft determine the day of maturity. Thus, if a draft is dated May 16, accepted May 20, and payable sixty days after sight, it would mature or be due 63 (including 3 days of grace) days after May 20, or July 22. If payable 60 days after *date*, it would mature 63 days after May 16, or July 18. It is not necessary to present for acceptance drafts drawn a certain time after date, but as a courtesy to the drawee, it is usually done.

8. Days of grace are allowed on drafts according to the custom of the place where they are payable. The statute of New York forbids grace on all sight drafts, no matter on whom drawn, and on all time drafts which appear on their face to be drawn "upon any bank, or upon any banking association or individual banker, carrying on the banking business under the act to authorize the business of banking."

EXAMPLES.

326. 1. How much would be due on Note 1, Art. 324. Jan. 1, 1882 (time by compound subtraction)?

2. How much would be due on Note 2, Art. 324, at its maturity? How much March 1, 1883? Supposing the rate of interest was omitted in the note, how much would be due May 4, 1883?

3. Ninety days after June 21 is what date?

OPERATIONS.				ANALYSIS.—Subtract from the
90	Or 9	June.		given number of days, the num-
9	June.	31	July.	ber of days remaining in June.
81		31	Aug.	and from this remainder, subtract
31	July.	71		successively the number of days
50		90		in the following months until the
31	Aug.	19	Sept.	remainder is equal to or less than
19	Sept.			the number of days in the next
				following month. The last re-
				mainder represents the required
				date.

Or, write the remaining number of days in June, and the number of days in a sufficient number of months to produce about the given number of days. Take their sum and subtract it (if possible) from the given number of days. The remainder will be the day of the following month representing the required date. If the sum is greater

than the given number, subtract the excess from the number of days in the last month written. The remainder will be the required date.

If the time be 30, 60, or 90 days, regard each 30 days as a calendar month, and correct by subtracting 1 day for each intervening month containing 31 days, and adding 2 days for February (in leap year 1 day). Thus 3 months after June 21 is Sept. 21, and by subtracting 2 days for July and August, the correct result is Sept. 19.

4. Supposing Note 3, Art. **324**, was payable 90 days from date, what would be its due date? The note as given not being paid at maturity, how much would be due Feb. 25, 1882, protest fees \$2.10?

5. How much would settle Note 4, Art. **324**, Dec. 30, 1882?

6. If Draft 6, Art. **324**, was accepted June 19, 1881, what was the date of maturity?

BANK DISCOUNT.

327. Bank Discount is simple interest of a note, paid in advance, for the number of days the note has to run. It may be computed by any of the methods given for simple interest.

On notes without interest (the usual case of notes discounted at banks), bank discount is reckoned on their face, the amount due at maturity; on notes with interest, it is reckoned on the amount due at maturity, or their face plus the interest for the full time of the note.

328. The Proceeds of a note is the amount received by the holder from the bank when the note is discounted. It is the amount on which the discount is reckoned less the discount.

329. Call Loans.—Banks in the City of New York loan large amounts of money upon stocks, bonds, etc., as collateral security, payable on demand or on giving one day's notice. Such loans are called "call" or demand loans, and interest on them is paid at the end of the time.

330. The time to be reckoned on a loan or note is exclusive of the day of date, but includes the day of maturity or payment. Thus, in discounting a note, Apr. 4, which would mature Apr. 24, the discount would be calculated for 20 days.

In Philadelphia, Baltimore, and other cities, it is the custom of banks in finding time to include both the day of discount and the day of maturity. Thus, the discount on the above note would be reckoned for 21 days.

EXAMPLES.

331. Find the date of maturity and proceeds of the following notes :

(1.)

\$10000.

NEW YORK, *July 16, 1881.*

Four months after date, I promise to pay to the order of FISK & HATCH, Ten Thousand Dollars, at the First National Bank, value received.

S. D. BABCOCK.

Discounted July 16, 1881, at 6%.

ANALYSIS.—The note is due 4 months (325, 5) and 3 days (days of grace, 325) after July 16, or Nov. 19. From the day of discount (July 16) to the day of maturity (Nov. 19) there are 126 days.

The interest of \$10000 for 126 days at 6%, if reckoned on the basis of 360 days to the year, is \$210, and the proceeds are \$10000 less \$210, or \$9790. The interest on the basis of 365 days to the year would be \$2.88 less, or \$207.12, and the proceeds would be \$9792.88.

If the note was discounted Sept. 1, the interest or discount would be reckoned for 79 days (Sept. 1 to Nov. 19).

(2.)

\$8000.

BROOKLYN, N. Y., *July 16, 1881.*

Ninety days from date, I promise to pay S. B. CHITTENDEN, or order, Eight Thousand Dollars, value received.

A. A. LOW.

Discounted Aug. 31, 1881, at 6%.

ANALYSIS.—The note is due 93 days (325, 5) after July 16, or Oct. 17. Compute the discount for 47 days Aug. 31 to Oct. 17) on \$8000.

If the note had been discounted July 16, the date of the note, the interest would have been computed for 93 days, the full time of the note.

NOTE.—The results of the following examples will be given on the basis of both 360 and 365 days to the year.

No.	Date of Note.	Time.	Face.	Date of Discount.	Rate of Discount
3	Jan. 24.....	90 days	\$1200	Jan. 24.....	6%
4	May 18.....	3 mo.	\$5280	May 18.....	6%
5	Aug. 31.....	60 days	\$2560	Aug. 31.....	8%
6	June 4.....	4 mo.	\$3756	June 4.....	7%
7	Oct. 16.....	30 days	\$6425	Oct. 16.....	5%
8	Mar. 13.....	6 mo.	\$8375	Mar. 13.....	5½%
9	May 29.....	3 mo.	\$4500	July 7.....	10%
10	July 27.....	60 days	\$8240	Sept. 2.....	6%
11	Mar. 28.....	90 days	\$4324	Apr. 14.....	5¼%
12	May 27.....	6 mo.	\$4885	Aug. 15.....	8%
13	Jan. 3.....	120 days	\$9000	Feb. 28.....	6%
14	Sept. 12.....	4 mo.	\$5000	Oct. 14.....	7%
15	Nov. 1.....	90 days	\$6000	Nov. 28.....	5½%

Required the proceeds and date of maturity of the following notes discounted (360 days to the year) through a broker, his commission being $\frac{1}{4}\%$ of the face of the notes.

No.	Date of Note.	Time.	Face.	Date of Discount.	Rate of Discount
16	Feb. 21.....	4 mo.	\$10000	Feb. 21.....	4¾%
17	June 8.....	4 mo.	\$6000	June 12.....	4½%
18	Jan. 10.....	4 mo.	\$6000	Jan. 10.....	4¼%
19	Mar. 3.....	6 mo.	\$8775	Apr. 30.....	4¾%

20. What were the proceeds of Note 3, Art. 324, if discounted Dec. 16, 1881, at the legal rate?

21. Find the date of maturity and proceeds of a note of \$5000, payable 60 days from date, dated and discounted at a Philadelphia bank, Aug. 3. (See Art. 330.)

22. Find the date of maturity and proceeds of a note of \$3750, payable 60 days from date, dated and discounted at a Maryland bank, Jan. 31, 1882.

23. A broker discounts a note payable in 4 months at $4\frac{3}{4}\%$, and charges $\frac{1}{4}\%$ brokerage. This is equivalent to what rate of interest per annum, making no allowance for the days of grace?

24. A merchant can discount a note at his bank at 6% , 365 days to the year, or through a broker at $4\frac{3}{4}\%$, 360 days to the year, broker's commission $\frac{1}{4}\%$. How much better is the latter method on a note of \$10000, payable in 4 months, dated and discounted May 21?

Find the date of maturity and proceeds of the following interest-bearing notes:

(25.)

\$3000.

ALBANY, N. Y., *September 16, 1881.*

Four months after date, I promise to pay W. J. KLINE, or order, Three Thousand Dollars, with interest at 5% , value received.

J. M. THOMAS.

Discounted Nov. 3, 1881, at 6% .

NOTE.—Compute the discount at 6% for 77 days (Nov. 3 to Jan. 19) on the amount due at maturity (\$3000 plus the interest of \$3000 for 4 months and 3 days at 5%).

26. A note dated May 27, 1879, payable in 3 months, for \$3750, with interest at 7% ; discounted May 21, 1879, at 8% .

27. A note dated Jan. 16, 1879, payable in 4 months, for \$1632, with interest at 6% ; discounted Mar. 5, 1879, at 7% .

28. A note dated Oct 12, 1878, payable in 6 months, for \$875, with interest at 7% ; discounted Jan. 10, 1879, at 10% .

29. For what amount must a note be given for 60 days to afford \$1000 proceeds, if discounted at 6% ?

ANALYSIS.—The proceeds of any note is as many times greater than the proceeds of \$1, as the face of the note is greater than \$1. If a note of \$1 is discounted for 63 days, at 6% , it will afford \$.9895 proceeds; to afford \$1000 proceeds, the face of the note must be as many times \$1, as \$.9895 is contained times in \$1000, or \$1010.61.

The following approximate method is generally used by business men : *To the given proceeds, add the interest for the given time.*

The interest of \$1000 for 63 days is \$10.50. $\$1000 + \$10.50 = \$1010.50$. Since the interest is reckoned on the proceeds instead of the face of the note, the error, 11 cents, is equivalent to the interest of the interest (\$10.50) for the given time.

Where greater accuracy is required, the necessary correction may be made. The interest of \$10.50 for 63 days is 11 cents. $\$1010.50 + \$.11 = \$1010.61$.

30. A owes B \$1500 ; how large a 90-day note must A give B that when discounted at a bank at 6%, the proceeds will be sufficient to pay the debt ?

31. A merchant having \$8000 to pay, gets a note for \$5000, that will mature in 40 days, discounted at a bank at 6%. How large a note must he draw, payable in 90 days, for discount at the same rate, that the proceeds of the two notes may enable him to meet his payment ?

PARTIAL PAYMENTS.

332. Partial Payments are payments in part of a note, mortgage, or other debt, made at different times.

333. Indorsements are the acknowledgments of the payments, written on the back of the note, mortgage, etc., and stating the amount and date of the payment.

Special receipts are sometimes given for such payments.

UNITED STATES RULE.

334. Ex. How much would be due Sept. 1, 1882, on a note of \$600, dated March 1, 1882, with interest at 6% ? Suppose a payment of \$100 be made Sept. 1, 1882, to pay the interest and part of the principal, how much would then be due ?

Ans. \$518.

Ex. How much would be required to settle the above note Jan. 1, 1883, the balance of \$518 remaining on interest at the same rate from Sept. 1, 1882 ?

Ans. \$528.36.

Ex. Find the amount due on the following note, Jan. 19, 1885:

\$1000.

BOSTON, MASS., *Aug. 1, 1881.*

One year after date, I promise to pay JORDAN, MARSH & Co., or order, One Thousand Dollars, for value received. with interest from date, at 6 per cent.

ALEXANDER H. RICE.

On this note are the following indorsements:

Received Apr. 21, 1882, \$200. Received Aug. 1, 1883, \$100.

Received Dec. 1, 1882, \$25. Received July 7, 1884, \$400.

The method given in the following operation, is that adopted by the Supreme Court of the United States, and has been made the legal method of nearly all the States. By the United States Rule, as this is generally called, settlements are made whenever the payments are equal to or exceed the interest due; if the payment exceeds the interest, it is applied first to discharge the interest, and the surplus is applied towards paying the principal; if the payment is less than the interest, it is not applied until the payments, taken together, are sufficient to pay all interest due; since no unpaid interest is added to the principal to draw interest, a new principal can never be greater than the preceding principal.

OPERATION.

Face of note, or principal, from Aug. 1, 1881.	.	.	\$1000
Interest from Aug. 1, 1881, to Apr. 21, 1882 (8 mo. 20 da.)			43.33
Amount, Apr. 21, 1882,	.	.	1043.33
First payment, Apr. 21, 1882,	.	.	200.00
New principal from Apr. 21, 1882,	.	.	843.33
Interest of \$843.33 from Apr. 21, 1882, to Dec. 1, 1882 (7 mo. 10 da.). (See note.)		\$30.92	
(Interest exceeds the payment, and a new principal is not formed.)			
Interest of \$843.33 from Dec. 1, 1882, to Aug. 1, 1883 (8 mo.),		33.73	64.65
[Payments \$125 (\$25 + \$100), now greater than the interest due (\$64 65).]			
Amount, Aug 1, 1883,	.	.	907.98
Second and third payments, \$25 + \$100,	.	.	125
New principal from Aug. 1, 1883,	.	.	\$782.98

New principal from Aug. 1, 1883,	\$782.98
Interest of \$782.98 from Aug. 1, 1883, to July 7, 1884 (11 mo. 6 da.),	43.85
Amount, July 7, 1884,	826.83
Fourth payment, July 7, 1884,	400
New principal from July 7, 1884,	426.83
Interest of \$426.83 from July 7, 1884, to Jan. 19, 1885 (6 mo. 12 da.),	13.66
Amount due Jan. 19, 1885, the final day of settlement, . .	\$440.49

NOTE.—In many cases it can be determined mentally in advance whether the payment is greater or less than the interest. In this case the interest could be taken at once from Apr. 21, 1882, to Aug. 1, 1883 (1 yr. 3 mo. 10 da.), since it is evident that the payment (\$25) is less than the interest of \$843.33 for 7 mo. 10 da. (The interest of \$800 for 7 mo. is $3\frac{1}{2} \times \$8$, or \$28, and it would be more on \$843.33 for 7 mo. 10 da.) If it is doubtful whether the payment is greater or less than the interest, perform all the work.

335. UNITED STATES RULE.—*Find the amount of the given principal to the time when the payment or the sum of the payments exceeds the interest due; subtract from this amount the payment or the sum of the payments. Treat the remainder as a new principal, and proceed as before, to the time of settlement.*

EXAMPLES.

336. NOTES.—1. In the following examples, find the time by compound subtraction.

2. In the first five examples, all the payments exceed the interest.

\$1680.

TRENTON, N. J., Oct. 9, 1880.

1. On demand, I promise to pay COOPER, HEWITT & Co., or order, Sixteen Hundred Eighty Dollars. Value received.

JOHN A. ROEBLING.

On this note were indorsed the following payments :

Dec. 21, 1881, received \$289.12. June 9, 1883, rec'd \$991.50.

How much was due Jan. 30, 1884?

2. On a note dated May 11, 1877, for \$2000, are the following indorsements :—Aug. 6, 1879, \$361 ; Feb. 11, 1880, \$901.60 ; Nov. 2, 1882, \$1000. What remained due Feb. 2, 1883, at 6% ? At 5% ?

3. On a note dated July 11, 1878, for \$2400, are the following indorsements :—Sept. 17, 1879, \$200 ; Jan. 29, 1880, \$400 ; Nov. 29, 1881, \$1150. What is the amount due Jan. 11, 1882, the interest being at 6% ? At 7% ?

4. On a mortgage for \$1700, dated May 28, 1880, there was paid Nov. 12, 1880, \$80 ; Sept. 20, 1881, \$314 ; Jan. 2, 1882, \$50 ; Apr. 17, 1882, \$160. What was due Dec. 12, 1882, at 6% ? At 8% ?

5. On a note dated May 30, 1879, for \$1666, are the following indorsements :—Apr. 9, 1880, \$314 ; Nov. 4, 1880, \$180 ; Aug. 25, 1881, \$575. What was due June 30, 1882, at 6% ? At 8% ?

6. A note for \$3600, dated May 12, 1880, bore the following indorsements :—Jan. 2, 1881, \$255 ; Mar. 15, 1881, \$225 ; June 3, 1881, \$120 ; Aug. 6, 1881, \$300 ; Feb. 3, 1882, \$30. How much had to be paid June 2, 1882, to take up the note, at 6% ? At 10% ?

7. A note for \$4000, dated Mar. 9, 1874, was indorsed as follows :—Jan. 18, 1876, \$300 ; June 4, 1876, \$400 ; Dec. 9, 1876, \$1800 ; Sept. 1, 1879, \$2000. How much was due Jan. 1, 1880, at 6% ? At 7% ?

8. A mortgage of \$6000 is dated May 9, 1877, on which there were the following payments :—July 15, 1878, \$500 ; Nov. 27, 1878, \$1000 ; June 1, 1879, \$100 ; May 9, 1880, \$275 ; Sept. 27, 1880, \$2000. What was due Nov. 9, 1880, the interest being at 6% ? At 12% ?

9. What remained due June 3, 1882, on a note dated June 21, 1880, for \$3300 with interest at the legal rate in Illinois, the following payments having been made ? Oct. 9, 1880, \$90 ; Jan. 15, 1881, \$60 ; Mar. 27, 1881, \$100 ; Aug. 6, 1881, \$60 ; Dec. 15, 1881, \$500. What remained due at the legal rate in Nevada ?

MERCANTILE RULES

337. The following methods are used by merchants in finding the balance due on a note where partial payments have been made. They are similar to the methods in general use for finding the balance due on an open account.

338. When the note runs for one year only, or less.

339. RULE.—*Compute the interest on the principal from the time it commenced to draw interest, and on each payment from the time it was made until the time of settlement, and deduct the amount of all the payments, including interest, from the amount of the principal and interest.*

NOTES.—1. This rule is used by some merchants when the note runs more than one year, although it is greatly to the disadvantage of the creditor, or holder of the note.

2. In solving examples by this rule, the different methods for finding time and interest, given in Art. 274, are used. The results of the following examples will be given for the first method (Compound Subtraction and 360 days to the year).

EXAMPLES.

340. 1. Find the balance due May 12, 1882, on a note for \$2400, dated July 12, 1881, on which the following payments have been made :—Dec. 16, 1881, \$40; Jan. 2, 1882, \$100; Mar. 15, 1882, \$150.

OPERATION.

Face of note, or principal, July 12, 1881,	\$2400.00
Interest on the same to May 12, 1882 (10 mo.),	120.00
Amount, May 12, 1882,	2520.00
First payment, Dec. 16 1881,	\$40.00
Interest on the same to May 12, 1882 (4 mo. 26 da.),97
Second payment, Jan. 2, 1882,	100.00
Interest on the same to May 12, 1882 (4 mo. 10 da.),	2.17
Third payment, Mar. 15, 1882,	150.00
Interest on the same to May 12, 1882 (1 mo. 27 da.),	1.42
Balance due May 12, 1882	\$2225.44

2. On a note dated Jan. 13, 1882, for \$1234, are the following indorsements:—May 17, 1882, \$234; June 16, 1882, \$345; July 27, 1882, \$123; Sept. 19, 1882, \$135. What remained due Nov. 13, 1882, at 6%? At 7%?

3. A note for \$1567, dated Jan. 14, 1881, bore the following indorsements:—Mar. 11, 1881, \$50; May 13, 1881, \$245; June 19, 1881, \$374; Aug. 30, 1881, \$412; Sept. 28, 1881, \$316.40. What was due Jan. 1, 1882, at 6%? At 5%?

4. On a note dated Aug. 17, 1881, for \$3300, were the following indorsements:—Dec. 18, 1881, \$320; Feb. 5, 1882, \$425; Apr. 13, 1882, \$550; June 29, 1882, \$630; July 16, 1882, \$375; Aug. 1, 1882, \$500. What amount was due Aug. 17, 1882, at 6%? At 10%?

5. On a note dated Mar. 16, 1883, for \$2468, are the following indorsements:—July 11, 1883, \$750; Aug. 4, 1883, \$428; Sept. 21, 1883, \$150; Nov. 12, 1883, \$170; Dec. 18, 1883, \$128; Jan. 16, 1884, \$224; Feb. 13, 1884, \$600. What is the amount due Mar. 6, 1884, at 6%? At 8%?

341. When the note runs for more than one year.

342. Since it is the custom of merchants and bankers to balance their accounts annually, the following method is used by them in computing the balance due on a note when it runs more than one year.

It is equivalent to finding the balance due yearly by the previous rule, and treating the balance as a new principal. The periodical settlements are made annually, semi annually, or quarterly, depending upon the custom of the merchant or banker in balancing his accounts. Some merchants make the end of the business year, Jan. 1 or July 1, the periodical rest, or date of settlement for notes and accounts.

When payments are made yearly greater than the interest due, this rule is the same as the New Hampshire rule for notes "with interest annually." (See complete edition, Art. 377.)

343. RULE.—*Find the amount of the principal for one year; also of each payment made during the year from the time the payment was made to the end*

of the year (1 yr. from the date of the note). From the amount of the principal, subtract the sum of the payments, including interest. With the remainder as a new principal, proceed thus for each entire year that follows, and for the interval between the end of the last year and the final date of settlement.

EXAMPLES.

344. 1. By the above rule, find the balance due Jan. 19, 1885, on a note for \$2400 dated Aug. 1, 1881, on which the following payments have been made:—Apr. 21, 1882, \$200; Dec. 1, 1882, \$25; Aug. 1, 1883, \$100; July 7, 1884, \$400. (Time by Compound Subtraction.)

OPERATION.			
Face of note, or principal, Aug. 1, 1881,	.	.	\$2400.00
Interest on the same for 1 year,	.	.	144.00
Amount, Aug. 1, 1882,	.	.	2544.00
First payment, Apr. 21, 1882,	.	\$200.00	
Interest on the same to Aug. 1, 1882 (3 mo. 10 da.),	.	3.33	203.33
Balance and new principal, Aug. 1, 1882,	.	.	2340.67
Interest on the same for 1 year,	.	.	140.44
Amount, Aug. 1, 1883,	.	.	2481.11
Second payment, Dec. 1, 1882,	.	\$25.00	
Interest on the same to Aug. 1, 1883 (8 mo.),	.	1.00	
Third payment, Aug. 1, 1883,	.	100.00	126.00
Balance and new principal, Aug. 1, 1883,	.	.	2355.11
Interest on the same for 1 year,	.	.	141.31
Amount, Aug. 1, 1884,	.	.	2496.42
Fourth payment, July 7, 1884,	.	\$400.00	
Interest on the same to Aug. 1, 1884 (24 da.),	.	1.60	401.60
Balance and new principal, Aug. 1, 1884,	.	.	2094.82
Interest on the same to date of settlement, Jan. 19, 1885			
(5 mo. 18 da.),	.	.	58
Balance due Jan. 19, 1885,	.	.	\$215.17

2-9. Solve Examples 2-9, Art. 336, according to the mercantile rule.

RATIO AND PROPORTION.

DEFINITIONS.

345. **Ratio** is the relation of two numbers as expressed by the quotient of the first divided by the second. Thus the ratio of 6 to 3 is $6 \div 3$, or 2.

1. There is no ratio between quantities of different kinds; as 6 *bu.* and 3 *ft.* But a ratio exists between quantities of the same kind though of different denominations; as 6 *ft.* and 8 *in.* To express the ratio in such cases, the quantities must first be reduced to the same denomination. Thus, the ratio of 6 *ft.* to 8 *in.* is $72 \text{ in.} \div 8 \text{ in.}$, or 9.

2. The ratio between two numbers is denoted by placing a colon (the sign of division without the horizontal line) between them. Thus, the ratio of 6 to 3 is expressed 6 : 3.

346. The numbers whose ratio is expressed are the **terms** of the ratio. The two terms of a ratio form a **couplet**, the first of which is the **antecedent**, and the second, the **consequent**.

347. **Proportion** is an equality of ratios.

The ratio of 6 *yd.* to 3 *yd.* is 2, and the ratio of \$24 to \$12 is 2; hence from the two equal ratios the following proportion can be formed — 6 *yd.* : 3 *yd.* = \$24 : \$12. This expression is read, "The ratio of 6 *yd.* to 3 *yd.* equals the ratio of \$24 to \$12." In place of the sign of equality (=), four dots (: :) are generally used; thus, 6 *yd.* : 3 *yd.* :: \$24 : \$12. The expression is also read, "6 *yd.* is to 3 *yd.* as \$24 is to \$12."

348. The first and fourth terms of a proportion are called the **extremes**; and the second and third, the **means**.

349. **PRINCIPLES.**—1. *The product of the means is equal to the product of the extremes.*

2. *A missing mean may be found by dividing the product of the extremes by the given mean.*

3. *A missing extreme may be found by dividing the product of the means by the given extreme.*

350. To solve examples by proportion.

Ex. If 24 hats costs \$27, what will 32 hats cost?

ANALYSIS.—For convenience, make the fourth term the missing term, or the required answer. Since the third and fourth terms must be of the same denomination and the denomination of the answer will be dollars, take \$27 as the third term. From the nature of the example, the answer will be more than \$27, the third term, therefore make 32 hats the second term, and 24 hats the first term. The proportion will then be stated as follows: 24 hats : 32 hats :: \$27 : x (Let x represent the unknown term). Multiplying 32 by 27, and dividing the product by 24, the fourth or missing term will be \$36.

351. RULE.—*For convenience, take for the third term the number that may form a ratio with, or is of the same denomination as, the answer. If from the nature of the example, the answer is to be greater than the third term, make the greater of the two remaining terms (which must be of the same denomination) the second term; when not, make the smaller the second term. Then multiply the means (the second and third) together, and divide their product by the given extreme (the first term).*

NOTE.—After the example is stated, any factor of the given extreme may be cancelled with an equal factor of either of the means.

EXAMPLES.

352. Find the missing term (represented by x) in each of the following proportions (See Principles, Art. 349):

- | | |
|---|---|
| 1. $16 : x :: 24 : 18.$ | 5. $\$48 : \$75 :: \$32 : x.$ |
| 2. $x : 27 :: 18 : 54.$ | 6. $\$375 : \$144 :: 625 \text{ lb.} : x.$ |
| 3. $32 : 27 :: x : 135.$ | 7. $\$1728 : \$288 :: \$666 : x.$ |
| 4. $24 \text{ bu.} : 32 \text{ bu.} :: \$27 : x.$ | 8. $144 \text{ yd.} : 175 \text{ yd.} :: \$18 : x.$ |
| 9. If 19 yd. of silk cost \$28.50, what will 37 yd. cost? | |

10. If 64 *yd.* of carpet 36 *in.* wide will cover a floor, how many yards 27 *in.* wide will be required to cover it?

11. A cane 3 *ft.* 3 *in.* high casts a shadow $5\frac{1}{2}$ *ft.* long; how long a shadow is cast by a steeple, 234 feet high?

12. If the freight of a long ton (167, 3) is 70 shillings, what is the freight of 16375 pounds?

13. The net assets of a bankrupt are \$27675, and the liabilities \$138375. How much must be paid to Mr. A, whom he owes \$4800?

14. A building is insured in several companies for \$28000. During a fire the building is damaged to the amount of \$13500. What is the loss of company A, whose risk is \$5000?

15. A invests in business \$8450, and B \$7200. The gain is \$3474.30, which is divided in proportion to the investments. What is each partner's share?

16. The assessed value of the property of a certain town is \$325000, and the total tax is \$10238. How much is the tax of Mr. A, whose property is valued at \$5700?

17. A bankrupt whose assets were \$43225, pays 44 cents on a dollar; what did his debts amount to?

18. A company with a capital of \$250000 divides \$8750 among its stockholders. How much will be received by a stockholder who owns 36 100-dollar shares?

19. If a long ton of coal is worth \$4.25, what is the value of a short ton?

20. If a farm valued at \$4500 is taxed \$26.24, what should be the tax on property valued at \$23500?

21. A piece of land 40 rods long and 4 rods wide contains an acre; what is the breadth of a piece 32 rods long, that is equivalent to an acre?

22. A merchant gains \$625 by selling \$12000 worth of goods; what amount must he sell to gain \$8000?

23. Find the value of 6 *T.* (2240 *lb.*) 7 *cwt.* 2 *qr.* 20 *lb.* of iron at 85s. per ton.

24. How many feet of boards will be required for a fence 764 *ft.* long, if 888 *ft.* of boards are required for 288 *ft.*?

INSURANCE.

DEFINITIONS.

353. Insurance is a contract by which one party (The Insurer or Underwriter) engages for a stipulated consideration (The Premium) to make up a loss which another may sustain.

Insurance is effected on property against loss or damage by fire and water, and on lives of persons. (For Life Insurance, see Art. 474.)

Insurance is also effected against accidents to persons, the breakage of plate-glass, and the loss of live-stock.

354. An Insurance Company is a company or corporation which insures against loss or damage.

Insurance companies usually make a specialty of a certain kind of insurance, as Fire, Marine, Life, Accident, etc. Certain companies combine Fire and Marine Insurance, while some of the large English companies have Fire, Marine, and Life departments.

355. Insurance companies may be classified according to principles of organization as follows:—1, Stock; 2, Mutual; 3, Mixed, or Stock and Mutual.

356. A Stock Insurance Company is one in which the capital is owned by individuals, called stockholders. They alone share the profits and are liable for the losses.

The business of a stock company and also of a mixed company, is managed by directors chosen by the stockholders. No policyholder, unless a stockholder, has any voice in any way in the election of the officers, or in the management of its business.

357. A **Mutual Insurance Company** is one in which there are no stockholders, and the profits and losses are shared among those who are insured (the policyholders).

Non-participating policies, the holders of which do not share in the profits or losses, are issued by certain mutual and mixed companies.

358. A **Mixed Insurance Company** is one which is conducted upon a combination of the stock and mutual plan.

Usually in a mixed company, all profits above a limited dividend to the stockholders are divided among the participating policyholders.

359. The **Policy** is the written contract between the Insurance Company (the Insurer or Underwriter) and the Insured. It contains a description of the property insured, the amount of the insurance, and the conditions under which the policy is issued, etc.

360. The **Premium** is the amount paid for the insurance.

1. Premium rates are expressed by giving the cost in cents of \$100 insurance. The rate is sometimes expressed as a certain per cent. of the amount of the risk. Thus, a rate of 75 cents per \$100 is equivalent to $\frac{3}{4}\%$.

2. The premium rates depend upon the nature of the risk, and the length of time for which the policy is issued.

3. A fee of \$1, or \$1.25, is sometimes charged for the policy in addition to the premium.

361. An **Insurance Agent** is a person who represents an insurance company or several companies, and acts for them in soliciting business, collecting premiums, adjusting losses, etc.

362. An **Insurance Broker** is a person who effects insurance, for negotiating which he receives a commission or brokerage from the company taking the risk.

Brokers are regarded as agents of the insured, and not of the insurance company.

FIRE INSURANCE.

363. Fire Insurance refers to insurance against loss or damage by fire.

Fire policies are usually issued for periods of from 1 to 5 years. Certain companies issue policies for longer periods.

364. Adjustment of Losses.—In an ordinary fire insurance policy, a person who insures will be paid the extent of his loss up to the amount of his insurance; but in policies containing the “average clause,” the payment is such proportion of the loss as the amount of the insurance bears to the total value of the property.

1. The usual form of the “average clause” is as follows: “It is a condition of this insurance, that if the whole value of the above described property, contained in any or all of the above mentioned buildings and premises, shall exceed the whole amount of insurance thereon, then, in case of loss or damage by fire, this policy shall contribute to the payment of said loss or damage in the proportion only that the whole amount of insurance on said property shall bear to the whole value of said property, in all of said buildings, at the time said loss or damage may occur.”

2. Under a policy containing the “average clause,” a person who insures \$5000 on property worth \$10000, would receive only \$2500 in case of an actual loss of \$5000; \$1500 in a loss of \$3000.

3. Insurance companies usually reserve the privilege of replacing or repairing the damaged premises.

365. A Floating Policy is one which covers property stored in several buildings or places. The name is applied more particularly to policies which cover goods whose location may be changed in process of manufacture or in the ordinary course of business. The “average clause” is a usual condition of policies of this class.

366. Short Rates are rates for a term less than a year.

If a policy is terminated at the request of the policy-holder, the company retains the customary “short rates” for the time the policy has been in force; if terminated by the company, a ratable proportion of the premium is refunded for the unexpired term of the policy.

MARINE INSURANCE.

367. Marine Insurance refers to insurance of vessels and their cargoes against the dangers of navigation.

1. Inland and Transit Insurance refer to insurance of merchandise while being transported from place to place either by rail or water routes, or both.

2. Policies on cargoes are issued for a certain voyage, or from port to port, and on vessels for a specified time or for a certain voyage.

3. The particular average clause is the clause which exempts the insurance company from the payment of any partial loss or particular average, unless it exceeds a certain per cent. of the value of the property. The particular average clause is sometimes applied to the value of each parcel or series of parcels, according to invoice numbers.

4. Insurance Certificates, showing that certain property has been insured, and stating the amount of the insurance and the name of the party abroad who is authorized to make the settlement, are issued by marine companies. They are negotiable, and are usually sent to the consignee of the merchandise to make the loss payable at the port of destination, and to otherwise facilitate the adjustment of the insurance in case of loss.

368. Adjustment of Losses.—In marine insurance, in case of loss or damage, the insurance company contributes such proportion of the loss as the amount of the insurance bears to the total value of the property.

The adjustment of marine losses is the same as the adjustment of fire losses on policies containing the "average clause" (**364, 1**).

369. An Open Policy is one upon which additional insurances may be entered at different times. It covers merchandise which may be shipped on "Vessel or Vessels" from "Ports and Places" to "Ports and Places," for amounts "as endorsed" and at rates "as agreed."

1. The date of the shipment, name of vessel, ports of shipment and destination, the amount of the insurance, rate, premium, and a description of the property are entered on the policy or in a pass-book, which is regarded as part of the policy. (See Ex. 23, Art. **370**)

2. Open policies with pass-books attached and insuring merchandise against loss or damage by fire, are issued by fire insurance companies.

3. Open policies, which cover all risks whether accepted and endorsed on the policy or not, are issued to merchants who are receiving merchandise from foreign countries, and who do not always have a definite knowledge of the time and mode of shipment. Such policies usually contain the following clause: "The company are to be entitled to premiums at their usual rates on all shipments reported or not. It is warranted by the assured to report every shipment on the day of receiving advice thereof, or as soon thereafter as practicable, when the rate of premium shall be fixed by the President of the Company.

The above policies cover the invoice cost and 10% additional until the amount of the risk is endorsed on the policy or pass-book.

4. Open policies are sometimes issued which cover only such risks as may be accepted and endorsed on the policy by the company.

EXAMPLES.

370. 1. A building was insured for \$2500 in one company at $1\frac{1}{4}\%$, and for \$5000 in another company at 125 cents. What was the total premium paid?

2. A cargo of goods was insured for \$9000 at $\frac{3}{4}\%$. Find the cost of the insurance, \$1.25 being charged for the policy.

3. What is the total premium of the following insurances: \$5000 at $1\frac{1}{2}\%$ for 2 years, \$7000 at 45¢ for 5 years, \$2000 at 5% for 7 years, \$3500 at 45¢ for 1 year, \$2000 at 70¢ for 4 years, \$4000 at $1\frac{1}{4}\%$ for 5 years, \$2000 at 60¢ for 4 years, \$4500 at 25¢ for 2 years, \$3600 at 125¢ for 1 year, and \$3000 at 240¢ for 4 years?

4. \$20 were paid for an insurance of \$2500; what was the premium rate?

5. \$25.20 were paid for an insurance at the rate of 70¢ per \$100. What was the amount of the risk?

6. A factory was insured for \$7500 for 1 year at $2\frac{1}{2}\%$, stock for \$2500 at $2\frac{1}{2}\%$, and raw material for \$2500 at $1\frac{1}{4}\%$. What was the total premium?

7. What is the cost of insuring a house for \$5000 at the rate of 45¢ per \$100?

8. A cargo of merchandise was insured for \$6500 at $\frac{4}{5}\%$, including the risk of fire while on wharf awaiting shipment. What was the premium?

9. A building was insured Jan. 1, 1880, for \$2000, for 7 years, at 5%; what was the value of the unearned premium, Jan. 1, 1882?

10. A shipment of goods was insured in the Pacific Mutual Insurance Co. for \$9600 at 75¢ less 20% in lieu of scrip and interest. What was the net cost of the insurance?

11. A house was insured for \$5000 for 1 year at \$2.40. The house was destroyed by fire. What was the actual loss of the company, making no allowance for interest?

12. Suppose the above house was worth \$8000. What was the actual loss of the owners?

13. A cargo of hides from Montevideo to New York having increased in value since the insurance was effected, the anticipated profits were insured for \$3000 at $1\frac{3}{4}\%$ less 20%. What was the premium?

14. A factory (worth \$3000) and its contents are insured for \$10000 as follows: \$2000 on building, \$3000 on machinery (worth \$5000), and \$5000 on stock (worth \$8000). The building is damaged by fire to the amount of \$1000, the machinery \$4000, and the stock is a total loss. How much is the claim against the insurance company?

15. A cargo of goods valued at \$20000 was insured for \$12000. If the goods were damaged to the amount of \$15000, how much of the loss would be paid by the insurance company? (368.)

16. What was paid for insuring a cargo of merchandise for \$8750 at $\frac{7}{8}\%$ less 20%?

17. A stock of goods was insured, May 1, for 1 year, for \$6000, at 90¢. The policy was cancelled Nov. 1, at the request of the insured. How much was the return premium, the short rate for 6 months being 63¢? How much would have been returned by the company, if the policy had been cancelled at its request?

18. A building is insured in several companies for \$60000, and is damaged by fire to the extent of \$24000. What per cent. of its risk is paid by each company?

19. A quantity of merchandise valued at \$6000 is insured for \$5000. It is damaged by fire to the amount of \$1728. How much of the loss is paid by the insurance company, the policy containing the "average clause" (367)?

20. A factory and its contents are insured for \$5000 in company M, \$5000 in N, \$5000 in O, \$4000 in P, and \$2500 in each of the following companies : Q, R, S, T, U, V, W, X, Y, and Z. What is the premium at 2% less 10%?

21. The above insurance covers the following property: \$4000 on building marked A on plan, \$4000 on B, \$5000 on C, \$500 on D, \$500 on E, \$3500 on stock and materials in building marked A on plan, \$8000 on machinery, etc., in A, \$11500 on stock and materials in B and C, \$4000 on machinery, etc., in B and C, \$2500 on horses in D, \$500 on harness, hay, feed, etc., in D. Suppose building A and its contents were totally destroyed by fire, what would be the loss of company M? Of P? Of T?

NOTE.—The above insurance is divided pro rata among the several companies, each policy designating the amount on each building.

22. What is the amount of the risk of company M on the building marked A on plan? On C?

23. Find the premium of the following "open policy."

[illegible]

EXCHANGE.

DEFINITIONS.

371. Exchange is the system by which merchants in distant places discharge their debts to each other without the transmission of money.

Suppose, for example, A of New York owes B of Chicago \$1000 for grain, and C of Chicago owes D of New York \$1000 for dry goods. The two debts may be discharged by means of one draft or bill of exchange without the transmission of money. Thus, B of Chicago draws on A of New York for \$1000, and sells the draft to C of Chicago who remits it to D of New York. D of New York presents the draft to A of New York for acceptance or payment, and thus both debts are cancelled. There is in effect a setting-off or exchange of one debt for the other.

The business of exchange is usually conducted through the medium of banks and bankers, who buy commercial bills and transmit them for credit to the places on which they are drawn. They also sell their own drafts on their correspondents in any amounts demanded.

372. A Bill of Exchange, or Draft, is an order or request addressed by one person (the Drawer) to another (the Drawee), directing the payment of a specified sum of money to a third person (the Payee) or to his order. It is issued at one place and payable at another. (See Art. 324, 5-6.)

For brevity, bills of exchange are frequently called "exchange."

According to the laws of most States, drafts drawn in one State and payable in another, are termed *foreign* bills of exchange. For the purposes of this book, the term "domestic exchange" will be applied to bills drawn and payable in the United States.

373. Bills of exchange are of two kinds, Inland or Domestic, and Foreign.

374. A Domestic or Inland Bill of Exchange is one which is payable in the same country in which it is drawn.

375. A Foreign Bill of Exchange is one which is payable in a different country from the one in which it is drawn; as a draft drawn in the United States and payable in England.

376. When drafts sell for more than their face value, exchange is above par or at a premium; when for less than their face, below par or at a discount.

DOMESTIC EXCHANGE.

377. Domestic or Inland Exchange relates to drafts drawn at one place on another in the same country.

378. The domestic exchanges on New York at the places named were quoted as follows, May 7, 1881: Savannah, $\frac{1}{8}$ @ $\frac{3}{8}$ premium; Charleston, $\frac{1}{8}$ @ $\frac{1}{4}$ premium; New Orleans, \$1.50 @ \$2.50 premium; St. Louis, 25 cents premium; Chicago, 50 @ 75 cents premium; and Boston, 25 cents discount.

1. At Savannah and Charleston the rates per cent. of the premium or discount are given. Thus, when exchange is quoted at $\frac{1}{4}$ premium, a draft of \$100 may be purchased for \$100 $\frac{1}{4}$ (\$100.25).

2. At New Orleans, St. Louis, Chicago, and Boston, the premium or discount per \$1000 is given. Thus, a draft of \$1000 at \$2.50 premium may be purchased for \$1002.50. \$2.50 per \$1000 premium is equivalent to $\frac{1}{4}$ % premium.

3. The rate of domestic exchange is limited by the cost of shipping gold or currency by express, and the premium or discount will not exceed this cost. Thus, if a merchant in Chicago is charged a premium of \$10 for a draft of \$10000, and he can send the currency by express for \$7.50, it will be to his advantage to remit by the latter method.

4. The preceding quotations refer to sight exchange. Time drafts are discounted in the same manner as promissory notes. In certain cases bankers in discounting notes and drafts payable in distant places charge interest for the time required for the return of the money when

the note or draft is paid; and in the case of drafts drawn a certain number of days after sight, bankers sometimes charge interest for the time required for the acceptance of the drafts. Thus, if a draft was drawn in New York on St. Louis and payable 60 days after sight, it would require, in the ordinary course of the mails, 3 days for the acceptance of the draft. The draft would be paid in 63 days (including the days of grace), and 3 days would elapse before the money would be returned to New York. The banker would be justified in charging interest for 69 days, the interval between the day he advanced the money in New York, and the day it was returned to him again. Between New York and San Francisco and other distant places, money is frequently transferred by telegraph.

EXAMPLES.

379. 1. What is the value in Savannah of a draft on New York for \$8150 at $\frac{3}{8}\%$ premium?

2. Find the cost in New Orleans of a draft on New York for \$8375 at \$2.50 premium.

Find the value of the following drafts:

Face.	Exchange.	Face.	Exchange.
3. \$5000,	$\frac{1}{8}\%$ premium.	7. \$4287.75,	15¢ discount.
4. \$4375,	$\frac{3}{8}\%$ discount.	8. \$3416.33,	25¢ premium.
5. \$8417,	$\frac{1}{8}\%$ premium.	9. \$2825.49,	\$1.25 discount.
6. \$9873,	$\frac{1}{2}\%$ premium.	10. \$9873.62,	\$2.50 premium.

11. A of Chicago buys cattle for B of New York to the amount of \$9858.07. How large a draft should be drawn on B, so that when sold at a discount of 50¢ ($\frac{1}{2}\%$), the proceeds would be sufficient to pay the bill?

NOTE.—To find the face of a draft, instead of dividing the value of the draft by the rate of exchange (in the above example, $99\frac{1}{2}\%$ or .9995), business men and bankers calculate the premium or discount on the value of the draft, and subtract or add it to the value as the case requires. Thus, in the above example, the discount would be $\frac{1}{2}\%$ of $\frac{1}{16}\%$ of \$9858.07, or \$4.93, which added to the given proceeds would produce the face \$9863. This method produces too small a result in all cases, the error being equivalent to the percentage of the premium or discount. In this example the error is less than $\frac{1}{2}$ cent.

For ordinary examples in business, the foregoing method is suffi-

cienly accurate. At $\frac{1}{2}\%$, or \$5.00 (a very high rate for domestic exchange) on a draft whose value is \$10000, the error would be only 25 cents. If greater accuracy is required, the necessary correction can be made by adding the percentage of the premium or discount. Thus, if the value of the draft is \$10000, and exchange is $\frac{1}{2}\%$ discount, the face would be $\$10000 + \50 ($\frac{1}{2}\%$ of \$10000) $+ \$0.25$ ($\frac{1}{2}\%$ of \$50) = \$10050.25. If at $\frac{1}{2}\%$ premium, the face would be $\$10000 - \$50 + \$0.25 = \9950.25 .

Find the face of the following drafts:

Value.	Exchange.	Value.	Exchange.
12. \$1876.16,	$\frac{1}{4}\%$ premium.	16. \$7375.	25¢ premium.
13. \$2437.75,	$\frac{1}{4}\%$ discount.	17. \$9218,	50¢ discount.
14. \$3342.38,	$\frac{1}{8}\%$ discount.	18. \$6438,	\$1.00 premium.
15. \$2238.42,	$\frac{1}{2}\%$ premium.	19. \$9243,	\$1.25 premium.

20. A of New Orleans being indebted to B of New York \$9316.75, forwards to him a check on a New Orleans bank for that amount, to cash which B is obliged to allow a discount of $\frac{1}{4}\%$. How much does A still owe B, and for what amount should the check have been drawn to net B the amount due?

21. What is the value of a draft on New York for \$3000, payable in 60 days (63 days) after date (325, 7), exchange being $\frac{1}{8}\%$ premium, and interest 6%?

NOTE.—From the face of the draft, subtract the interest, and to the result add the exchange.

22. Find the proceeds of a draft drawn at Chicago on New York for \$12000, and payable 90 days after sight, exchange 50¢ discount, interest 5%, and allowing 3 days additional for the acceptance of the draft.

23. A merchant paid \$6920.64 in Charleston for a sight draft of \$6912; what was the rate of exchange?

24. A commission merchant sold 13475 pounds of leather at $26\frac{1}{4}$ cents a pound. If his commission is 5%, and exchange $\frac{1}{8}\%$ premium, how large a draft can he buy to remit to the consignor?

FOREIGN EXCHANGE.

380. Foreign Exchange relates to drafts or bills of exchange drawn in one country and payable in another.

Foreign bills of exchange are usually drawn in the moneys of account of the countries in which they are payable. Thus, drafts on England are usually drawn in pounds, shillings, and pence; on France, Belgium and Switzerland, in francs; on Germany, in marks; on the Netherlands (Holland), in guilders.

Foreign bills of exchange are usually drawn at sight (3 days) or at sixty (63 days) days' sight. Sight drafts are frequently called "short" exchange, and 60 day drafts, "long" exchange. "Long" exchange is sold at a rate below that for "short" exchange, sufficient to equalize the difference in interest between the dates of maturity of the two classes of bills.

381. To secure safety and speed in the transmission of foreign bills of exchange, they are drawn in sets of two or three of the same tenor and date. The separate bills are sent by different steamers, and when any one of them is paid, the others become void. Some merchants send only the first and second, and preserve the third.

SET OF EXCHANGE.

(1.)

EXCHANGE FOR £1000.

NEW YORK, *May 16, 1882.*

Sixty days after sight of this FIRST of Exchange (Second and Third unpaid). Pay to the order of H. B. CLAFLIN & Co., One Thousand Pounds Sterling, value received, and charge the same to account of

No. 1738.

BROWN BROTHERS & Co.

To BROWN, SHIPLEY & Co., }
London, England. }

The above is the form of the *first bill*. In the *second* the word "FIRST" is changed into "SECOND," and "Second and Third unpaid" into "First and Third unpaid." The *third* is changed similarly.

382. The Intrinsic Par of Exchange is the value of the monetary unit of one country expressed in that of

another, and is based upon the comparative fineness and weight of the coins, as determined by assay.

The intrinsic par of exchange between different countries and the United States, is given in Art. 187.

383. The Commercial Par of Exchange is the market value in one country of the *coins* of another.

384. The Commercial Rate of Exchange is the market or buying and selling value in one country of the *drafts* on another.

1. In giving quotations of foreign exchange, no reference is made to the par value, the quotations being given by means of equivalents.

2. Premium or discount for exchange can not long exceed the transportation charges and insurance of shipping coin; for, if a merchant can ship gold cheaper than he can buy a bill of exchange, he will choose the former method of paying his indebtedness. When sight exchange is 4.84, gold can be imported at a small profit; and when sight exchange is 4.89½, gold can be exported at a profit.

385. The quotations of foreign exchange, Apr. 20, 1881, were as follows:

Where payable.	60 days.	Sight.
London:		
Prime bankers'	4.81½	4.84
Good bankers' and prime commercial	4.81	4.83½
Documentary commercial	4.78½	4.81½
Cable transfers	4.84½	
Paris (francs)	5.27½	5.24½
Antwerp (francs)	5.27½	5.24½
Swiss (francs)	5.26¾	5.23½
Amsterdam (guilders)39½	.397½
Hamburg (reichsmarks)93¾	.94¾
Frankfort (reichsmarks)93¾	.94¾
Bremen (reichsmarks)93¾	.94¾
Berlin (reichsmarks)93¾	.94¾

In the above quotations, exchange is below par. (See intrinsic par values below, or in Art. 187.) When exchange is above par, we are exporters of gold; when below par, we are importers of gold.

386. Exchange on England (Sterling exchange) is quoted by giving the value of £1 in dollars and cents.

Thus, when exchange is 4.84, a draft of £1 will cost \$4.84; of £100, \$484. The intrinsic par value of £1 is \$4.8665 (187).

387. Exchange on France, Belgium, and Switzerland is quoted by giving the value of \$1 in francs and centimes (hundredths of a franc).

Thus, when exchange is $5.27\frac{1}{2}$, \$1 will buy a bill of 5 francs and $27\frac{1}{2}$ centimes; a draft of 1000 francs will cost \$189.57 ($1000 \div 5.27\frac{1}{2}$). The intrinsic par value of 1 franc is $19\frac{3}{10}$ cents (187); of the equivalent exchange, $5.18\frac{1}{8}$ ($1.00 \div .193$).

In French, Belgian, and Swiss exchange, the higher the apparent rate, the less the value of the draft. Thus, when exchange is 5.13, a draft of 1000 francs is worth \$194.93, and each franc is worth $19\frac{4}{10}$ cents. When exchange is $5.26\frac{3}{8}$, the same draft would be worth \$189.98, and each franc 19 cents.

388. Exchange on Amsterdam is quoted by giving the value of one guilder (gülden) or florin in U. S. cents.

The intrinsic par value of 1 guilder is $40\frac{2}{10}$ cents (187).

389. Exchange on Germany is quoted by giving the value of 4 reichsmarks in cents.

The intrinsic par value of 1 mark is $23\frac{8}{10}$ cents (187); of 4 marks $95\frac{2}{10}$ cents.

390. Documentary Exchange is a bill drawn by a shipper upon his consignee against merchandise shipped, accompanied by the bill of lading, "to order," the insurance certificates, covering the property against which the bill is drawn, and the letter of hypothecation.

391. Exchange on London in the countries named, and a London on the same countries, is quoted as follows:

United States, by giving the value of £1 in dollars and cents.

France and Belgium, by giving the value of £1 in francs and centimes.

Germany, by giving the value of £1 in marks and pfenniges.

Austria, by giving the value of £1 in florins and kreutzers.

India, by giving the value of 1 rupee in shillings and pence.

EXAMPLES.

392. 1. Find the cost of a bill of exchange on London for £225 at $4.81\frac{3}{4}$.

ANALYSIS.—If £1 costs $\$4.81\frac{3}{4}$, £225 will cost 225 times $\$4.81\frac{3}{4}$.

2. What is the value of a draft for £324 16s. at $4.87\frac{1}{2}$?

NOTE.—Write one-half of the greatest even number of shillings as tenths of a pound, and if there be an odd shilling write 5 hundredths. Thus, £324 16s. = £324.8; £324 17s. = £324.85. (See Art. 199, Ex. 6, Note.) The value of £324 16s. at $4.87\frac{1}{2}$ is found by multiplying $\$4.87\frac{1}{2}$ by 324.8.

3. Find the value of a draft on London for £379 12s. 7d., at $4.86\frac{3}{8}$.

OPERATION.

379.6

$4.86\frac{3}{8}$

949

475

22776

30368

15184

14

1846.420

ANALYSIS.—If each penny be regarded as 2 cents, the result will be sufficiently accurate. For 11d. the maximum number of pence in any example, and exchange at 4.91, the error would be only $\frac{1}{2}$ cent. $\$4.86\frac{3}{8} \times 379.6 = \1846.28 . $\$1846.28 + \$0.14 = \$1846.42$. To save one addition, add the 14 cents to the partial products as in the operation. Or,

£379 12s. 7d. = £379.629. $\$4.86\frac{3}{8} \times 379.629 = \1846.42

Find the value of

4. £500 at $4.81\frac{1}{2}$.

8. £512 13s. at $4.84\frac{3}{8}$.

5. £775 at $4.85\frac{1}{4}$.

9. £834 6s. 6d. at $4.88\frac{5}{8}$.

6. £837 at $4.83\frac{3}{4}$.

10. £675 11s. 8d. at $4.87\frac{1}{2}$.

7. £84 8s. at 4.85.

11. £225 7s. 5d. at $4.82\frac{3}{4}$.

12. Find the cost of a bill of exchange on Liverpool, for £875 12s. 6d. at the par value.

13. What are the proceeds of a draft of £959 5s. 4d., sold through a broker, at $4.79\frac{1}{2}$, brokerage $\frac{1}{4}\%$?

14. An exporter sold a draft for £540 3s. on Manchester, payable in London, at 4.84, brokerage $\frac{1}{4}\%$. What were the proceeds?

30. What is the value of a draft on London for £416 16s. 3d., at $4.85\frac{3}{8}$?

31. What is the cost of a draft on Paris for 12420 francs. at $5.19\frac{3}{4}$, brokerage on exchange $\frac{1}{8}\%$?

32. What will it cost to remit to Antwerp 8750 francs at the par value?

33. Sold through a broker a draft on Geneva for 7324 francs. What were the proceeds, exchange being $5.18\frac{3}{8}$, brokerage $\frac{1}{8}\%$?

34. What will be the face of a bill of exchange on Geneva that can be bought for \$15372, exchange selling at $5.22\frac{1}{2}$?

35. Paid for a draft on Paris \$3460.32; what was the face of the draft, exchange being $5.19\frac{3}{8}$, and brokerage $\frac{1}{8}\%$?

36. A merchant paid \$6272 for a bill of exchange of 32512.48 francs; what was the rate of exchange?

37. Find the cost of a bill of exchange on Hamburg for 14400 marks (Reichsmarks) at $94\frac{1}{8}$.

OPERATION.

4) 14400

3600

.94 $\frac{1}{8}$

3388.50

ANALYSIS.—Since 4 marks cost $\$0.94\frac{1}{8}$, 14400 marks will cost 3600 ($14400 \div 4$) times $\$0.94\frac{1}{8}$, or \$3388.50.

Find the value of

38. 7200 marks at 94.

41. 1237 marks at $93\frac{5}{8}$.

39. 8416 marks at $93\frac{1}{2}$.

42. 9894 marks at $95\frac{3}{8}$.

40. 3456 marks at $95\frac{1}{4}$.

43. 6515 marks at $94\frac{3}{8}$.

44. What is the cost of a bill of exchange on Frankfort for 16200 marks at $95\frac{1}{2}$?

45. Sold a bill of exchange on Hamburg for 13200 marks, at $94\frac{1}{8}$; what was the amount received, brokerage $\frac{1}{8}\%$?

46. An importer purchased a bill of exchange on London for £318 10s. 7d., at $4.85\frac{3}{4}$; what did it cost?

47. What were the proceeds of a draft, sold through a broker, for 8748 marks, at $94\frac{3}{8}$, brokerage $\frac{1}{8}\%$?

48. An exporter sold a draft on Paris for 12275 francs, at 5.19 $\frac{5}{8}$; what were the proceeds, brokerage $\frac{1}{8}\%$?

49. What is the face of a bill on Hamburg that cost \$816, exchange 94 $\frac{1}{8}$?

ANALYSIS.—Since \$.94 $\frac{1}{8}$ will buy 4 marks, \$816 will buy 4 times as many marks as \$0.94 $\frac{1}{8}$ is contained times in \$816.

50. What is the face of a 3 days' draft on Bremen, that was purchased in New York for \$3261.60, exchange 94 $\frac{3}{8}$?

51. The cost of a draft of 12320 marks was \$2922.15; what was the rate of exchange?

52. Find the cost of a bill of exchange on Amsterdam, for 7240 guilders, at 40 $\frac{1}{8}$.

53. Find the cost of a bill of exchange on Amsterdam, for 12480 guilders, exchange 39 $\frac{7}{8}$, brokerage $\frac{1}{8}\%$.

54. An exporter received \$1890.86 for a bill of exchange on Amsterdam; what was its face, exchange being 41 $\frac{1}{2}$, brokerage $\frac{1}{8}\%$?

55. At 40 $\frac{3}{8}$, how much exchange on Amsterdam will \$2877.93 buy?

56. The value of a draft of 5280 guilders is \$2145; what is the quotation?

57. The dividends of the N. Y. C. and H. R. R. Co., are paid in London at the rate of 49 $\frac{1}{8}$ pence to the dollar. What is the equivalent rate of exchange?

58. Find the value in U. S. money of 16319 bushels of wheat at 4s. 4 $\frac{1}{2}$ d. per bushel, exchange 4.86 $\frac{1}{2}$.

59. A merchant sent a messenger with a bill of exchange of 20000 francs to two bankers, A and B, with instructions to sell it to the best advantage. A offered 5.27 and B 5.27 $\frac{1}{2}$. The messenger imprudently accepted the latter offer. How much did the merchant lose by the ignorance of the messenger?

60. A commission merchant wishes to remit \$2475 to his principal in England. How large a draft must he purchase, exchange being 4.83 $\frac{1}{4}$?

EQUATION OF ACCOUNTS.

DEFINITIONS.

393. Equation of Accounts (also called Equation of Payments and Averaging Accounts) is the process of finding the time when several debts due at different dates may be paid in one amount without loss of interest to either party. It is also the process of finding the time when the balance of an account having both debits and credits may be paid without loss of interest to either party. This time is called the *equated* or *average time*.

394. To find the equated time when the items of the account are all on the same side, i. e., all debits or all credits.

ANALYTICAL STEPS.—By assuming a certain date as the time of settlement, we find what the loss or gain of interest would be to the payer if all the bills were paid by him on that date. We next find in how many days the total amount of the bills would produce a sum equivalent to this loss or gain of interest, and find the true day of settlement by counting forward or backward this number of days from the assumed date. Thus, if the sum of the several bills is \$1000, and the loss of interest to the payer at the assumed date of settlement is \$10 (the interest of \$1000 at 60 days at 6%), it is evident that the true date of settlement, or the time when there would be no loss of interest to either party, must be 60 days after the assumed date.

NOTES.—1. The interest on the bills paid after they became due would equal the interest on the bills paid in advance, the former being a gain to the payer, and the latter, a loss.

2. Any date may be assumed as the time of settlement. For convenience, the earliest or latest date is generally used.

In Equation Tables, Dec. 31 or Jan. 1 is taken for all examples.

The assumed date is sometimes called the *focal date*.

395. Ex. At what date may the following bills of merchandise be paid in one amount without loss of interest to either party? Due Apr. 10, \$114; due Apr. 26, \$140; due May 22, \$320; due June 6, \$976.

OPERATION.

Due Apr. 10,	\$114	\times	0	=	0
“ “ 26,	140	\times	16	=	2240
“ May 22,	320	\times	42	=	13440
“ June 6,	976	\times	57	=	55632
	1550)	71312 (46 days
					after Apr. 10, or May 26.

ANALYSIS.—For convenience, assume Apr. 10, the earliest due date, as the time of settlement. If the first bill, which is due Apr. 10, is paid on that date, there will be no loss or gain of interest to either party. If the second bill, which is due Apr. 26, is paid Apr. 10, 16 days before it is due, there will be a loss to the payer of the interest or the use of \$140 for 16 days, or \$2240 for 1 day. On the third bill, there will be a loss of the interest of \$320 for 42 days, or \$13440 for 1 day. On the fourth bill, there will be a loss of the interest of \$976 for 57 days, or \$55632 for 1 day. If all the bills are paid Apr. 10, there will be a loss to the payer of the interest of \$71312 for 1 day, or of \$1550 for 46 days. Since the loss of interest to the payer is equivalent to the interest of the total amount of the bills for 46 days, it is evident that the day when there would be no loss of interest must be 46 days after Apr. 10, or May 26. The payer is entitled to defer payment 46 days after the assumed date as a compensation for the estimated loss.

The gain of interest to the payer on the first three bills, which are paid after they are due, equals the loss of interest on the fourth bill, which is paid before it is due.

PROOF.

The interest of \$114 for 46 days at 6%	\$0.874
“ “ “ 140 “ 30 “ “	.70
“ “ “ 320 “ 4 “ “	.213
Total gain of interest to the payer	1.787
The interest (a loss to the payer) of \$976 for 11 days	1.789

NOTES.—1. In finding the number of days from the assumed date to the other dates, instead of calculating from the assumed date each

time, find the interval from one date to the next and add it to the last number of days. Thus, from Apr. 10 to May 22 is 42 days, and from May 22 to June 6, 15 days; hence, from Apr. 10 to June 6 is 57 ($42 + 15$) days. (See Art. 205, Ex. 3.)

2. To determine the due date, find the number of days in the operation nearest to the quotient, and add or subtract, as may be necessary, the difference between it and the quotient, to its corresponding date. Thus, in the above example, the number of days in the operation nearest to the quotient is 42; hence the due date is 4 ($46 - 42$) days after May 22, or May 26.

3. If the fraction of the quotient is less than $\frac{1}{2}$, disregard it; if more than $\frac{1}{2}$, add 1 day to the integral number of days in the quotient.

396. RULE.—*Assume the earliest due date as the day of settlement for all the items. Multiply each item by the number of days intervening between the assumed date of settlement and the date of the item; and divide the sum of the several products by the sum of the account. Count forward from the assumed date the number of days obtained in the quotient. The result will be the equated time.*

EXAMPLES.

397. 1. At what date may the following bills be paid in one amount? Due Sept. 10, \$145; Sept. 28, \$144; Oct. 8, \$75; Oct. 23, \$512.

2. Find the average due date of the following bills, each being due at the date given: Jan. 5, \$127.85; Jan. 26, \$134.18; Feb. 5, \$249.40; Feb. 23, \$418.73; Feb. 28, \$176.25.

NOTE.—The result will be practically the same if the nearest dollar is used in multiplying or in calculating the interest. Thus, regard the above amounts as 128, 134, 249, 419, and 176 respectively.

When there are several items in the example, some accountants omit the cents and units of dollars, and use the nearest number of tens. Thus, if the above account were of sufficient length, the numbers might be regarded as 13, 13, 25, 42, and 18 respectively. In this example the result is the same, but in some examples, containing the same number of items, there would be a discrepancy of one or more days.

3. Sold a customer bills at the due dates and to the amounts specified: June 1, \$152.73; June 15, \$114.28; July 16, \$247.84; July 25, \$88.90; Aug. 18, \$735.42; Aug. 29, \$416.34. When may the whole indebtedness be equitably discharged at one payment?

4. Average the following account:

NEW YORK, *July 1, 1882.*

MESSRS. RICE, STIX & Co.,

To LORD & TAYLOR, *Dr.*

1882.									
April	4	Mdse.	30	days	per	bill	rendered.	\$816	37
"	21	"	30	"	"	"	"	724	25
May	13	"	30	"	"	"	"	342	46
"	25	"	30	"	"	"	"	535	84
June	16	"	30	"	"	"	"	628	62
Due by equation June *, 1882.								****	**

NOTE.—When several bills are sold on a common term of credit, first find the average date of purchase, and to the result add the common term of credit.

5. A. Hamilton bought of F. A. Leggett & Co., several bills of goods, as follows:

May 16,	a bill of	\$212.46	on 60 days' credit.
" 28,	"	318.40	" 60 " "
June 6,	"	275.64	" 60 " "
" 21,	"	187.83	" 60 " "
July 13,	"	835.60	" 60 " "

A 60-day note for the whole amount is given in settlement. What must be its date, no allowance being made for the days of grace?

6. Sold on a credit of 90 days the following bill of goods: Mar. 4, \$194.13; Mar. 27, \$222.36; Apr. 12, \$538.72; May 3, \$432.64; May 28, \$303.10. What is the equated time of payment? How much will settle the account Aug. 1, at 6%? How much July 1?

NOTE.—When monthly statements are sent to customers the accounts are frequently averaged. (See Ex. 5.) When the account is averaged, the simplest method of finding the cash balance due at a certain date, is to calculate the interest on the total amount from the average date to the time of payment, and add it, if the time of settlement is after the average date, and subtract it, if before.

Since a fraction of a day is not considered in determining the average date, this method of finding the cash balance is not as accurate as that of Art. 408, in which the interest is reckoned on each item separately.

7. A commission merchant sold several bills of goods, on a credit of 4 months, as follows: Aug. 16, 1881, \$387; Sept. 4, 1881, \$243.60; Sept. 18, 1881, \$637.75; Oct. 28, 1881, \$165.50; Dec. 10, 1881, \$856.45. What is the equated time of payment?

NOTE.—The above account may be averaged by first finding the average date of purchase, and adding the common term of credit; or by finding the due date of each bill separately, and determining the average due date from the dates thus found. Since the months have not uniformly the same number of days, the results by the two methods sometimes differ by one or more days, when the common term of credit is expressed in months.

8. Bought goods on 6 months' credit as follows: Feb. 16, 1881, \$376.50; Mar. 12, 1881, \$287.40; Mar. 19, 1881, \$612.87; Apr. 5, 1881, \$345.60; Apr. 26, 1881, \$134.80; June 1, 1881, \$612.35. What is the average time of maturity? How much would balance the account Jan. 1, 1882? How much Oct. 1, 1881?

9. Park and Tilford sold to R. M. Bishop & Co. the following bills of merchandise on 60 days' credit: Feb. 24, \$176.82; Feb. 28, \$327.49; March 16, \$282.75; Mar. 28, \$512.14; Apr. 7, \$438.36; Apr. 14, \$109.70; May 1, \$632.65. What is the equated time of payment, and how much would be required to balance the account June 1? How much July 1?

10. The following bills of merchandise were purchased on 4 months' credit: June 1, \$237.16; June 18, \$146.75; June 30, \$333.84; June 5, \$416; July 16, \$535.62; July 27, \$912.33; Aug. 13, \$345.60. A note payable in 4 months was given in settlement. What was its date, no allowance being made for the days of grace?

11. Bought goods on 60 days' credit as follows: Aug. 11, \$487.60; Aug. 20, \$398.30; Sept. 1, \$411.26; Sept. 13, \$283.36; Sept. 22, \$112.43; Sept. 30, \$555.55; Oct. 20, \$342.48; Nov. 4, \$337.64. What is the average due date?

12. Bought several bills of goods as stated below:

June 3,	a bill of \$375 on 30 days' credit.
“ 28,	“ 420 “ 60 “ “
July 16,	“ 560 “ 4 months' “
Sept. 4,	“ 228 “ 90 days' “

What is the equated time of payment?

NOTE.—When the bills are sold on different terms of credit, first find the due date of each bill separately as in the following operation.

OPERATION.

Date of purchase.	Credit.	Due date.	Amount.	Days.	Products.
June 3,	30 days,	July 3,	\$375 ×	0 =	0
“ 28,	60 “	Aug. 27,	420 ×	55 =	*****
July 16,	4 mo.,	Nov. 16,	560 ×	*** =	*****
Sept. 4,	90 days,	Dec. 3,	228 ×	*** =	*****
				****) ***** (** da

13. What is the equated time for the payment of the following bills?

July 5,	1882,	\$516.60 on 4 months' credit.
“ 28,	“	327.35 “ 60 days' “
Aug. 15,	“	147.84 “ 4 months' “
Sept. 8,	“	485.42 “ 30 days' “
“ 25,	“	230.39 “ 60 “ “

14. Sold several bills of goods as follows :

May 4,	a bill of \$418.75	on 30 days	credit.
" 16,	" 322.86	" 60 "	"
June 1,	" 513.44	" 4 months'	"
" 12,	" 118.76	" 60 days'	"
" 30,	" 786.30	" 6 months'	"
July 16,	" 274.85	" 60 days'	"

What is the average time of payment, and how much would balance the account Sept. 1? How much Oct. 1?

15. What is the average time of maturity for the payment of the following bills?

Mar. 4, 1883,	\$117.26	on 4 months'	credit.
" 21,	" 97.43	" 30 days'	"
" 29,	" 243.84	" 60 "	"
Apr. 16,	" 376.14	" 4 months'	"
" 30,	" 182.75	" 90 days'	"
May 18,	" 412.50	" 60 "	"
June 1,	" 518.65	" 30 "	"

16. A commission merchant made the following sales for a consignor :

May 10,	\$175,	on a credit of 4 months,	or 30 days less 5%.
" 18,	243,	" 4 "	30 " "
" 31,	364,	" 4 "	30 " "
June 18,	387,	" 4 "	30 " "
July 1,	216,	" 4 "	30 " "

What is the average due date?

NOTE.—Since each of the above bills was sold on two different terms of credit, the account may be averaged on two different bases producing different results. The average date of purchase is June 5. If the account is settled on the first term of credit, the total amount of the bills, \$1385, will be due 4 months after June 5, or Oct. 5. If the account is settled on the second term of credit, there will be \$1315.75 (\$1385 less 5%) due 30 days after June 5, or July 5. Since money is always worth less than 20% (4 × 5%), the second method is in favor of the commission merchant. Probably most of his buyers settle their bills on the second terms, and thus take advantage of the discount.

17. A commission merchant made the following sales: Aug. 1, 1881, \$387.40; Aug. 10, 1881, \$416.75; Sept. 5, 1881, \$583.28; Sept. 20, 1881, \$144.13; Oct. 3, 1881, \$582.76; Oct. 24, 1881, \$327.41. What is the net amount and the average due date if the goods were sold on the following time? "60 days, or 2% discount if paid in 10 days."

398. To find the equated time for the payment of the balance of an account having both debit and credit items.

399. Ex. At what date may the balance of the following account be paid without loss of interest to either party?

Dr. JOHN ROACH in account with GEO. H. STUART. *Cr.*

1881.				1882.			
June 6	Mdse. 30 da.	456	00	July 26	Cash. . .	400	00
" 20	" 60 da.	384	00	Aug. 10	" . .	375	00
July 5	" 3 mo.	216	00	" 10	Mdse. 60 da.	288	00
" 26	" 3 mo.	552	00				

OPERATION.

Due		<i>Dr.</i>		Due		<i>Cr.</i>	
July 6.	\$456 × 0 =	0		July 26,	\$400 × 20 =	8000	
Aug. 19,	384 × 44 =	16896		Aug. 10,	375 × 35 =	13125	
Oct. 5,	216 × 91 =	19656		Oct. 9,	288 × 95 =	27360	
" 26,	552 × 112 =	61824			1063	48485	
	1608	98376					
	1063	48485					
	545) 49891 (92 da. after July 6, or Oct. 6.					

ANALYSIS.—First find the due date of each item. For convenience, assume July 6, the earliest due date, as the day of settlement for all the items on each side of the account. (See Art. 394, Note 2.) If the balance of the account is paid July 6, the assumed date of settlement, there would be a loss to the payer, on the debit side of the account, equivalent to the interest of \$98376 for 1 day, and on the credit side of \$48485 for 1 day; or a net loss of \$49891 for 1 day, or of \$545 for 92 days. Since the loss of interest to the payer by settling

the account July 6 is equivalent to the interest of the balance, or the amount paid, for 92 days, it is evident that the day when there would be no loss of interest must be 92 days after July 6, 1882, or Oct. 6, 1882.

If the greater sum of the products had been on the credit side, there would have been a gain to the payer by settling the account July 6, and the day that the balance of the account would commence to draw interest would have been 92 days before July 6, or Apr. 5, 1882.

400. RULE.—*First find the due date of each item. Assume the earliest due date as the day of settlement for all the items on both sides of the account. Multiply each item by the number of days intervening between the assumed date of settlement and the due date of the item, and find the sum of the products on each side of the account. Divide the balance (the difference between the sums of the debit and credit products) of the products by the balance of the account. The quotient will be the number of days intervening between the assumed date and the true date of settlement.*

To find the true date of settlement, count forward from the assumed date, when the balance of the account and the balance of the products are on the same side (both debits or both credits); and count backward, when on opposite sides.

NOTE.—1. The rule for counting backward and forward is the reverse of the above, when the latest date or a date after the latest date is taken as the assumed date of settlement.

2. Although the principles of equation of accounts are theoretically correct, they are not always practicable and can not be legally enforced. Thus, if a debt of \$4000 is due Feb. 1, no merchant would accept a payment of \$3600, Jan. 1, with the understanding that the remaining \$400 would remain unsettled 9 months after Feb. 1, or until Nov. 1. The merchant would undoubtedly be willing to allow a discount equivalent to the interest of \$3600 for the unexpired time, or 1 month.

3. In finding the equated time, reject the cents when less than 50; and add 1 dollar to the dollars when the cents are more than 50. The results will be sufficiently accurate.

EXAMPLES.

401. 1. At what date may the balance of the following account be paid without loss to either party?

<i>Dr.</i>				ISAIAH B. PRICE.				<i>Cr.</i>	
1882.				1882.					
May 16	To Mdse.	.	437 00	May 23	By Cash	.	400 00		
" 31	" "	.	324 00	June 16	" "	.	300 00		

2. Find the average date of maturity for the balance of the following account:

<i>Dr.</i>				WILLIAM C. DOUGLAS.				<i>Cr.</i>	
1881.				1881.					
Jan. 4	Mdse.	30 da.	516 00	Feb. 1	Cash	.	500 00		
" 28	"	60 da.	325 00	" 1	Note 60 da.		300 00		
Feb. 4	"	4 mo.	437 00						

3. Average the following account:

<i>Dr.</i>				JOSEPH H. WRIGHT.				<i>Cr.</i>	
1882.				1882.					
Mar. 27	Mdse.	4 mo.	716 48	Apr. 16	Cash	.	300 00		
Apr. 16	"	60 da.	325 75	May 2	"	.	400 00		
May 1	"	4 mo.	413 40	July 8	"	.	500 00		
June 4	"	4 mo.	716 87						

4. What is the equated time for the payment of the balance of the following account?

<i>Dr.</i>				A in account with B.				<i>Cr.</i>	
1882.				1882.					
Mar. 16	Mdse.	4 mo.	444 57	July 1	Cash	.	400 00		
" 30	"	60 da.	376 82	" 20	"	.	375 00		
Apr. 20	"	30 da.	712 19	Aug. 16	"	.	700 00		
May 17	"	4 mo.	628 75	" 30	"	.	600 00		
" 28	"	4 mo.	419 31						

5. When will the balance of the following account commence drawing interest? How much would be due March 1, 1883?

Dr. ANDREW CARNEGIE, Pittsburg, Pa. Cr.

1882.			1882.		
Sept. 4	Cash	100	Aug. 16	Mdse. 4 mo.	647 13
" 4	Note 4 mo.	900	" 29	" 4 mo.	322 85
Oct. 31	Cash	250	Sept. 4	" 4 mo.	412 90
Dec. 28	"	600	" 17	" 4 mo.	588 33
			" 17	" 30 da.	246 12
			Nov. 4	" 4 mo.	683 45

402. To find the equated time for the payment of the net proceeds (257) of an account sales (258).

403. 1. The sales form the credit side of the account, and the charges and advances the debit side.

2. The charges for transportation, cartage, and other items paid by the commission merchant are considered due at the time of the payment of the same.

3. The commission and other after-charges of the commission merchant are considered due by some at the average *due* date of the sales; and by others, at the average date of the sales. Since the commission is so small compared with the gross sales, in many examples, it makes no difference which date the commission is considered due. Certain merchants enter the commission at the date the account sales is rendered, and, by so doing, produce a result sufficiently accurate.

4. Many commission merchants, when the consignments are not separated and numbered, enter the sales and commission only on the account sales (See Ex. 3, Art. 405), and enter the advances and the general charges in the account current (See Ex. 6, Art. 412). Accounts sales, when the shipments are continuous, are rendered monthly to the manufacturers or consignors, and "sketches" weekly or whenever a sale is made.

5. With the exception of finding the date for the commission and other after-charges, the process of averaging an account sales is exactly the same as that of averaging an account containing both debit and credit items.

404. Ex. What is the equated time for the payment of the net proceeds of the following account sales?

NEW YORK, Dec. 1, 1881.

Account sales of Seed

For account of WILLIAM STEPHENS & Co.

By FRANKLIN EDSON & Co.

1881.							
Nov.	4	45 ²⁰	bu. Timothy Seed	30 da.	1 ¹⁵	79	53
"	18	50	" Mammoth Cl. Seed	60 da.	9 ⁰⁰	450	
"	28	494 ²	" Clover Seed	Cash.	8 ⁴⁰	418	32
						947	85
CHARGES.							
Oct.	31	Transportation,			60.00		
Dec.	1	Commission 5% as Dec. 22, 1881,			47.39	107	39
Net proceeds due Dec. 26, 1881,						840	46

ANALYSIS.—The average *due* date of the sales is Dec. 22, 1881, which is taken as the due date for the commission.

The account sales to be averaged will now be as follows:

<i>Dr.</i>		<i>Cr.</i>	
Due Oct. 31, 1881,	\$60.00	Due Dec. 4, 1881,	\$79.53
" Dec. 22, "	47.39	" Jan. 11, 1882,	450.00
		" Nov. 28, 1881,	418.32

By averaging the above, we find the net proceeds, \$840.46, are due Dec. 26, 1881.

If the commission is considered due Nov. 21, 1881, the average date of the sales, the net proceeds will be due Dec. 28, 1881.

NOTE.—If the same assumed date, or focal date, be taken in finding the average due date of the sales as in finding the average due date of the net proceeds, the operation of the former will form the debit side of the latter operation.

EXAMPLES.

405. Find the proceeds and equated time of the following accounts sales. (Unless otherwise stated, consider the commission due at the average due date of the sales.)

1. Sales of 400 bbls. flour received per N. Y. C. & H. R. R. R., for account of A. W. ARCHIBALD, Ottumwa, Iowa.

1881.								
May	11	125 bbls. " Kirkwood " cash, .	675	***	**			
"	12	150 " " Iowa " 4 mo.,	650	***				
"	18	125 " " Kirkwood " 4 mo.,	700	***		****	**	
CHARGES.								
May	3	Transportation and Cartage, . .	425					
		Commission and Guaranty 5%, .		***	**	***	**	
		Net pro. due per average —, 1881.				****	**	

E. & O. E.

E. R. LIVERMORE.

NEW YORK, May 20, 1881.

What would be the equated time for the payment of the above proceeds, if the commission and guaranty were considered due at the average due date of the sales? At the average date of the sales?

2. Account sales of 900 sides hemlock sole leather by MASSEY & JANNEY, for account of GRANT & HORTON, Ridgway, Pa.

PHILADELPHIA, PA, Aug. 22, 1881.

1881.		Sides.	Description.	Terms.	Weight.	Price.		
Aug.	14	400	" Ridgway " 27	4 mo.	9407	27	****	**
"	18	300	" " 27	4 mo.	6875	274	****	**
"	21	200	" " 28	30 da.	4712	274	****	**
CHARGES.								
Aug.	2		Transportation \$70, Cartage \$9. .			**.	**	
			Commission and Guaranty 5%, .			***.	**	****
			Proceeds due —, 1881.				****	**

3. Sales by JAMES TALCOTT, New York, for account of Phenix Mills, Cohoes, N. Y. March 31, 1882.*

Date.	Cases.	Description.	Time.	Yards.	Price.	Amount.
Mar. 1	2	Fancy Cassimere.	30 <i>da.</i>	966 ²	1.35	****.**
" 10	4	" "	10 <i>da.</i>	1994	1.70	****.**
" 13	3	" "	30 <i>da.</i>	1506 ¹	2.30	****.**
" 17	4	" "	4 <i>mo.</i>	1936 ³	1.65	****.**
" 26	2	" "	Cash.	978	1.85	****.**
						*****.
Less Commission 5%, . . .						****.**
Proceeds due —, 1882, . . .						*****.

4. Account Sales of merchandise by JOHN F. COOK, for account of Excelsior Packing Co., Cincinnati, Ohio.

1881.							
Oct.	16	50 Bbls. Mess Beef, .	Cash.	11 ²⁵	***	**	
"	24	100 " N. M. Pork, . .	"	17 ⁵⁰	****		
"	31	25 " Hams 6376 lbs.,	10 <i>da.</i>	13 ¹ / ₂ ¢	***	**	
Nov.	9	25 " Sh'lders 5717 lbs.,	60 <i>da.</i>	9¢	***	**	
"	18	75 " C. M. Pork, . .	4 <i>mo.</i>	13 ¹⁵	****	**	
CHARGES.						****	**
Oct.	13	Transportation,		325.			
"	15	Cartage,		37.50			
"	15	Cooperage,		15.			
"	15	Inspection,		13.75			
Nov.	18	Storage,		48.75			
		Commission 5%,		***.	***	**	
Net proceeds due —, 1881, .						****	**

E. & O. E.

JOHN F. COOK.

NEW YORK, N. Y., Nov. 20, 1881.

* If the commission is considered due at the average due date of the sales, and since there are no other changes, the net proceeds will be due at the same date.

ACCOUNTS CURRENT.

DEFINITIONS.

406. An **Account Current** is an itemized account of the business transactions between two houses, showing the balance or amount due at the current date. The amount due is sometimes called the *cash balance*.

1. An account current is a transcript of the ledger account with the addition of certain details taken from the books of original entry, and is arranged in a different form.

2. Interest is charged, or not, according to the custom of the business, or the agreement between the parties. This chapter treats only of accounts in which interest is charged. When interest is not charged, the balance due is the difference between the two sides of the account as originally entered in the ledger. The interest may be reckoned according to any of the methods of Art. **274**. In the illustrative example the exact time in days is found, and the days are regarded as 360ths of a year. In the examples for practice, unless otherwise stated, the interest is reckoned on the same basis.

3. Accounts current are rendered by merchants, bankers, and brokers annually (Ex. 4), semi-annually (Ex. 1), quarterly (Ex. 5), or monthly (Ex. 6). Since the interest draws interest after the account is balanced, the oftener the account is balanced, or the interest is added to the account, the greater the amount due. Some merchants render partial accounts current monthly, but do not carry the interest to the main column until the end of the year (Ex. 10). The twelve partial accounts current make, when combined, the complete account current for the whole year.

4. There are two methods in common use for finding the amount due on an account, including interest, at a certain date, both of which are presented in the following illustrative example: 1. By interest; 2. By products.

407. Ex. Find the amount due, including interest at 6%, on the following account Jan. 1, 1882.

Dr. GEO. W. CHILDS in account with A. A. Low. *Cr.*

1881.				1881.			
Oct. 1	Balance.	1800		Oct. 31	Cash.	1000	
" 16	Mdse. 30 d.	360		Nov. 16	Note 30 d.	600	
Nov. 27	" 30 d.	432		Dec. 4	Cash.	240	
Dec. 18	Bill of H. C.	420		" 26	"	300	

408. OPERATION.—INTEREST METHOD.

<i>Dr.</i>				<i>Cr.</i>			
Due.	Amount.	Days.	Interest.	Due.	Amount.	Days.	Interest.
Oct. 1,	\$1800	92	\$27.60	Oct. 31,	\$1000	62	\$10.33
Nov. 15,	360	47	2.82	Dec. 19,	600	13	1.30
Dec. 27,	432	5	.36	" 4,	240	28	1.12
" 18,	420	14	.98	" 26,	300	6	.30
	<u>\$3012</u>		<u>\$31.76</u>		<u>\$2140</u>		<u>\$13.05</u>
	2140		13.05				
	872	+	18.71 = 890.71.				

ANALYSIS.—First find the due date of each item of the account. Each item will draw interest from its due date until the day of settlement, or Jan. 1, 1882. The total interest on the debit side of the account is \$31.76, and on the credit side, \$13.05. The balance of interest, \$18.71, is therefore in favor of the debit side, or is due Mr. Low.

Since both the balance of the account (\$872) and the balance of interest (\$18.71) are due the same party, the net amount due Jan. 1, 1882, is \$872 plus \$18.71, or \$890 71.

If the balance of interest had been on the credit side of the account the net amount due would have been \$872 *minus* \$18.71, or \$853 29.

NOTES.—1. It will sometimes happen that certain items will fall due after the day of settlement. The interest on such items should be transferred to the opposite side of the account. (See Ex. 8.)

2. If the account has been averaged, the amount due at a given date may be found by calculating the interest on the balance of the account from the time it is due to the date of settlement. If the date of settlement is earlier than the average date, subtract the interest from the balance of the account ; if later than the average date, add the interest. (See Art. 397, Ex. 6, Note.)

3. The interest method is generally used in business. Since it gives the interest on each item and is readily understood, it is more satisfactory to those to whom accounts current are sent than the product method. When interest tables are used, it is shorter than any other method.

409. The following is a common form of an account current including interest:

Dr. GEO. W. CHILDS in % current with A. A. Low. Cr.

1891.		Days	Int.	Amounts.	1891.		Days	Int.	Amounts.
Oct. 1	Balance.	92	27.60	1800.00	Oct. 31	Cash.	62	10.33	1000.00
" 16	Mdse. Nov. 15.	47	2.82	360.00	Nov. 16	Note Dec. 19.	13	1.30	600.00
Nov. 27	" Dec. 27.	5	.36	432.00	Dec. 4	Cash.	28	1.12	240.00
Dec. 18	Bill of H. C.	14	.98	420.00	" 26	"	6	.30	300.00
1892.					1892.				
Jan. 1	Bal. of Int.			18.71	Jan. 1	Bal. of Int.		18.71	
					" 1	Bal. of Acct.			890.71
			31.76	3030.71				31.76	3030.71
1892.									
Jan. 1	Balance.			890.71					

410. RULE FOR THE INTEREST METHOD.—*First find the due date of each item of the account. Then find the interest on each item from the date it becomes due to the day of settlement. The difference between the sums of the debit and the credit interest will be the balance of interest.*

To find the net amount due, when the balance of interest and the balance of items are on the same side, take their sum ; when on opposite sides, take their difference.

411. OPERATION.—PRODUCT METHOD.

<i>Dr.</i>				<i>Cr.</i>			
Due.	Am't.	Days.	Products.	Due.	Am't.	Days.	Products.
Oct. 1,	\$1800	× 92 =	165600	Oct. 31,	\$1000	× 62 =	62000
Nov. 15,	360	× 41 =	16920	Dec. 19,	600	× 13 =	7800
Dec. 27,	432	× 5 =	2160	“ 4,	240	× 28 =	6720
“ 18,	420	× 14 =	5880	“ 26,	300	× 6 =	1800
	\$3012		190560		\$2140		78320
	2140		78320		\$872 + \$18.71 =		\$890.71.
	872	6)	112240				
			\$18.706				

ANALYSIS.—By multiplying the number of dollars by the number of days, and taking the sum of the products on each side of the account, we find that the total debit interest is equivalent to the interest of \$190560 for 1 day, and the total credit interest to the interest of \$78320 for 1 day. The balance of interest is therefore equivalent to the interest of \$112240 for 1 day. The interest of \$1 for 1 day is $\frac{1}{360}$ of a mill (286, 3), and of \$112240, 18706 ($\frac{1}{360}$ of 112240) mills, or \$18.71. Since the balance of items (\$872) and the balance of interest (\$18.71) are both due the same party, the net amount due is their sum, or \$890.71.

EXAMPLES.

412. 1. Find the balance due on the following account. Jan. 1, 1883, interest being reckoned at 6%.

<i>Dr.</i>				HOWARD THORNTON.				<i>Cr.</i>	
1882.				1882.					
July 1	Balance.	1830	45	Sept. 13	Net Proc'ds.	876	40		
Aug. 24	Mdse.	448	00	Oct. 31	“ “	912	36		
Oct. 18	Draft C. C.	387	40	Nov. 5	Cash.	1000	00		
Dec. 12	Draft H.L.	516	88						

2. Find the amount due Aug. 4, at 6%, on the account represented in Ex. 6, Art. 397. (See Note, Ex. 6, Art. 397.)

3. Find the amount due Oct. 1, 1882, at 6%, on the account represented in Ex. 4, Art. 401.

4. What is the net amount due on the following account, July 1, 1882, at 6%?

Dr. C. H. MILLS in % current with T. W. SUFFERN. *Cr.*

1881.				1881.			
July 1	Balance.	1275	46	Nov. 14	Mdse. 4 mo.	587	19
Sept. 13	Draft #12.	871	52	1882.			
1882.				Mar. 13	" 30 da.	612	35
Jan. 4	" #17.	913	27	Apr. 27	" 60 da.	846	93
May 17	" #24.	345	63	June 3	Cash.	500	00

5. What is the balance of the following account, Apr. 1, 1882, at 6%?

Dr. W. J. HILLIS in account with LANGRAVE SHULTS. *Cr.*

1882.				1882.			
Jan. 16	Dft. M. C.	937	64	Jan. 1	Balance.	3456	75
" 31	" B. D.	856	75	" 14	Mdse. 60 d.	1225	19
Mar. 3	" W. Y.	1749	30	Feb. 4	" 30 d.	673	75
" 24	" V. C.	912	38	" 17	" 30 d.	2428	35

6. Find the balance due Apr. 1, 1882, at 6%, on the following account current :

PHENIX MILLS in % current with JAMES TALCOTT. New York, Apr. 1, 1882.

Date.	Dr.	Amounts.	Date.	Cr.	Amounts.
1882.			1882.		
Mar. 1	Balance.	45108 34	Mar. 31	Net Proceeds of Account Sales due Apr. 26, 1882.	12505 70
" 16	Draft #76.	1000		(See Ex. 3, Art. 405)	
" 18	" #75.	2000			
" 24	" #78.	5000			
" 28	Cotton Bill.	3176 42			
" 30	Freight.	875 10			

7. Find the gain or loss at 6%, accurate interest, on the following consignment account, Jan. 29, 1881. (The day the draft for the balance of the account was drawn.)

Cons. F. L. BRUCKMANN, \$14.

1880.		<i>Dr.</i>	Days.	Interest.		Amounts.	
Apr.	25	Mdse. Net Cash.	279	300	17	6544	92
May	10	Insurance.	***	*	**	40	
1881.							
Jan.	29	Balance of Interest to debit.				***	**
"	29	<i>Gain.</i>				***	**
				***	**	*****	**
1880.		<i>Cr.</i>					
May	7	Draft 18000 Reichsmarks.	***	***	**	4258	42
Nov.	20	" 2000 "	**	*	**	468	75
1881.							
Jan.	29	" 9998 "	0			2368	28
"	29	<i>Balance of Interest to debit.</i>		***	**		
				***	**	*****	**

8. Find the amount due at 5%, accurate interest, on the following account Feb. 13, 1881, the estimated due date of a sight draft drawn Jan. 29, 1881, for the balance.

F. L. BRUCKMANN on account of Consignment \$14.

1880.		<i>Dr.</i>		Days.	Interest.		Amounts.	
Oct.	25	Account Sales	due Jan. 9, 1881	35	44	80	9344	82
Dec.	31	" "	" Mar. 7, 1881				22417	54
1881.								
Feb.	13	<i>Balance of Interest to credit.</i>			***	**		
					***	**	*****	**
1880.		<i>Cr.</i>						
June	30	Freight	due May 14, 1880	***	**	**	1176	32
May	6	Draft 60 days' sight	" July 18, 1880	***	***	**	18000	
Nov.	19	" 60 " "	" Feb. 1, 1881	**	*	**	2000	
1881.								
Feb.	13	Interest Recd. 22417.54	" Mar. 7, 1881	**	**	**		
"	13	Balance of Interest to credit.					***	**
Jan.	29	Draft at sight to balance	due Feb. 13, 1881				****	**
					***	**	*****	**

NOTES.—1. The interest on all items falling due after the day of settlement should be entered in the interest column on the opposite side of the account.

Some accountants enter these items of interest on the same side of the account in *red ink* so that they will not be added to the other items, and transfer the “red interest” in one amount to the opposite side.

2. The foregoing represents an account in German marks (reichsmarks) kept in an auxiliary book by a consignor of merchandise to a commission merchant at Hamburg, Germany.

The due dates of drafts, accounts sales, and other items are obtained from accounts sales and memoranda rendered by the commission merchant. The corresponding consignment account as entered in the books of the commission merchant is represented in Ex. 7.

9. What was the balance due Mar. 1, 1883, at 6%, on the account represented in Ex. 5, Art. 401.

10. Calculate the interest, Jan. 1, 1883, in the following partial account current, and find the total amounts. (Interest 6%, 365 days to the year.) (See Art. 406, 3.)

G. D. SLOCUM in account with W. B. McMECHAN.

		Dr.	Da.	Interest.		Amounts.	
1882.							
May	1	From statement of May 1, 1882.		1387	63	28765	72
"	6	Draft H. B. Claffin & Co.	240	50	71	1285	43
"	9	" Austin, Nichols & Co.	***	**	**	674	89
"	13	" W. H. Shieffelin & Co.	***	**	**	346	27
"	25	" Early & Lane.	***	**	**	418	43
"	28	" Mitchell, Vance & Co.	***	**	**	576	80
				***	**	*****	**
				—	—	—	—
		Cr.					
1882.							
May	1	From statement of May 1, 1882.		973	42	22413	71
"	5	Sales as June 28, 1882.	***	***	**	7316	84
"	12	" " Aug. 1, 1882.	***	**	**	2110	92
"	18	" " July 13, 1882.	***	***	**	13446	85
"	25	Cash.	***	**	**	2000	
				***	**	*****	**
				—	—	—	—

STOCKS AND BONDS.*

DEFINITIONS.

413. The term "Stock" is applied to the share capital of a company, and represents an interest in its property over and above its liabilities, and in the profits of its business after the expenses and interest on its bonds have been paid. This profit, when divided among the stockholders, is known as a *dividend*. The dividend is a certain amount per share, or a certain per cent. of the par value of the stock.

1. The Capital Stock of a company is divided into shares usually of \$100 each. Shares of \$50 and \$25 are called half-stock and quarter-stock respectively.

2. A *Stock Certificate* is a written instrument issued by a company, and signed by the proper officers, certifying that the holder is the owner of a certain number of shares of its Capital Stock.

3. The sum for which the shares or certificates were issued is called the *Par Value*, and the amount for which they can be sold, the *Market Value*.

414. A **Preferred Stock** is one on which a stated per cent. is payable annually, out of net earnings, before any dividend can be declared on the common stock.

Thus, the holders of preferred stock of a certain railroad are entitled to 6% on their stock out of any one year's earnings, before the common stock can receive any dividend. After such payment, the balance of earnings, if any remain, may be divided to the common stock.

Preferred stocks are generally the result of a reorganization of a railroad. For instance, the holders of the common stock may save the road from passing out of their hands by the payment of a certain sum of money, for which preferred stock is issued.

* Condensed from "Memoranda concerning Government Bonds, etc.," by Fisk & Hatch. Bankers, New York, 1882.

415. A Bond is the obligation of a Corporation, City, County, State, or Government to pay a certain sum of money at a certain time, with a fixed rate of interest payable at certain periods.

1. Bonds of business corporations are usually secured by a mortgage on the whole or some specified portion of their property ; although certain classes of bonds are issued without mortgage security, and are dependent on the good faith or solvency of the company issuing them, having the same force as a promissory note.

2. Bonds are issued with coupons attached representing the different installments of interest payable at the different periods specified, during the time the bond has to run, which are to be cut off and collected from time to time as the interest becomes due.

3. Bonds are also issued without coupons, in what is known as the registered form. In this case the bond is only payable to the registered owner, or his assignee, and the interest is paid by check or in cash, to the owner or his attorney.

4. Bonds are sometimes issued with coupons attached payable to bearer, but the principal of which may or may not be registered at the choice of the owner.

5. Bonds are known as First Mortgage, Second Mortgage, etc., Consols, Convertible, Income or otherwise, according to their priority of lien, the class of property upon which they are secured, or other characteristics. Income bonds are generally bonds on which the interest is only payable if earned, and ordinarily are not secured by a mortgage.

Bonds are also named from the rate of interest they bear, or from the dates at which they are payable or redeemable, or from both ; as, U. S. 4's 1907, Virginia 6's, Western Union 7's, coupon, 1900, Lake Shore reg. 2d, 1903.

6. The bond of a company may be a perfectly safe investment, when the stock is not ; and the stock of a prosperous and successful company, paying large dividends or having a large surplus, may sell at a higher price than the bonds of the same company, the income from which is limited to the agreed rate of interest which they bear. A much closer scrutiny should be made of a company's standing, when one thinks of investing in its share capital, than when it is the intention to loan the company money on its mortgage bond.

7. *Convertible Bonds* are those which are issued with provisions whereby they can be exchanged for stock, lands, or other property.

8. Bonds are issued in denominations of \$50 to \$50000.

GOVERNMENT BONDS.

416. Statement of the Public Debt of the United States, January 1, 1882.

INTEREST-BEARING DEBT.		Principal.	Interest.
Bonds at 6%, continued at 3½%.		\$149,682,900.00	\$2,619,448.11
Bonds at 5%, continued at 3½%.		401,503,900.00	2,379,103.91
Bonds at 4½%.		250,000,000.00	1,394,399.62
Bonds at 4%.		738,772,550.00	8,149,645.31
Refunding Certificates (4%).		575,250.00	61,880.90
Navy-Pension Fund (3%).		14,000,000.00	210,000.00
		1,554,534,600.00	14,814,378.85
DEBT ON WHICH INTEREST HAS CEASED SINCE MATURITY.	{	11,528,265.26	714,985.31
DEBT BEARING NO INTEREST.			
Old Demand Notes,		59,920.00	
Legal-Tender Notes. (184.)		346,681,016.00	
Certificates of Deposit.		9,500,000.00	
Gold Certificates,		5,188,129.00	
Silver Certificates, (181.)		68,675,230.00	
Fractional Currency,*		7,075,926.92	
Unclaimed Pacific Railroad Interest.			7,256.51
		2,003,333,078.18	15,536,619.67
			2,002,333,078.18
TOTAL DEBT.			2,018,869,697.85
TOTAL CASH IN THE TREASURY.			253,377,980.76
DEBT, LESS CASH IN THE TREASURY, JAN. 1, 1882.			1,765,491,717.09

BONDS ISSUED TO THE PACIFIC RAILWAY COMPANIES, INTEREST PAYABLE BY THE UNITED STATES.

Principal outstanding,		\$64,623,512.00
Interest accrued and not yet paid,		1,938,705.36
Interest paid by the United States,		51,467,272.02
Int. repaid by transportation of mails, \$14,707,886.34		
By cash payments 5% net earnings, 655,198.87		15,363,085.21
Balance of interest paid by the U. S.,		36,104,186.81

417. Continued 6's, 6's of 1881. Authorized by Acts of July 17 and August 5, 1861, and March 3, 1863. Redeemable at the option of the Government after June 30, 1881. During the year 1881, at the request of the holders, these bonds were *continued at 3½ per cent.* The amount

* Amount of fractional currency estimated as lost or destroyed, \$8,375,994.

outstanding Jan. 1, 1882, was \$149,682,900, all registered. Interest is payable Jan. 1, and July 1. Although these bonds can be called at any time, the interest ceasing at the date of the call, it is the custom of the Secretary of the Treasury to give 60 days' notice. (See Art. 423.)

418. Continued 5's, 5's of 1881. These bonds were authorized by the "Funding Acts" of July 14, 1870 and Jan. 20, 1871, and were issued for the purpose of funding the 5-20 and 10-40 bonds. Redeemable at the option of the Government after May 1, 1881. During the year 1881, at the request of the holders, these bonds were *continued at 3½ per cent.* The amount outstanding Jan. 1, 1882, was \$401,503,900, all registered. Interest is payable Feb. 1, May 1, Aug. 1, and Nov. 1. These bonds may be called at any time, but the interest will not cease till three months after the date of the call. (See Art. 423.)

419. 4½'s of 1891. Authorized by the Acts of July 14, 1870, and Jan. 20, 1871, and issued for the purpose of funding the 5-20 and 10-40 bonds. Redeemable at the option of the Government after Sept. 1, 1891. The amount outstanding Jan. 1, 1882, was \$250,000,000, of which \$181,486,000 were registered and \$68,514,000 coupon bonds. Interest is payable Mar. 1, June 1, Sept. 1, and Dec. 1.

420. 4's of 1907. Authorized by the Acts of July 14, 1870, and Jan. 20, 1871, and issued for the purpose of funding the 5-20 and 10-40 bonds. Redeemable at the option of the Government after July 1, 1907. The amount outstanding Jan. 1, 1882, was \$738,772,550, of which \$547,760,700 were registered, and \$191,011,850 coupon bonds. Interest is payable Jan. 1, Apr. 1, July 1, and Oct. 1.

421. Refunding Certificates. Authorized by Act of Feb. 26, 1879. These certificates are of the denomination of \$10, bear interest at 4%, and are convertible at any time, with accrued interest, into 4% bonds. The amount outstanding Jan. 1, 1882, was \$575,250.

422. Currency 6's. These bonds were issued to aid in the construction of the Pacific railroads, and were authorized by the Acts of July 1, 1862, and July 2, 1864. Principal and interest are payable in lawful money of the United States. Payable 30 years after date, and maturing at different dates from 1895 to 1899. The amount outstanding Jan. 1, 1882, was \$64,623,512, all registered.

423. 3's. These bonds were authorized by the Act of July 12, 1882, and were issued in exchange for bonds bearing $3\frac{1}{2}\%$ interest (**417, 418**). Payable at the pleasure of the United States; provided that the bonds shall not be called in and paid so long as any bonds of the United States heretofore issued, bearing a higher rate of interest than 3%, and which shall be redeemable at the pleasure of the United States, shall be outstanding and uncalled. Interest is payable Feb. 1, June 1, Aug. 1, and Oct. 1. They are all registered.

424. Denominations. The coupon bonds of the various issues are in denominations of \$50, \$100, \$500, and \$1000. The registered bonds are in denominations of \$50, \$100, \$500, \$1000, \$5000, and \$10000. Of the funded loans, viz., the 5's of 1881, the $4\frac{1}{2}$'s of 1891, and the 4's of 1907, there are, in addition to the above, registered bonds of the denominations of \$20,000 and \$50,000.

425. All the issues of U. S. bonds now outstanding are exempt from taxation, and with the exception of the Currency 6's, are payable in coin.

426. Coupon bonds, being payable to bearer, pass by delivery without assignment, and are therefore more convenient for sale and delivery than registered bonds, which must be assigned by the party in whose name they are registered. The interest coupons being also payable to the bearer, will be cashed by any bank or banker in any part of the United States.

1. The interest on registered bonds is paid by checks, made to the order of the registered owner and sent to him by mail. These checks, when properly endorsed, will be cashed by any bank or banker.

2. Coupon bonds may be converted into registered bonds of the same issue, but there is no provision of law for converting registered bonds into coupon bonds.

3. Coupon bonds forwarded to the Treasury Department for conversion into registered bonds should be addressed to "The Secretary of the Treasury, Washington, D. C."

4. Registered bonds forwarded to the Treasury Department for transfer, and requests for a change in the address to which interest checks are to be sent, should be addressed to the "Register of the Treasury, Washington, D. C."

NEW YORK STOCK EXCHANGE.

427. The New York Stock Exchange is an incorporated body of brokers, whose business it is to buy and sell stocks, bonds, and other representatives of value.

1. The present number of members is eleven hundred, the maximum allowed under the by-laws. Therefore, membership is only to be obtained by the purchase of the seat of a deceased or retiring member. Seats have been sold within the last few months (1882), for from \$26000 to \$31000.

2. The floor of the Exchange is open for business from 10 A. M. to 3 P. M. There are two regular calls of Stocks daily; three of State and Railroad Bonds; and three of United States Bonds. Transactions are not, however, confined to the regular calls, but are continually taking place on the floor of the Exchange between the hours named above.

3. In Wall Street, there are what are known as strictly commission houses, who take and execute orders for securities, charging the regular commission, and, when customers desire, loaning funds on the securities on a deposit of 10 to 20% of market value being made. This is what is known as buying on a margin (**432**), where the customer merely buys for speculative purposes. Such houses will usually sell stocks "short" (**433**, 11) for their customers on a similar margin.

There are other houses which make no advances, and require customers to pay outright for securities when bought.

Then, again, there are houses which combine a banking and brokerage business, taking deposits and loaning money on any securities marketable at the Exchange, and buying and selling stocks on commission. Some of these extend the privilege of marginal business to their customers, while others do not.

There are other members and firms who operate exclusively for their own account.

428. Quotations are made at so much per cent. on the basis of a par value of \$100 per share of stock, except in the case of mining securities and Sutro Tunnel stock, which are quoted at so much per share, without reference to their par value.

For example, the par value of Morris and Essex stock is \$51, but the quotation, if the stock were worth just par in the market, would be 100% ; or, if the quotation is 110, it means \$110 for \$100 worth of the par value, which, in the case of this stock, would be two shares, while in the case of a stock the par value of which is \$100 per share, it would be for one share.

On the other hand, if Sutro Tunnel, the par value of which is \$10 per share, is quoted at 2, it means \$2 per share.

429. Commission. The regular charge for buying and selling securities dealt in at the Stock Exchange, except mining stocks, is one-eighth of one per cent. ($\frac{1}{8}\%$) on par value, or \$12.50 on 100 shares of stock of the par value of \$100 each.

430. Stocks are usually bought and sold either "cash," "regular way," "seller three" or "buyer three." A stock sold "cash" is deliverable the day sold ; a stock sold "regular way" is deliverable the next day, or bought "regular way" is to be paid for the next day. Where nothing else is specified, "regular way" is always understood. When a stock is reported as bought "seller three," it is meant that the seller of the stock can deliver it on either of the three days at his option, but is not required to deliver until the third day. On the other hand, when a transaction is

made "buyer three," the buyer can demand delivery of the stock at any time within three days, but must take it and pay for it by the third day.

1. Transactions on any of the above terms carry no interest.
2. If the option is over three days, six per cent. on the selling value of the stock is paid by buyer to seller.
3. One day's notice is required of intention to terminate an option of a longer period than three days.
4. The Stock Exchange does not recognize any contract for over sixty days. Should a stock pay a dividend during the pendency of a contract, the dividend belongs to the purchaser of the stock, unless otherwise previously agreed.

431. There are two lists of securities admitted to dealings at the Stock Exchange, viz.: (1) That which is regularly called every day; (2) that which is only called at request. The first list is known as the regular list, and the second as the free list.

432. A Margin is a deposit made with a broker, by a person who wishes to buy or sell stock for speculation to enable the broker to "carry" the stock and protect himself against loss. It is usually 10% of the par value of the stock.

1. A person desiring to speculate in stocks, deposits with his broker \$1000 as a margin, and directs him to purchase 100 shares of a certain stock at 90. The broker would pay for the stock \$9000, \$1000 of which being furnished by the speculator, and the remainder, \$8000, by the broker. The broker charges legal interest on the amount furnished by him for "carrying" the stock. (See Ex. 42, Art. 434.)

2. The margin deposited with the broker is simply to protect the broker against losing any money should the stock move in the wrong direction. In case of its so doing, the margin must be made good by the deposit of an additional amount, otherwise the broker will sell the stock to protect himself from losing any of the money he has advanced.

433. Explanation of Words and Phrases used in Wall Street.

1. *Bear*. An operator who is "short" of stock. He wishes to buy at a lower rate, and therefore tries to depress the price of the stock of which he is "short."

2. *Bull*. An operator who is holding stock for an advance. He is said to be "long" of the stock. Bulls try to advance the prices of the stocks of which they are "long."

3. *b. 3 (Buyer 3), 10, 20, 30, etc.* Meaning at the buyer's option, within three days, ten days, etc. When in a stock transaction, the buyer has the privilege of taking the stock at any time during the number of days mentioned. In buyer's options, when the option is for more than three days, six per cent. interest is charged the buyer, and the seller is entitled to one day's notice.

4. *b. c., "between calls."* The sale not taking place on the call of the stock, but after the first call and before the second.

5. *Collaterals*. Stocks, bonds, notes, or other value given in pledge as security, when money is borrowed.

6. *Cover, to "cover one's shorts."* Where stock has been sold short, and the seller buys it in to realize his profit, or to protect himself from loss, or to make his delivery. This is "covering short sales."

7. *Differences*. When the price at which a stock is bargained for and the rate on day of delivery are not the same, the broker against whom the variation exists, frequently pays the "difference" in money, instead of furnishing or receiving the stock.

8. *Ex-Div., Ex-Dividend*. When the price or quotation of a stock does not include, and the stock does not carry to the buyer a recently declared dividend.

9. *Hypothecating*. Putting up collaterals.

10. *Seller, 3, 10, 20, 30, etc.* Sold deliverable at seller's option, within the number of days named. When seller's options are for more than three days, the buyer pays six per cent. interest, unless "flat" is specified in the contract, and the seller must give one day's notice of delivery.

11. *Short*. When one has sold stock which he does not own, hoping to realize a profit by buying in at lower prices, he is said to be "short."

12. *Syndicate*. A combination of bankers who together undertake the placing of a loan.

EXAMPLES.

434. 1. A bank with a capital (**413**) of \$250,000, declares a semi-annual dividend of $3\frac{1}{2}\%$. What is the amount of the dividend, and how much will a stockholder receive who owns 16 shares of \$100 each (**413**, 1)?

2. An insurance company divides among its stockholders \$18000. What is the rate of the dividend, the capital stock being \$225000? How much is paid to Mr. A., who has a certificate (**413**, 2) for 25 shares?

3. A gas company declares a dividend of 5%, and divides among its stockholders \$125000. What is its capital stock?

4. The board of directors of a mining company declared a dividend of \$100,000, being five cents per share (par value \$10) on the capital stock of the company. What was the capital stock, and in how many shares was it divided? The dividend was what per cent. of the capital stock?

5. An installment of 10% was called on the capital stock of a new railroad company. How much was paid by Mr. B. who had subscribed for 50 shares (par value \$100)?

6. A railway company, whose capital stock is \$1,750,000, declares a dividend of $3\frac{1}{2}$ per cent. What was the amount of the dividend?

7. The Union Pacific Railway paid to its stockholders, in 1879, \$2,204,700. What was the par value of its stock, the rate of the dividend being 6%?

8. A quarterly dividend of $3\frac{1}{2}\%$ was declared by a manufacturing company. What was the capital stock, the amount of the dividend being \$2100?

9. If stock is quoted at $116\frac{5}{8}$, what is the market value of 200 shares?

10. How many shares of W. U. Tel. can be bought for \$43725 at $79\frac{3}{4}$, brokerage $\frac{1}{4}\%$?

11. Find the proceeds of \$15000 U. S. 4's, registered, 1907 (**420**), b. 3, at $117\frac{1}{2}$, and \$10000 U. S. 4 $\frac{1}{2}$'s coupon (**419**) at $114\frac{1}{2}$, usual brokerage.

12. What is the total par value (413, 3) and the total market value of 100 shares Lake Shore at $118\frac{5}{8}$ (428), 300 sh. N. J. Central at $89\frac{3}{4}$, 500 sh. W. U. Telegraph at $78\frac{1}{2}$, and 500 sh. N. Y., L. E. & W. com. at $40\frac{1}{2}$, and 800 sh. N. Y., L. E. & W. pref. (414) at $90\frac{3}{8}$?

13. Find the cost of 250 shares Tex. & Pac. at $50\frac{5}{8}$ and 100 shares Ohio & Miss. at 104, brokerage $\frac{1}{8}\%$ (429).

14. What are the proceeds of 600 shares Morris and Essex (half stock, 413, 1) sold through a broker at $121\frac{1}{2}$?

15. What are the proceeds of the following stocks sold through a broker? 200 shares Union Pacific at $117\frac{3}{8}$, 2000 shares N. Y., O. & W. at $27\frac{1}{4}$, 800 shares A. & T. H. pref. at 88, and 600 shares Chi. & Alton at $131\frac{1}{4}$.

16. Find the cost of 10 shares Manhattan Bank at 135, \$5000 Erie 7's (415, 5) cons. gold bonds (415) at 128, \$1000 Toledo and Wabash 2d, s. 3 (415, 5—433, 10) at $108\frac{1}{4}$, \$5000 C. R. I. & P. 6's, 1907, coupon (415, 2) at 129, and \$5000 Ohio Southern Income (415, 5) at 45, usual brokerage.

17. How much must be invested in U. S. $4\frac{1}{2}$'s, 1891, to produce a quarterly income of \$675, bonds selling at $114\frac{3}{8}$?

18. When Ohio 6's, 1886, are sold at $109\frac{1}{2}$, what is received for six \$500 bonds, brokerage $\frac{1}{4}\%$?

19. When Pittsburg, Fort Wayne and Chicago 2d 7's, 1912, are worth 135, what will \$12000 in bonds cost?

20. How many \$500 bonds shall I receive for \$4735 invested in U. S. 4's at $118\frac{3}{8}$?

21. How much must be sent to a broker that he may purchase \$8000 U. S. continued fives (418) at $102\frac{3}{4}$, commission $\frac{1}{8}\%$?

22. An executor sold Central of New Jersey stock at $52\frac{1}{4}$, and purchased with the proceeds \$42000 in U. S. 4's, 1907, at $100\frac{1}{4}$. What was the par value of the stock sold, usual brokerage?

23. How many shares of Ill. Cen. bought at $129\frac{3}{8}$ and sold at $132\frac{3}{4}$, usual brokerage, will produce a gain of \$1375?

24. A broker bought on his own account 200 sh. Nor. Pac. pf. at $69\frac{3}{4}$, and sold the same the same day at $73\frac{3}{4}$. What was his gain?

25. What income will be produced by investing \$235250 in 4% bonds at $117\frac{3}{8}$?

26. The common stock of a railroad company is \$46,000,000, and the preferred stock (414) \$8,000,000. The company declares a dividend of $3\frac{1}{2}\%$ on the preferred stock, and 2% on the common stock. What is the surplus, if the net earnings are \$1,317,645?

27. Bought June 4, 800 sh. Ohio & Miss. pref. at $35\frac{1}{2}$, s. 30. The stock was delivered June 24. What was the amount paid including interest (433, 10)?

28. Bought May 16, 200 sh. Lake Shore at $116\frac{3}{4}$, b. 60, and called for the stock July 5. What was the cost including interest (433, 3)?

29. Jan. 10, sold 100 sh. Phil. & Read. at $65\frac{1}{4}$, s. 3. Jan. 13, the stock was quoted at $68\frac{1}{2}$. How much was the difference (433, 7) paid by the seller in settlement?

30. Sept. 1, 1865, the interest-bearing debt of the United States was as follows: 4 per cents., \$618,127.98; 5 per cents., \$269,175,727.65; 6 per cents., \$1,281,736,439.33; $7\frac{3}{8}\%$ per cents., \$830,000.00. What was the total annual interest charge?

31. The interest-bearing debt of the United States was as follows, Jan. 1, 1881: 6's, \$202,266,550; 5's, \$469,651,050; $4\frac{1}{2}$'s, \$250,000,000; 4's, \$739,347,800; 3's, \$14,000,000. What was the decrease during the year 1881 in the annual interest charge? (See statement of Jan. 1, 1882, Art. 416.) What was the interest of the debt for one day ($\frac{1}{365}$ yr.) Jan. 1, 1882?

32. The population of the United States and Territories Jan. 1, 1881, was 50,152,554, and the public debt was \$1,899,181,735. What was the debt per capita? What was the average monthly decrease of the debt during the year 1881? (See statement, Art. 416.)

33. A gentleman bought bank stock, paying regular annual dividends of 6%, at 120. What was the rate per cent. of his income, or what per cent. did he receive on the money invested?

ANALYSIS.—Since dividends are reckoned on the par value of the stock, the dividend on 1 share of \$100 would be \$6. Since each share costs \$120, and pays \$6 income, the per cent. will be $\$6 \div \120 , or 5%.

NOTE.—The above analysis will not apply to bonds that mature at a certain fixed time, unless the investor expects to sell the bonds before maturity at the cost price. If 6% bonds that mature in 1891 are purchased in 1881 at 120, and are sold at the same rate before maturity, they will pay 5% on the investment, or cost. If the bonds are held until maturity (1891), or for 10 years, the owner would receive from the government the par value only, or \$100 for a bond of that amount, and the bonds would yield less than 5%. If 6% bonds, maturing in 10 years, are purchased at $107\frac{7}{10}$ and held until maturity, they will pay 5% on the investment (See Ex. 48). If 6% bonds, that mature in 2 years, are purchased at more than 112, there would be a loss of interest to the purchaser instead of a gain.

34. Which is the better investment, stock paying a regular annual dividend of 5% and bought at 80, or stock paying 8% dividends and bought at 120?

35. If insurance stock paying regular dividends of 10% annually is bought at $137\frac{1}{4}$, brokerage $\frac{1}{4}$ %, what per cent. of income will it produce?

36. Which investment will produce the greater annual income and how much, \$20,000 invested in Chemical Bank stock at 2000 which pays dividends of 15% every 2 months, or the same amount invested in Chatham Bank stock at 125 which pays regular semi-annual dividends of 3%?

37. What rate can you afford to pay for stock paying regular annual dividends of 10%, in order to realize 6% on the investment?

38. Stocks bought at 80 pay regular dividends of 5%. What is the rate per cent. on the investment? At what rate should they be purchased to afford 4% on the investment? To afford 8%?

39. At what price must 8% stocks be purchased to afford 5% on the investment? To afford 6%?

40. I sell 200 sh. H. & St. J. pf. at $111\frac{3}{8}$, and \$10000 N. Y. Elevated 1st mortgage bonds at 119. What will be the net proceeds of the sale, allowing usual brokerage?

41. Purchased 400 shares Lake Shore at $118\frac{1}{2}$, and 200 shares Chesapeake and Ohio 2d pref., at $24\frac{3}{8}$. Sold the Lake Shore at $113\frac{5}{8}$, and the Chesapeake and Ohio at $22\frac{3}{4}$. What was the loss, usual brokerage, no interest?

42. July 26, a broker received from a customer a remittance of \$1000 as a margin (432) and purchased for him 100 shares of St. Paul at 59. On Aug. 2, the broker sold the stock at $64\frac{1}{2}$. What was the customer's profit?

OPERATION.

<i>Dr.</i>					
July 26.	To 100 shares St. Paul 59.	\$5900			
	Commission $\frac{1}{8}\%$. .	12.50	5912	50	
Aug. 2.	Interest \$5912.50, 7 days.		*	**	**** **
<i>Cr.</i>					
July 26.	By margin deposited . .		1000		
Aug. 2.	" 100 shares St Paul $64\frac{1}{2}$	\$6450			
	Commission $\frac{1}{8}\%$. .	12.50	6437	50	
Aug. 2.	Interest \$1000, 7 days . .		*	**	**** **
	Balance				**** **

The profit is equal to the balance less \$1000, the original deposit.

43. Aug. 30, a broker purchased for the account of a customer 300 shares Northwestern Railroad stock at 78. He deposited as a margin \$3000. On Sept. 22, the stock was sold at $74\frac{3}{4}$. What was the loss? (Interest 6%, usual commission.)

44. May 10, a speculator deposited with his broker \$5000 as a margin, and directed him to purchase for his account 500 shares N. Y., L. E., & W., pref. at $90\frac{3}{8}$. May 20, the stock was sold at $94\frac{1}{8}$. What was the gain, interest 6%, usual brokerage?

45. An operator, supposing Erie would decline in value, ordered his broker to sell short 100 shares at 50, and at the same time deposited with him as a margin \$1000. The broker on receiving the order sold for his account 100 shares at 50, and borrowed the stock for delivery. When the market price declined to 45, he ordered the broker to "cover his short sale" (buy the stock for delivery), and return the stock to the party from whom it was borrowed. What was the gain, usual brokerage? (No interest.)

OPERATION.

<i>Cr.</i>			
By margin deposited.	.	.	\$****.**
" 100 shares Erie borrowed and sold at 50	.	****.**	\$****.**
<i>Dr.</i>			
To 100 shares Erie bought and returned at 45	.	\$****.**	
" commission for selling the stock $\frac{1}{8}\%$.	**.**	
" " " buying the stock $\frac{1}{8}\%$.	**.**	****.**
" amount to credit	.	.	****.**

The net profit equals the balance less the margin deposited.

46. A broker sold "short" for me 400 sh. C. B. & Q., at 135 $\frac{3}{4}$, and 100 sh. C. R. I. & P., at 132 $\frac{1}{2}$. My "short" sale on C. B. & Q. was "covered" at 131 $\frac{1}{2}$, and C. R. I. & P. at 133 $\frac{3}{4}$. What was my net profit, usual brokerage? (No interest.)

47. Sold Aug. 11, 500 shares Chicago & Alton, s. 3, at 94 $\frac{1}{2}$, and covered my short sale Aug. 14, at 91. What was my profit, allowing the usual brokerage?

48. At what price may 6% bonds, maturing in 10 years, be purchased, so that the investment will pay 5%?

NOTE.—Tables have been constructed on various plans, and different methods are used by bankers and financiers, for the solution of problems relating to bond investments; two of which are given below.

ANALYSIS. 1. In the following method, it is presumed that the accruing interest is not reinvested, but that a sufficient part of it is set aside as a sinking fund to make up the amount which was originally paid out as premium.

A \$1000 bond in 10 years at 6% would amount to \$1600 (\$1000 + $10 \times \$60$). \$1 in 10 years at 5% would amount to \$1.50. To amount to \$1600, the principal, or the amount paid for the bond, must be as many times \$1 as \$1.50 are contained times in \$1600, or \$1066.66 $\frac{2}{3}$ (106 $\frac{2}{3}$ %).

If a \$1000 bond is purchased at 106 $\frac{2}{3}$, it will be necessary to set aside as a sinking fund each year \$6.66 $\frac{2}{3}$ ($\frac{2}{3}$ %) to make up the premium in 10 years. The annual interest, \$60, less \$6 $\frac{2}{3}$, the annual sinking fund, is \$53 $\frac{1}{3}$, which is 5% of \$1066 $\frac{2}{3}$, the cost of the bond or the amount invested.

If the amount set aside as a sinking fund is placed at interest, either simple or compound, 6% bonds, maturing in 10 years and purchased at 106 $\frac{2}{3}$, would pay a little more than 5%.

2. The following method anticipates compound interest throughout; *i.e.*, the interest is immediately reinvested at compound interest.

The holder of a \$1000 bond would receive \$60 interest annually, and \$1000, the face of the bond, in 10 years. If money is worth 5%, the several interests in the 10 years at compound interest would amount to \$754.674 (\$1 placed at compound interest at the beginning of each year would amount in 9 years to \$11.5779 (314). \$11.5779 plus \$1 of the last interest = \$12.5779. \$60 would amount to 60 times \$12.5779, or \$754.674. \$1000, the principal, plus \$754.674, the compound amount of the interest, equals \$1754.674, the total value of the bond at maturity, money being worth 5%. The present worth of \$1754.64, due in 10 years, at 5% compound interest, is \$1754.64 \div \$1.6289 (313), or \$1077.19. Hence the bonds must be purchased at 1.07 $\frac{19}{100}$ to pay 5% on the investment. See Ex. 33, Note.

49. What must I pay for 6% bonds, maturing in 15 years, that my investment may yield 3 $\frac{1}{2}$ %? (Both methods.)

50. 6% bonds, maturing in 10 years and bought at 106 $\frac{2}{3}$, pay what per cent. on the investment? (See 1st analysis. Ex. 48.)

ANALYSIS.—A \$1000 bond would amount in 10 years at 6% to \$1600. If \$1066.66 $\frac{2}{3}$ is paid for the bond, the net interest for 10 years is \$1600 - \$1066.66 $\frac{2}{3}$, or \$533.33 $\frac{1}{3}$; and for one year \$533.33 $\frac{1}{3} \div 10$, or \$53.33 $\frac{1}{3}$. An income of \$53.33 $\frac{1}{3}$ on an investment of \$1066.66 $\frac{2}{3}$ is equivalent to 5% (\$1066.66 $\frac{2}{3} \div$ \$53.33 $\frac{1}{3}$).

51. What rate of interest do I receive on my investment if I buy 7% bonds maturing in 20 years at 133 $\frac{1}{4}$?

T A X E S .

DEFINITIONS.

435. A **Tax** is a sum of money assessed on persons and property to defray the expenses of a state, county, town, corporation, or district.

1. In certain states all citizens above 21 years of age are required by law to pay a certain tax on the person. This tax is called a *Capitation* or *Poll Tax*.

2. The expenses of states, counties, towns, etc., are paid by a direct tax upon the property or polls of the same. The methods of assessing taxes differ in the several states. In some states, a certain percentage of the whole tax is assessed upon the polls, while in others the poll tax is a fixed amount for each citizen. In certain states, the whole tax is paid by the owners of the property of the same.

3. The expenses of the United States government are paid by duties on imports; the internal revenue (the tax upon distilled spirits, fermented liquors, tobacco, snuff, and cigars, proprietary medicines, perfumery and cosmetics, playing cards, matches, etc.); sales of public lands; tax on circulation, deposits, and capital of national banks; customs fees, fines, penalties, and forfeitures; fees, consular, letters patent, and land; profits on coinage, etc.

The receipts of the United States for the year ended June 30, 1881, were as follows: Customs, \$198,159,676; internal revenue, \$135,264,385; public lands, \$2,201,863; miscellaneous, \$25,156,366.

436. **Real Estate** is fixed property; as land, houses, etc.

437. **Personal Property** is movable property, as money, stocks, bonds, mortgages, furniture, merchandise, etc.

438. An **Assessor** is a person appointed or elected to estimate the valuation of all property liable to taxation.

439. A **Collector** or **Receiver** of taxes is a person appointed or elected to collect or receive the taxes of a city, town, village, or district.

Collectors receive a commission on the amount collected or a fixed salary.

EXAMPLES.

440. 1. For the fiscal year 1879, the N. Y. State tax levy was at the rate of $2\frac{8\frac{6}{100}}{1000}$ mills. How much would this rate produce, the valuation of the taxable property being \$2,686,140,000?

2. The rate of taxation of a certain county was $3\frac{1}{2}$ mills, and the amount of the tax \$40,653.48. What was the valuation of the property?

3. The following were the rates of taxation of New York for state purposes, 1880:—schools, 1.085 mills; general purposes, 1.475 mills; new capitol, .6 mills; canals, .34 mills. What was the total rate of taxation, and how much was raised by a county whose valuation was fixed by the state board of equalization at \$11,047,534?

4. The state tax of a certain county was \$38,666.37, and the valuation of the county, \$11,354,880. How much of this tax was paid by a town whose valuation was fixed by the board of supervisors at \$3,938,663.17?

5. The total county expenses of the same county were \$25,063.35. How much should be apportioned to the above town?

6. Taxes were levied in a certain town for the following purposes:—support of poor, \$2,000; roads and bridges, \$500; accounts audited by town auditors, \$2,876.10; accounts audited by supervisors, \$19.48; county expenses, \$9,774.72 less a surplus of \$6,055.90 in the county treasury; state and school tax, \$15,079.88; surplus tax, \$868.98. What was the rate of taxation, the total valuation of the property, as made by the town assessors, being \$4,321,252? What was the tax of Mr. A., whose valuation was \$7,300?

NOTE.—To save labor in the calculation of taxes, a table similar to the following is usually prepared by the accountant.

TAX TABLE.—Rate, 5.8 mills on \$1.

	0	1	2	3	4	5	6	7	8	9
1	.0580	.0638	.0696	.0754	.0812	.0870	.0928	.0986	.1044	.1102
2	.1160	.1218	.1276	.1334	.1392	.1450	.1508	.1566	.1624	.1682
3	.1740	.1798	.1856	.1914	.1972	.2030	.2088	.2146	.2204	.2262
4	.2320	.2378	.2436	.2494	.2552	.2610	.2668	.2726	.2784	.2842
5	.2900	.2958	.3016	.3074	.3132	.3190	.3248	.3306	.3364	.3422
6	.3480	.3538	.3596	.3654	.3712	.3770	.3828	.3886	.3944	.4002
7	.4060	.4118	.4176	.4234	.4292	.4350	.4408	.4466	.4524	.4582
8	.4640	.4698	.4756	.4814	.4872	.4930	.4988	.5046	.5104	.5162
9	.5220	.5278	.5336	.5394	.5452	.5510	.5568	.5626	.5684	.5742

7. Find from the preceding table the tax on \$16750.

OPERATION.

Tax on \$16000 is \$92.80
 “ 750 “ 4.35
 “ 16750 “ 97.15

ANALYSIS.—By looking in the ta-

ble opposite 1 and under 6, we find that the tax on \$16 is \$.0928, and by removing the point 3 places to the right, we find the tax on \$16000 to

be \$92.80. In the same manner, the tax on \$750 is found to be \$4.35. The tax on \$16750 is \$92.80 plus \$4.35, or \$97.15.

8. How much was paid by Mr. B. on an assessment of \$6400, the collector charging a commission of 1% additional?

9. Mr. D. being delinquent was charged 5% additional. How much was he obliged to pay on a valuation of \$9500?

10. What was the total tax, including commission of 1% of Mr. C., whose real estate was assessed at \$24000, and personal property at \$15500?

11. Find the total tax on 8375 pounds tobacco at 16¢, 4360 gallons distilled spirits at 70¢, 2165 barrels beer at \$1.

12. How much is the semi-annual tax of a national bank whose circulation is \$225,000 at $\frac{1}{2}\%$, average deposits \$1,416,387 at $\frac{1}{4}\%$, capital stock \$400,000 at $\frac{1}{4}\%$?

DUTIES.

DEFINITIONS.

441. Duties or Customs are taxes assessed by the Government upon imported merchandise for the purpose of revenue for the support of the government and for the protection of home industry.

1. The waters and shores of the United States are divided into collection districts ; in each of which there is a port of entry and one or more ports of delivery. Thus, the district of Boston and Charlestown comprises all the waters and shores within the counties of Middlesex, Suffolk, and Norfolk. Boston (including Chelsea) is the port of entry, and Medford, Cohasset, Hingham, Weymouth, Cambridge, Roxbury, and Dorchester, the ports of delivery. All ports of entry are also ports of delivery.

2. All cargoes chargeable with duties shall be entered and the duties paid, or secured to be paid, at the port of entry, before permission shall be given to discharge the same at the port of delivery.

3. The principal officer of every district is the collector, who is assisted by deputy-collectors, surveyors, appraisers, weighers, gaugers, inspectors, etc. The duties of the above vary in the several collection districts and ports. There is also in the leading ports of entry, a "naval officer," whose department is a check upon that of the collector. He receives copies of all invoices and entries, estimates duties, countersigns permits, clearances, certificates, debentures, and other documents, granted by the collector.

4. The surveyor usually superintends and directs the inspectors, weighers, and gaugers, within his port.

5. An importer desiring a permit to land merchandise, presents his invoice, with the consular certificate, bill of lading, and the formal entry attached (See Ex. 21, Art. **451**), to the entry clerk at the custom-house, and makes the necessary oath before the collector or his deputy. The duties, if any, are estimated in the departments of the collector and the naval officer. The amount of the estimated duties having been paid, or secured by a bond, the collector, together with the naval officer, where there is one, grants a permit to land the merchandise.

It is the custom of custom-house brokers and many merchants to calculate the duties and enter the same on the entry.

The permit is presented to the inspector in charge of the vessel, who allows the merchandise to be landed. The collector indicates on the permit by numbers what packages shall be sent to the public store for examination.

When the merchandise is examined by the appraiser (447), he enters on the invoice (446) or manifest the rate of duty to be collected. The invoice and the accompanying papers are then sent to liquidators in both the collector's and naval officer's departments for adjustment. The liquidators check the calculations on the entry, or again calculate the duty if the appraiser has changed the rate or the dutiable value, or if the returns of the weigher or gauger differ from the weight or measurement in the invoice. The amount of duty to be refunded or collected is marked on the entry. If the difference between the duty as estimated and as liquidated is less than \$1, it is disregarded, and the liquidator approves the original estimate.

442. A Custom-House Broker is a person who makes entries, secures permits, and transacts other business at custom-houses for merchants. He is familiar with the tariff laws and the details and regulations of custom-house business, and usually acts under a power of attorney.

443. Duties are of two kinds, *ad valorem* and *specific*.

444. An Ad valorem Duty is a tax assessed at a certain per cent. on the dutiable value of the merchandise; as silks at 60%, watches at 25%, linens 30, 35 and 40%.

1. The dutiable value of merchandise is its market value at the port of export, but not less than its invoiced cost, commission added, whether paid or not. It is usually the original cost plus all charges, excepting the consul's fee, to the vessel on which the shipment is made. The charges include the transportation to the place of export, the value of the sack, box, *etc.*, in which the merchandise is contained, commission at the usual rates, but in no case less than 2½%, brokerage and all other charges, except the consul's fee. There is no duty on the freight or transportation from the port of export. The appraised value is sometimes greater than the invoice value (446).

2. In reducing foreign money to U. S. money for the purpose of calculating duties, if the cents of the result are less than 50, they are rejected; if more than 50, \$1 is added to the dollars.

445. A **Specific Duty** is a tax assessed at a certain sum per ton, pound, foot, yard, gallon, or other weight or measure, without reference to the value; as leaf tobacco at 35¢ per pound, ale and beer (not bottled) 20¢ per gallon, clay \$5 per ton, plate glass per square foot, playing cards 25 and 35 *cts.* per pack, brandy \$2 per proof gallon, lumber per M feet board measure, salt (in bulk) 8 *cts.* per 100 *lbs.*, flaxseed 20 *cts.* per bushel (56 *lbs.*), cotton goods per square yard.

1. Before specific duties are calculated, allowances are made for tare (the weight of the box, barrel, or cask), leakage (of liquids in barrels), and breakage (of liquids in bottles, usually 5%).

2. The U. S. Custom House ton contains 2240 *lbs.* (**167**, 3), the hundred-weight 112 *lbs.*, and the quarter 28 *lbs.*

3. On certain goods, there is both a specific and an ad valorem duty (sometimes called a combined duty); as iron wire \$20 3½ *cts.* per pound and 15%, tobacco pipes (except common clay) \$1.50 per gross and 75%, statuary marble \$1 per cubic foot and 25%, woollen goods 50 *cts.* per pound and 35%.

446. An **Invoice** (**252**) is a statement made by the seller or shipper of merchandise giving a description of the same, and showing marks, numbers, quantity, value, charges, and other details. (See Ex. 22, Art. **451**.)

1. All invoices are made out in the weights and measures of the country from which the importation is made.

2. All invoices of merchandise subject to a duty ad valorem, are made out in the currency of the country or place from whence the importation is made.

3. When the value of the foreign currency is fixed by law (see Art. **187**), the value is to be taken in estimating the duties; when the value is not fixed by law, the invoice must be accompanied by a consular certificate showing its value.

4. All invoices of importations must, before the shipment of the merchandise, be produced to and authenticated by the U. S. consular officer, where there is such an officer. In countries without a U. S. consular officer, the authentication is made by a consul of a country in amity with the United States; or, if there be no such consul, then by two respectable resident merchants.

447. An Appraiser is an officer of the customs who examines imported merchandise and determines the dutiable value and the rate of duty of the same.

1. The place where the examinations are usually made is called the "Public Store."

2. One package of every invoice and one package at least out of every ten similar packages, shall be sent to the public store for examination. Certain bulky and heavy articles are examined at the wharf where unloaded. Weighable and gangeable goods on which the duties are specific, are not sent to the public store for examination.

3. When the appraised value of any merchandise subject to an ad valorem duty is 10% more than the invoice value as entered by the importer, then in addition to the duty imposed by law on the same, there shall be collected 20% of the duty imposed on the same.

448. A Bonded Warehouse is a place for the storage of merchandise on which the duties or taxes have not been paid.

1. If an importer does not desire to place his goods at once in the market, or anticipates exporting the same, by giving a bond for the payment of the duties and making the entry in the proper form, he may have the merchandise stored at his own risk in a bonded warehouse, and thus defer the payment of the duties.

2. Merchandise may be withdrawn from a bonded warehouse for exportation to Canada or other foreign country, without the payment of the duty on the same.

3. Merchandise that may be in warehouse under bond for more than one year, will be liable when withdrawn for 10% additional duty.

4. Any goods remaining in public store or bonded warehouse beyond three years shall be regarded as abandoned to the government, and sold under certain regulations and the proceeds paid into the Treasury.

449. Drawback.—When distilled spirits, fermented liquors, medicines, and perfumery, upon which an internal revenue tax has been paid, and foreign merchandise upon which an import duty has been paid, are exported, the tax or duty upon the same is refunded. Such return of the tax or duty is called a Drawback.

450. The **Free List** is a list of articles which are exempt from duty.

In making entries of free goods, the value as given in foreign money must be reduced to U. S. money, permits must be obtained to land the goods, and certain packages are sent to the public store for examination.

EXAMPLES.

451. 1. A merchant imported from Lyons an invoice of silk, the dutiable value (**444**, 1) of which was 48765 francs. What was the dutiable value of the same in U. S. money, and what was the duty at 60% (**444**)?

NOTES.—1. For foreign moneys of account and their values in United States money, see Art. **187**.

2. 48765 francs at 19.3¢ = \$****. (See Art. **444**, 2.) 60% of \$**** = \$****.**.

2. Find the duty on 1617 pounds of almonds, at 6 *cts.* per pound.

3. An invoice of woollen cloth weighing 516 pounds, and valued at £327 16*s.*, was imported from England. What was the duty at 50 *cts.* per pound and 35%?

4. An importer on making his entry at the custom-house, paid the duty on 38716 pounds (Invoice weight) of tobacco, at 35 *cts.* per pound. According to the return of the custom-house weigher, the net weight was 38472 pounds. How much of the duty was refunded when the entry was liquidated?

5. The duty on 28432 pounds of sugar was paid at the rate of $2\frac{3}{4}$ *cts.* per pound. According to the weigher's return, the net weight was 28218 pounds. How much additional duty was collected, the appraiser having fixed the duty at $3\frac{1}{4}$ *cts.* per pound?

6. Find the duty on an invoice of linens from Ireland. Dutiable value £424 15*s.* 6*d.*, at 35%?

7. What is the duty on an invoice of porcelain vases from Paris at 50%, dutiable value 9843 francs?

8. Find the duty on 475 *cu. ft.* of statuary marble imported from Italy, dutiable value 16425 lire, at \$1 per cubic foot, and 25%.

9. What is the duty on 37420 pounds of pig iron at \$7 per ton (445, 2)?

10. Find the duty on an invoice of leather goods from Vienna, dutiable value 6429 florins, at 35%?

11. What is the duty on an importation of toys from Germany, dutiable value 8437 marks, at 50%?

12. What is the duty at 28 cents per *sq. yd.* and 35%, on 1248 yards of Brussels carpet, 27 *in.* wide, invoiced at 3*s.* 6*d.* per yard, shipping charges (less consul's fee) £2 16*s.* 9*d.*?

13. Find the duty on an importation from Canada of 5284 bushels of potatoes, invoiced at 45 *cts.* per bushel, and 37475 pounds of hay, invoiced at \$12.50 per ton, the duty on potatoes being 15 *cts.* per bushel, and on hay 20%.

14. On a certain invoice of 34216 pounds of pepper, there are discounts for damage as follows: 12% on 6190 pounds, 8% on 6438 pounds, and 5% on 9642 pounds. After deducting the discount, what would be the duty on the remainder at 5 cents per pound?

15. The duty on burlaps is 30% *ad valorem*. What is the amount chargeable on a bale containing 50 webs, each being 54 *yds.* and 16 *in.* long, and 27 *in.* wide, and valued at 30 cents per *sq. yd.*?

16. What is the amount of duty chargeable on 2465 pounds of wool, valued at £171 8*s.*, when the rate of duty is 10 *cts.* per pound and 11% *ad valorem*?

17. The duty on certain glass plates being 35 cents per *sq. ft.*, find the duty on 316 boxes, each containing 20 plates, and each plate being 24 *in.* by 30 *in.*

18. Find the duty at 25%, on one engraving, cost in London £34 5*s.*, case and shipping charges 15*s.*, commission 2½%.

19. What is the duty at \$1 per *cu. ft.* and 25%, on a block of marble $2 \times 3 \times 7$ *ft.*, imported from Italy, dutiable value 3450 lire?

20. Find the duty on 4175 *lbs.* cloves at 5¢ per *lb.*, 476 *lbs.* cinnamon at 20¢, and 5437 *lbs.* rice at $2\frac{1}{2}$ ¢.

Find the dutiable value, and calculate the duty on the following invoices and entries:

21. Invoice of one package merchandise, purchased by GLADHILL & Co. for account of D. BUCKLEY & Co., New York, and forwarded for shipment to D. & C. MACIVER, Liverpool.

		£	s.	d.
D. B.	4 Pieces Cotton Pantaloon 32 <i>in.</i> wide, .			
207	#1729 79 <i>yd.</i> ,			
	30 80,			
	31 $77\frac{1}{2}$,			
	32 79, 315 $\frac{1}{2}$ (less $\frac{1}{3}$) 307 @ 2s. 2d.,	**	*	*
	1 $\frac{1}{2}$ % discount,		**	
		**	**	*
	Consul's Certificate,		14	10
	2 $\frac{1}{2}$ % Commission,		16	5
		**	*	*
	Less Consul's Certificate (not dutiable),		14	10
		33	11	7

Entry of merchandise, imported by D. BUCKLEY & Co. in the Str. "Catalonia" from Liverpool. New York, Jan. 12, 1882.

		35%.
D. B.	One case cotton,	33-11-7
207	@ 4.8¢65 =	\$***.
	Duty 35% of \$*** =	\$***.**

22. Invoice of 700 bales leaf tobacco shipped by F. B. DEL RIO & Co., per Str. "Niagara" for New York, and consigned to FREDERICK DE BARY & Co.

F. B.	700 bales 83077 lbs.,	\$35000	
	CHARGES.		
3328/4027	Baling,	\$525.	
	Export duties,	3407.39	
	Consul fee,	2.75	
	Small charges,	49	**** *
			***** **
	Commission $2\frac{1}{2}\%$,		*** **
	Spanish gold,	\$*****	**

HAVANA. Dec. 27, 1881.

Custom House, New York, Collector's Office, Jan. 4, 1882.
Bond No. 9817.

Entry of merchandise, imported on the third day of January, 1882, by FREDERICK DE BARY & Co., in the Str. "Niagara" from Havana.

Marks.	Nos.	Packages and Contents.	T.C.	
F. B.	3328/4027	700 bales Leaf Tobacco,	84240 lbs.	\$39958.74
		Duty 84240 lbs. @ 35¢ =		@ .93,2 =
		† Weighers return 83675 lbs. at 35¢ =		*****.
		Refund,		*****.
		† Added by the liquidator.		

23. What is the duty on an invoice of crockery invoiced at £1275 16s. 6d. *f. o. b.* (free on board), at 40%?

24. What is the duty on 28916 pounds of steel rails at 14¢ per pound, and 11438 pounds of tin plates at $1\frac{1}{10}$ ¢ per pound?

PARTNERSHIP.

DEFINITIONS.

452. Partnership is the association of two or more persons who join their capital and services for the purpose of conducting business, the gains or losses being shared in such proportion as may be stipulated in the agreement.

The business association is called a Firm, House, or Company; and each individual of the association is called a Partner.

453. A Special Partner is one who takes no active part in the business, and whose liability is limited to the amount of his investment. In order to thus limit his liability, the amount of his investment must be duly advertised, and he must take no active part in the business.

The partners who conduct the business are called General Partners. Their private property is liable for the debts of the partnership.

454. The Capital or Capital Stock is the money or other property which is invested in a business.

The partners' accounts are used to show the amounts invested. |

In most firms, the investments are entered in the partners' "stock accounts," and the amounts withdrawn by the partners during the year and their salaries are entered in their "private accounts."

455. A Resource or Asset is any kind of property belonging to the concern having a financial value.

456. A Liability is a debt owing by the concern

457. The Net Worth of a concern is the excess of its resources over its outside liabilities.

458. The Net Insolvency of a concern is the excess of its outside liabilities over its resources. The concern being unable to pay its debts in full, it is said to be insolvent.

459. Gains or Losses, how shared.—In most partnerships, the gains or the losses are divided according to certain fractions or percentages; the inequalities of the investments are adjusted by allowing interest upon the same; and the partners receive salaries for their services rendered. (See Ex. 26, Art. **461**.) Sometimes the net gain or net loss is shared in proportion to the investments (Ex. 9, Art. **461**), or the average investments. (Ex. 20, Art. **461**.) In joint stock companies the gains (dividends) and the losses (assessments) are shared in proportion to the investment or the amount of stock held.

460. Gains or Losses, how found.—When the books have been kept by “Single entry,” and when no books have been kept, the gain is found by subtracting the net worth (**457**) at commencing, or the investment, from the net worth at closing; and the loss, *vice versa*.

When the books have been kept by “Double entry,” the gain may be found as above, or by subtracting the sum of the separate losses from the sum of the separate gains. The results by the two methods should be the same and should prove each other.

EXAMPLES.

461. 1. A and B are partners, A sharing $\frac{2}{3}$ of the gain or loss and B $\frac{1}{3}$. A invests \$5000, and B \$2350. At the end of the year their resources and liabilities are as follows: merchandise on hand, per inventory, \$2000; real estate, \$7000; cash, on hand and in bank, \$1532; due on personal accounts, \$1642.25; notes on hand, \$1000; notes out-

standing, \$800 ; owing by the concern to sundry persons, \$1471.69. What is the amount of net resources belonging to each partner ?

FIRST OPERATION.

RESOURCES.

Merchandise on hand, . . .	\$2000	
Real estate,	7000	
Cash on hand,	1532	
Personal accounts,	1640.25	
Bills receivable,	<u>1000</u>	\$13172.25

LIABILITIES.

Bills payable,	\$800	
Personal accounts,	<u>4471.69</u>	5271.69
Present worth,		\$7900.56
Investments (subtracted),		<u>7350.</u>
Total net gain,		\$550.56

$\frac{2}{3}$ of \$550.56 = \$367.04, A's share of the gain.

$\frac{1}{3}$ of \$550.56 = \$183.52, B's share of the gain.

A's investment,	\$5000	
Plus his gain,	<u>367.04</u>	
Equals his present worth,		\$5367.04
B's investment,	\$2350	
Plus his gain,	<u>183.52</u>	
Equals his present worth,		\$2533.52
Total present worth, as above,		\$7900.56

SECOND OPERATION.

ANALYSIS.—Theoretically, all the resources of a business belong to the creditors and the partners (proprietors), the partners' investments being regarded as liabilities ; hence, the resources and liabilities—including the partners' accounts—should be equal. If in a statement of the condition of a business, the resources and liabilities thus considered should not be equal, it is evident that the partners' accounts do not show their true interests, and the inference is that a gain or loss has occurred which has not been entered to their accounts. The

excess of resources over liabilities would in such case show the gain, as would the excess of liabilities over resources show the loss. In order to restore the equilibrium, the gain should be credited or the loss debited to the partners' accounts.

1. STATEMENT BEFORE ADJUSTING PARTNERS' ACCOUNTS.

RESOURCES.		LIABILITIES.	
Merchandise, . . .	2000	Bills payable, . . .	800
Real estate, . . .	7000	Personal accounts, . .	4471.69
Cash,	1532	A's investment, . . .	5000
Personal accounts, .	1640.25	B's do.	2350
Bills receivable, . .	1000		12621.69
	<u>13172.25</u>		
	12621.69		
Excess res'rce's (net gain),	550.56.	A's $\frac{2}{3}$, \$367.04; B's $\frac{1}{3}$, \$183.52.	

2. STATEMENT AFTER ADJUSTING PARTNERS' ACCOUNTS.

RESOURCES.		LIABILITIES.	
Merchandise, . . .	2000	Bills payable, . . .	800
Real estate, . . .	7000	Personal accounts, . .	4471.69
Cash,	1532	A's investment and gain,	5367.04
Personal accounts, .	1640.25	B's do.	2533.52
Bills receivable, . .	1000		
	<u>13172 25</u>		<u>13172.25</u>

2. A and B are partners, A sharing $\frac{2}{3}$ of the gain or loss and B $\frac{1}{3}$. A invested \$5000, and B \$2350. During the year the concern gained on merchandise, \$955.56; on real estate, \$315. The expense account showed a loss of \$675; the interest account, \$45. What was the net gain, and balance of each partner's account?

NOTE.—The above example is the complement of Ex. 1. The books having been kept by double entry, the separate gains and losses are given, and the net gain thus found. The loss and gain account and the partners' accounts are shown in the following operation in "skeleton ledger" form.

OPERATION.

A.				B.			
Balance,	5367 04	Investment,	5000	Balance,	2533 52	Investment,	2350
		Gain, . .	367 04			Gain, . .	183 52
	5367 04		5367 04		2533 52		2533 52
		Balance, .	5367 04			Balance, .	2533 52

LOSS AND GAIN.

Expense, . .	675		Mdse., . .	955	56
Interest, . .	45		Real Estate.	315	
A's Gain $\frac{2}{3}$, .	367	04			
B's " $\frac{1}{3}$, .	183	52			
	1270	56		1270	56

3. A and B started in business July 1, 1881. Each put into the concern \$2200. The resources on Jan. 1, 1882, were as follows: goods, \$4000; bills receivable, \$1500. The liabilities were \$580. A has drawn out cash, \$3000; and B, \$2000. How much is due each partner, the gain or loss being divided equally?

NOTE.—It must be borne in mind that the amounts drawn out by the partners are as fully resources of the business as if charged to an outside party.

4. On Jan. 1, my brother and I started a business in which I invested \$900, and he \$400. We now propose to separate, and the business stands as follows: stock in store, \$1800; cash on hand and in bank, \$1200; outstanding accounts, considered good, \$1200. According to the agreement, I am entitled to $\frac{3}{5}$ of the net gain, and my brother $\frac{2}{5}$. During the time of the copartnership, I have drawn \$4000 and he, \$2800. Of the assets given above, how much are we each entitled to?

5. C, D, and E are partners, each investing \$10000, and each to share $\frac{1}{3}$ of the gain or loss. The resources and liabilities at the close of business are found to be as follows.

viz.: Merchandise on hand, per inventory, \$8159.50; cash on hand, \$5012.88; personal accounts due the firm, \$4235; notes and accepted drafts (bills receivable) on hand, \$5000; real estate, \$8000; bonds and stocks, \$12000; owing by the firm to sundry persons, \$5505; firm's notes outstanding (bills payable), \$3000. C has withdrawn during the year \$1247.87; D, \$1400; and E, \$1489. What is each partner's interest in the concern at closing?

6. C, D, and E are partners, sharing the gains and losses equally. C's net investment was \$8752.13; D's, \$8600; and E's \$8511. During the year the firm's gains were as follows: Merchandise, \$8529; stocks and bonds, \$650; interest, \$985.25. The cost of conducting the business was \$2125. What was each partner's interest at closing?

7. M and N are partners, M sharing $\frac{3}{4}$ of the gain or loss and N $\frac{1}{4}$. M invested \$15000 and N \$5000. At the close of the business year, the resources and liabilities of the concern are as follows: cash on hand, \$2128; bills payable, \$4000; bills receivable, \$3000; the firm owes sundry persons, \$8375; due the firm from sundry persons, \$16427; rent paid in advance, \$375; mortgage held by the concern on the property of A. G. Pope, \$5000; accrued interest on the same, \$150; store fixtures valued at \$835; merchandise on hand, \$9416; accrued interest on firm's notes outstanding, \$112; accrued interest on notes held by the firm, \$75. M has withdrawn \$2465; and N, \$2275. According to the agreement, each partner is to receive a salary of \$2500. What are the separate interests at the close of the business?

8. X, Y, and Z commence business without capital. According to the partnership contract, X is to receive a salary of \$3000; Y, \$2500; and Z, \$2000; the gain or loss to be divided equally. During the year, X withdraws \$3000; Y, \$2800; and Z, \$1800. What is the balance due each partner at the end of the year, if the gain, without taking into account the partners' salaries, is \$9000?

9. A and B are partners in business, the gain or loss to be divided in proportion to their investments. A invested \$8750; B invested \$4000. The net gain is \$2726.15. What is each partner's share?

FIRST OPERATION.—FRACTIONAL METHOD.

ANALYSIS.—Since A's investment, \$8750, is $\frac{8750}{12750}$ of the total investment, he is entitled to $\frac{8750}{12750}$ of the gain; and for a similar reason, B is entitled to $\frac{4000}{12750}$ of the gain.

$$\frac{8750}{12750} = \frac{35}{51}; \frac{35}{51} \text{ of } \$2726.15 = \$1870.89, \text{ A's gain.}$$

$$\frac{4000}{12750} = \frac{16}{51}; \frac{16}{51} \text{ of } \$2726.15 = \$855.26, \text{ B's gain.}$$

SECOND OPERATION.—BY PROPORTION.

ANALYSIS.—The total investment is to each partner's investment as the total gain is to each partner's gain.

$$\$12750 : \$8750 :: \$2726.15 : \$1870.89, \text{ A's gain.}$$

$$\$12750 : \$4000 :: \$2726.15 : \$855.26, \text{ B's gain.}$$

NOTE.—Cancel any factor common to the given extreme and either of the means.

THIRD OPERATION.—BY PERCENTAGE.

ANALYSIS.—\$2726.15, the gain, is 21.3816% of \$12750, the total investment. The partners' gains are therefore 21.3816% of their respective investments.

$$21.3816\% \text{ of } \$8750 = \$1870.89, \text{ A's gain.}$$

$$21.3816\% \text{ of } \$4000 = \$855.26, \text{ B's gain.}$$

NOTE.—In order to produce exact results by this method, it is necessary to extend the number expressing the rate per cent. of the gain or loss to several decimal places.

10. E, F, G, and H enter into a joint speculation. E advances \$5000, F \$7000, G \$8000, and H \$10000, the gain or loss to be divided according to their investments. They gain \$14285. What is the share of each?

11. A lot, whose front is 240 feet and whose depth is 100 feet, is bought by A, B, and C, who pay respectively \$3000, \$4000, and \$5000. How many feet front is each entitled to, if it is divided in proportion to their investments?

12. Four merchants ship goods on joint account. A puts in \$6000, B \$5500, C \$4200, and D \$4800. What will be each man's share, if the gain is \$9200?

13. Five persons having claims against the government, placed their claims in the hands of an agent for collection; A's claim amounted to \$500, B's to \$425, C's to \$300, D's to \$250, and E's to \$175; but, after the agent had deducted his fees, there remained only \$1237.50. How much did each claimant receive?

14. A and B are partners. They have cash and notes on hand to the amount of \$6475.28. A has drawn from the concern \$2478.30, and B has drawn \$1016.48. A invested \$4287.46, and B, \$1037.75. The firm owe sundry persons \$5016.82. What is each partner's present interest in the concern, if they share equally in gains and losses?

15. A and B are partners, A investing $\frac{2}{3}$ of the capital, and B $\frac{1}{3}$; the gains or losses to be shared in the same proportion. The following is an exhibit of the business, excepting the partners' accounts, at the close of a certain period: *Resources*, cash, \$3775; Stone & Co., \$150; A. R. Mead, \$1200; bills receivable, \$5500; interest on the same, \$125; merchandise, \$5140. *Liabilities*, L. Blair, \$500; W. H. Rice, \$723; Martens & Bultman, \$517.64; bills payable, \$3300; interest on the same, \$169. The net gain during the year was \$3174. What was each partner's original investment?

16. Upon a close valuation of the personal accounts due the firm in the preceding example, the partners are convinced that Stone & Co.'s is worth no more than 50% of its face; and A. R. Mead's, 25% of its face. Upon this valuation what would be the gain, and what the condition of the partners' accounts at closing?

17. P and Q are partners, each to receive interest on his net investment at the rate of 6% per annum, and the net gain or loss to be divided equally. P invests, Jan. 1, \$5000; Mar. 1, \$4000; June 16, \$1500; and draws out Apr. 16,

\$2500. Q invests, Jan. 1, \$3000; Sept. 16, \$2000; and draws out June 1, \$1500; Nov. 11, \$500. At the close of the year, the net gain is found to be \$4475.25, without taking into account the interest on the partners' accounts. What is the amount due each partner after the gain is adjusted? (Time by Compound Subtraction.)

NOTE.—From the given gain subtract the interest due the partners on their net investments. Credit the partners with their investments, with their net interest, and with their respective shares of the gain.

18. A and B have been doing business as partners, A sharing $\frac{3}{5}$ and B $\frac{2}{5}$ of the gains and losses. A invested \$4500, average date Mar. 25, 1882; and drew out \$2700, average date Sept. 12, 1882. B invested \$7200, average date June 17, 1882; and drew out \$3750, average date Oct. 25, 1882. At the time of their dissolution, Jan. 1, 1883, the debts of the firm were all paid and they had on hand belonging to the firm \$8750 in cash. How shall the money be divided, each being allowed interest at 6% on his investment and charged with interest at the same rate on the amounts drawn? (Time by exact days. Interest 360 days to the year.)

19. A and B are partners, A having $\frac{3}{5}$ and B $\frac{2}{5}$ interest. A advanced in business \$12000, average date Jan. 12, 1883; and drew out \$1265, average date Oct. 20, 1883. B advanced \$7500, average date Apr. 5, 1883; and drew out \$2560, average date Nov. 25, 1883. Jan. 1, 1884, A purchases B's interest in the business, and at that date the assets are as follows: Cash, \$5800; merchandise, \$6250; notes on hand, \$1300; accrued interest on the same, \$387.14; personal accounts, \$5700. The liabilities are as follows: Notes outstanding, \$4200; accrued interest on the same, \$227.65; personal accounts, \$2500. How much is B entitled to, 5% of the personal accounts being considered uncollectible, and interest being reckoned on the partners' account at 6% per annum (365 days to the year)?

20. A and B are partners, gain or loss to be divided in proportion to average investment. A invests, Jan. 1, \$4000; Mar. 1, \$2000; Oct. 1, \$3000; and withdraws July 1, \$1500; Dec. 1, \$1000. B invests, Jan. 1, \$6000; Sept. 1, \$3000. They close their books Jan. 1 of the following year and find they have gained \$3456. What is each partner's share?

NOTE.—An *Average Investment* is an investment for a certain period of time equivalent to several investments for different periods of time.

OPERATION.

A invested	Jan. 1,	$\$4000 \times 12 =$	$\$48000$	
"	Mar. 1,	$2000 \times 10 =$	20000	
"	Oct. 1,	$3000 \times 3 =$	9000	77000
A withdrew	July 1,	$1500 \times 6 =$	9000	
"	Dec. 1,	$1000 \times 1 =$	1000	10000
A's average investment for 1 month.				67000

Or,

A invested	Jan. 1,	$\$4000 \times 2 =$	$\$8000$	
"	Mar. 1,	2000		
		$6000 \times 4 =$	24000	
withdrew	July 1,	1500		
		$4500 \times 3 =$	13500	
invested	Oct. 1,	3000		
		$7500 \times 2 =$	15000	
withdrew	Dec. 1,	1000		
		$6500 \times 1 =$	6500	
A's average investment for 1 month,				\$67000

ANALYSIS.—By the first operation, we suppose each investment to be made for the remainder of the time. To find the average investment, multiply each investment and withdrawal by the interval between its date and time of settlement. Subtract the products obtained from the withdrawals from the products obtained from the investments. The remainder will be the average investment for 1 month, if the time is found in months. A's investment of Jan. 1 is in the business 12 months (Jan. 1 to Jan. 1); the use of \$4000 for 12 months is equivalent to the use of \$48000 for 1 month. Treating the

other investments in like manner, we find A's total investments are equivalent to \$77000 for 1 month. A's withdrawals are equivalent to \$10000 for 1 month. A's net average investment is therefore equivalent to \$67000 for 1 month.

By the second operation, we find the actual amount in the business for each month of the year. Jan. 1, A invested \$4000, which was in the business until Mar. 1, or for 2 months. Mar. 1, he added \$2000, making his total investment \$6000, which was in the business until July 1, or for 4 months. July 1, he withdrew \$1500, leaving in the business \$4500 until Oct. 1, or 3 months, etc. The several net investments as found in this manner are equivalent to \$67000 for 1 month.

B's average investment, as found by either of the above methods, is \$84000 for 1 month.

A's average investment for the year is \$5583.33 $\frac{1}{3}$; and B's, \$7000. To avoid fractions, divide the gain in proportion to the average investments for 1 month. After the average investments are found for a common time, the gain may be divided according to either of the methods under Ex. 9. By the fractional method, A would be entitled to $\frac{67}{151}$ of the gain, and B to $\frac{84}{151}$.

21. C and D are partners, gain or loss to be divided in proportion to average investment. C puts in \$6000 for one year, and \$7000 for one and a half years; D puts in \$6000 for two and a half years. The net loss is \$1565.40. What is each one's share?

22. A, B, and C are partners. A puts into the concern \$3000, but withdraws half of it at the end of 6 months; B puts in \$2000, and adds \$500 to it at the end of 4 months; C puts in \$2500 for the whole year. The gain during the year is \$1700. What is each one's share?

23. Three contractors agree to build a road for \$10000. A has 25 men at work for 16 days and 30 men for 34 days. B has 40 men for 10 days and 45 men for 40 days. C has 48 men for 50 days. C receives \$200 extra for superintending the work. How much is each contractor entitled to?

24. J, K, and L are partners, gain or loss to be divided according to average investment. J invests as follows: Jan. 1, \$6000; Apr. 1, \$4000. K invests, Jan. 1, \$8000;

L invests, Jan. 1, \$7000; Apr. 16, \$2500; and draws out June 16, \$3500. At the end of the year the net gain is found to be \$4135.60. What is each partner's share? (Time by Compound Subtraction.)

25. A, B, and C form a copartnership under the following conditions: A is to manage the business, and to receive therefor \$2400 per annum, which amount is to be credited as July 1. He is to receive interest on his salary and to pay interest on sums withdrawn at the rate of 6% per annum. B and C are to furnish the capital, and to receive interest therefor at the rate of 6% per annum. The net gain or loss to be divided equally. B invests, Jan. 1, \$10000; Apr. 1, \$5000. C invests, Jan. 1, \$10000; July 1, \$5000; and draws out Sept. 16, \$500. A draws out, Feb. 1, \$200; Mar. 1, \$400; July 11, \$500; Oct. 1, \$200; Nov. 21, \$100. At the end of the year, the gain—without taking into account either the salary to be paid to A or the interest on the partners' accounts—is \$8437.16. What will be the balance of each partner's account, when all the items have been properly entered?

26. A, B, and C are partners, A sharing $\frac{2}{5}$ of the gain or loss, B $\frac{2}{5}$, and C $\frac{1}{5}$. Interest is to be reckoned at the rate of 6% per annum (365 days to the year) on the partners' accounts, and each partner is to receive a salary of \$1800, to be credited as July 1. A invested Jan. 1, \$16000; and withdrew during the year \$4875, average date, Aug. 21. B invested, Jan. 1, \$20000; and withdrew \$6224, average date, June 18. C invested, Jan. 1, \$5000; and withdrew \$2625, average date, July 31. Jan. 1, of the following year, the merchandise account shows a gain of \$18437.16; the interest account (not including the interest on the partners' accounts) a gain of \$586.38; sundry consignment accounts show a net gain of \$1287.14. The expense account (not including the partners' salaries) shows a loss of \$3424.75. What is each partner's interest in the business at closing?

NATIONAL BANKS.

DEFINITIONS.

462. A National Bank is a bank organized under the laws of, and chartered by, the United States.

1. Associations for the purpose of carrying on the business of banking may be formed by any number of persons, not less in any case than five (R. S. § 5133).

2. No association shall be organized with a less capital than \$100,000; except that banks with a capital of not less than \$50,000 may, with the approval of the Secretary of the Treasury, be organized in any place the population of which does not exceed 6,000 inhabitants. No association shall be organized in a city the population of which exceeds 50,000 persons with a less capital than \$200,000. The capital stock shall be divided into shares of \$100 each.

3. Every national bank, before it shall be authorized to commence business, shall transfer and deliver to the Treasurer of the United States, any U. S. registered bonds, bearing interest, to an amount not less than one-third of the capital stock paid in; except that national banks having a capital of \$150,000 or less, shall not be required to deposit U. S. bonds in excess of one-fourth of their capital stock, as security for their circulating notes.

4. National banks are authorized to discount and negotiate notes, drafts, etc.; to receive deposits; to buy and sell exchange; to loan money on personal security; and to issue circulating notes.

5. National banks are prohibited from making loans on real estate (R. S. § 5137), or on security of their own shares of capital except to secure debts previously contracted (R. S. § 5201).

They are also prohibited from making loans to one person or association, excepting on business paper representing actually existing value as security, in excess of one-tenth of the capital of the bank.

6. The stockholders of a national bank are individually liable (equally and ratably, and not one for another) for an amount equal to the par value of the capital stock held by them.

463. Circulation.--Upon a deposit of registered bonds, the association making the same shall be entitled to receive from the Comptroller of the Currency circulating notes equal in amount to 90% of the current market value, not exceeding par, of the U. S. bonds so deposited, and at no time shall the total amount of such notes exceed 90% of the amount actually paid in of its capital stock.

1. Any national bank desiring to decrease its circulation may deposit lawful money (specie or legal-tenders) with the Treasurer of the United States in sums of not less than \$9,000, and withdraw a proportionate amount of bonds held as security for such notes.

No national bank which makes any deposit of lawful money in order to withdraw its circulating notes, shall be entitled to receive any increase of its circulation for the period of six months from the time it made such deposit. Not more than \$3,000,000 shall be deposited during any calendar month for this purpose.

2. The State bank circulation wholly ceased after Congress had imposed a penalty of 10% in the form of a tax every time it should be issued. This act took effect Aug. 1, 1866.

464. Redemption.--The circulating notes of national banks are redeemed in lawful money by the banks which issued them and by the Treasurer of the United States at Washington, D. C.

1. Every national bank shall, at all times, keep and have on deposit in the Treasury of the United States in lawful money of the United States, a sum equal to 5% of its circulation, to be held and used for the redemption of such circulation.

2. All national banks which go into voluntary liquidation shall, within six months thereafter, deposit in the Treasury an amount of lawful money equal to the amount of their circulating notes outstanding. The law also requires that full provision shall be made for the redemption of the circulating notes of any insolvent bank before a dividend is made to its creditors. Thus it will be seen that no association can close up its business without first providing for the payment of all its circulating notes, and that the amount deposited for their redemption must remain in the Treasury until the last outstanding note shall have been presented. It is therefore plain that the government, and not the bank, receives all the benefit arising from lost or unredeemed circulating notes.

465. Reserve.—The national banks in the reserve cities* are required by law to hold a reserve of 25% of their deposits; all other national banks 15%. The excess above legal requirements is called “surplus reserve.”

The reserve is made up of specie, legal-tender notes (184), gold certificates, silver certificates, balances due from reserve agents, and the 5% redemption fund, with the U. S. Treasurer (464, 1).

466. Surplus Fund.—The law provides that a surplus fund shall be accumulated, by setting aside, before the usual semi-annual dividend is declared, one-tenth part of the net profits of the bank for the preceding half-year, until the surplus fund shall amount to 20% of its capital stock.

467. Taxation.—The national banks pay to the United States a tax of 1% annually upon the average amount of their notes in circulation, $\frac{1}{2}$ % annually upon the average amount of their deposits, and $\frac{1}{2}$ % annually upon the average amount of capital not invested in U. S. bonds.

The banks, other than national, pay taxes to the United States on account of their deposits and capital, at the same rates as are paid by the national banks.

EXAMPLES.

468. 1. The impairment of the capital stock (\$300000) of an insolvent national bank was \$216000. What was the rate per cent. of the assessment made upon the stockholders for the purpose of making good the deficiency (462, 6)? How much was Mr. A. obliged to pay, who owned 80 shares?

2. What amount of bank notes is issued to a national bank that deposits \$180000 in U. S. bonds to secure circulation (463)? How much is its redemption fund (464, 1)?

* The reserve cities are New York, Boston, Philadelphia, Baltimore, Albany, Pittsburgh, Washington, New Orleans, Louisville, Cincinnati, Cleveland, Chicago, Detroit, Milwaukee, Saint Louis, and San Francisco.

3. A national bank, desiring to reduce its circulation, deposits with the Treasurer of the United States \$27000 in legal-tenders, and sells the bonds withdrawn (463, 1), in the market at 118 $\frac{3}{4}$. What were the proceeds?

4. How much is the redemption fund of a bank whose circulation is \$427500? What is the amount of bonds on deposit to secure its circulation?

5. The New York associated banks, according to the statement of Saturday, Mar. 25, 1882, held \$58,602,100 in specie and \$16,150,900 in legal-tenders. Their deposits on the same date were \$285,659,600. What was the excess of reserve (465) above legal requirements?

6. Oct. 1, 1881, the national banks of Boston had \$8,286,182 in specie, \$3,457,379 in legal-tenders, \$75,000 in U. S. certificates of deposit, \$11,735,499 due from reserve agents, and a redemption fund with U. S. Treasurer of \$1,603,628. Their deposits were \$95,776,386. What amount of reserve was required? What was the surplus reserve?

7. What amount of reserve was required by the national banks of the State of Maine, their deposits being \$9,558,878?

8. The net earnings of a bank, whose surplus (466) is less than 20% of its capital (\$300000), are \$10475.38. What amount must be carried to the surplus account, and what are the undivided profits after declaring a dividend of 3%?

9. What is the semi-annual tax (467) upon a national bank whose average circulation is \$462,730, average deposits \$1,185,952, capital \$1,500,000?

10. Find the semi-annual tax upon a banker whose capital is \$5358, and whose average deposits are \$18368.

11. A bank having a capital of \$250,000, and a surplus of \$50,000, earned \$58693, and declared a dividend of \$30000. What was the rate of the dividend? The dividend is what % of the capital and surplus? The earnings are what % of the capital and surplus?

SAVINGS BANKS.

DEFINITIONS.

469. Savings Banks are institutions for the deposit and safe keeping of small sums of money.

470. Interest is usually declared Jan. 1st and July 1st of each year, and when declared is carried at once to the credit of each depositor on the books of the bank, where it stands as a deposit, and is entitled to interest the same as any other deposit. Savings banks, therefore, pay compound interest.

No interest is allowed on the fractional parts of a dollar, nor is any interest allowed on any sum withdrawn previous to the first day of January or July, for the period which may have elapsed since the last dividend.

471. Deposits are practically payable on demand, though the right to require a notice of 60 or 90 days is reserved.

In some savings banks, deposits commence to draw interest Jan. 1st, April 1st, July 1st, and Oct. 1st; in others, deposits made on or before the first of any month draw interest from the first days of those months respectively.

472. According to the laws of the State of New York.

No person shall have a deposit larger than the sum of three thousand dollars.

Savings banks are restricted to 5% per annum regular interest. They must, however, declare an extra dividend at least once in three years, when their surplus earnings amount to 15% of their deposits.

Savings banks are allowed to pay interest on all sums deposited during the first ten days of January and July, and the first three days of April and October from the first of those months respectively.

EXAMPLES.

473. Perform the following examples according to both methods mentioned in Art. **471**. Where no rate is mentioned, 4% is understood.

1. Mr. A. deposited in a savings bank, Jan. 1, 1882, \$145. How much interest should be credited to him July 1, 1882?

OPERATION.

145	ANALYSIS.—In any savings bank, he would be credited
.02	for the interest of \$145 from Jan. 1 to July 1, or 6 <i>mo.</i> at 4%
2.90	<i>per annum.</i> 4% per annum is equivalent to 2% for 6 <i>mo.</i>

2. A person deposited Dec. 30, 1881, \$150; Feb. 20, 1882, \$40; April 1, 1882, \$120; May 30, 1882, \$60. What amount was due July 1, 1882, nothing having been withdrawn?

ANALYSIS.—If interest begin on the first of each quarter, the first deposit, \$150, will draw interest from Jan. 1, or for 6 *mo.*; the second and third deposits, \$160, will draw interest from April 1, or for 3 *mo.*; the last deposit, made May 30, will draw no interest July 1.

If interest begin on the first of each month, the first deposit, \$150, will draw interest from Jan. 1, or for 6 *mo.*; the second deposit, \$40, made Feb. 20, will draw interest from March 1, or for 4 *mo.*; the third deposit, \$120, made April 1, will draw interest from April 1, or 3 *mo.*; the fourth deposit, \$160, made May 30, will draw interest from June 1, or for 1 *mo.*

3. The following deposits were made in a savings bank: July 1, 1881, \$100; July 16, \$40; Aug. 1, \$75; Aug. 29, \$45; Sept. 30, \$75; Oct. 28, \$200; Nov. 25, \$30; Dec. 31, \$100. What was due Jan. 1, 1882?

4. How much interest was due on the following account July 1, 1883? Deposits, Oct. 1, 1881, \$200; Dec. 31, 1881, \$160; Mar. 24, 1883, \$100.

5. What is the balance of the following account July 1, 1879, interest being reckoned at 6% until July 1, 1877, and

at 5% thereafter: Deposits, Oct. 14, 1876, \$200; Mar. 30, 1878, \$135; April 1, 1879, \$90.

6. How much is due on the following account July 1, 1879, interest being reckoned at 6% until Jan. 1, 1877, and at 5% thereafter: Deposits, Jan. 31, 1876, \$100; Apr. 1, 1876, \$100; Oct. 28, 1878, \$30; Nov. 30, 1878, \$30; Feb. 1, 1879, \$25; Mar. 1, 1879, \$25.

7. What is the balance of the following account July 1, 1882? Balance due Jan. 1, 1882, \$103. Deposits, Jan. 28, \$40; Mar. 30, \$125; May 26, \$80. Drafts, Feb. 20, \$20; April 18, \$15; May 3, \$25; June 16, \$100.

ANALYSIS.—In order to more readily determine the amounts that are entitled to interest, arrange the account in the following form, and find the balance after each draft or after two or more drafts made without any intermediate deposit.

Date.	Deposits.	Drafts.	Balances.
Jan. 1,	103		
“ 28,	40		
Feb. 20,		20	123
Mar. 30,	125		
Apr. 18,		15	
May 3,		25	208
“ 26,	80		
June 16,		100	188

The smallest balance found is \$123, the amount remaining on deposit after the draft of Feb. 20; of this balance, \$103 was on deposit Jan. 1, and the remaining \$20 was deposited Jan. 28. (It is the custom to deduct the drafts from the last deposits made.) Since the balance, June 16, \$188, is less than the balance, May 3, \$208, it is evident that the excess, \$20, has been withdrawn, and therefore is not entitled to interest. Of the \$188, interest has already been allowed on \$123, and the remaining \$65, it is seen by inspection, was deposited Mar. 30.

If interest commence the first of each quarter, the several amounts will draw interest as follows: \$103 from Jan 1, or 6 months; \$20, deposited Jan. 28, and \$65 deposited Mar. 30, making \$85 from April 1, or 3 months.

If interest commence the first of each month, the several amounts will draw interest as follows: \$103 from Jan. 1, or 6 months; \$20, deposited Jan. 28, from Feb. 1, or 5 months, \$65, deposited Mar. 30, from April 1, or 3 months.

8. Find the balance of the following account July 1: Balance Jan. 1, \$30; deposits, Feb. 16, \$50; Apr. 1, \$185. Drafts, Mar. 12, \$60; May 10, \$50; June 20, \$60.

9. Find the balance of the following account, Jan. 1, 1883. Deposits, July 1, 1882, \$175; Aug. 1, \$40; Sept. 16, \$280. Drafts, Oct. 18, \$90; Nov. 27, \$125.

10. Balance the following account, Jan. 1, 1882. Deposits, July 28, 1881, \$100; Aug. 16, 1881, \$15; Oct. 17, 1881, \$50; Oct. 30, 1881, \$20. Drafts, Sept. 30, 1881, \$25; Nov. 30, 1881, \$100.

11. Balance the following Jan. 1, 1881. Balance due July 1, 1880, \$300. Deposits received, Aug. 1, \$150; Sept. 27, \$60; Oct. 12, \$325. Drafts paid, July 16, \$150; Sept. 1, \$150; Nov. 17, \$70; Dec. 18, \$140.

12. What is the balance of the following account July 1, 1882? Balance due Jan. 1, 1882, \$364.48. Deposits, Jan. 24, 1882, \$50; Feb. 16, 1882, \$80; Apr. 30, 1882, \$40; June 28, 1882, \$100. Drafts, Mar. 30, 1882, \$75; May 19, 1882, \$10.

13. How much was due July 1, 1882, on the following pass-book?

Dr. FRANKLIN SAVINGS BANK in account with F. G. SNOOK. *Cr.*

1881.				1881		
Jan. 1	Five Hundred Dollars.	500		July 20	One Hund. Dollars	100
Mar. 30	Ninety Dollars.	90		1882.		
1881.	Interest to July.	**	**	Jan. 2	Forty Dollars.	40
Oct. 1	One Hund. Sixty Dollars.	160		June 1	Sixty Dollars.	60
1882.	Interest to January.	**	**			
Feb. 20	Eighty Dollars.	80				
Mar. 28	Forty Dollars.	40				

LIFE INSURANCE.

DEFINITIONS.

474. Life Insurance is a contract by which a company (the insurer), in consideration of certain payments, agrees to pay to the heirs of a person, when he dies, or to himself, if living at a specified age, a certain sum of money.

475. The principal kinds of policies issued by Life Insurance Companies are the following: **Ordinary Life, Limited Payment Life, and Endowment.**

476. Ordinary Life Policies.—On this kind of policy, a certain premium is to be paid every year until the death of the insured, when the policy becomes payable to the persons named in the policy as the beneficiaries.

A policy of this kind gives more insurance, for the same sum of money paid annually, than any other, though it is necessary to continue the payments longer; as according to its terms the payment of the premiums annually continues during the life-time of the insured.

477. Limited Payment Life Policies.—On a policy of this kind, premiums are paid annually for a certain number of years fixed upon at the time of insuring—or, until the death of the insured, should that occur prior to the end of the selected period. The policy is payable on the death of the insured, whenever that may occur.

The payments on this class of policies may all be made while the insured is still young; then if he lives to old age the policy is not a continual expense, but, on the contrary, the dividends afford a yearly income in cash; or they may be used to increase the amount assured.

These policies are issued with single payments, or with 5, 10, 15, 20, or 25 annual payments.

478. Endowment Policies.—An Endowment Policy provides (1) **insurance** during a stipulated period, payable, like that of any other policy, at the death of the insured should he die within the period ; and (2) an **endowment**, of the same amount as the policy, payable at the end of the period if the insured survive until that time.

The Endowment Policy gives the insured the advantage of a limited term as to payments ; provides insurance during the period in which his death would cause most embarrassment to his family ; and, if he lives to the stipulated age, the amount of the policy is paid to him at a time when he may need it.

479. A Non-Forfeiting Policy is one which does not become void on account of non-payment of premiums.

1. According to the laws of the State of New York, after three full annual premiums have been paid, the legal reserve of the policy shall, on surrender of the policy within six months after the lapse, be applied as a single payment in either of two ways, at the option of the assured. (1) To the continuance of the full amount of the insurance so long as such single premium will purchase term insurance for that amount, or (2) to the purchase of a non-participating paid-up policy.

2. According to the Massachusetts limited forfeiture law of 1880, after two full annual premiums have been paid, and without any action on the part of the assured, the net value of the policy less a surrender charge of 8% of the present value of the future premiums which the policy is exposed to pay in case of its continuance, shall be applied as a single payment to the purchase of paid-up insurance.

3. Certain companies voluntarily apply all credited dividends to the continuance of the insurance ; others voluntarily apply the legal reserve to the purchase of term insurance at the regular rates.

4. In some companies, all limited payment life policies and all endowment policies, after premiums for three (or two) years have been paid and the original policy is surrendered within a certain time, provide for paid-up assurance for as many parts (tenths, fifteenths, etc., as the case may be), of the original amount assured, as there shall have been complete annual premiums received in cash by the Company.

480. The Surrender Value of a policy is the amount of cash which the company will pay the holder on the surrender of the policy. It is the legal reserve less a certain per cent. for expenses.

TABLE OF RATES.

481. Annual premium for an Insurance of \$1,000, with profits.

LIFE POLICIES. Payable at Death, only.					ENDOWMENT POLICIES. Payable as Indicated, or at Death, if Prior.			
AGE.	ANNUAL PAYMENTS.				AGE.	In 10 Years.	In 15 Years.	In 20 Years.
	For Life.	10 Years.	15 Years.	20 Years.				
25	\$19 89	\$42 56	\$32 34	\$27 39	25	\$103 91	\$66 02	\$47 68
26	20 40	43 37	32 97	27 93	26	104 03	66 15	47 82
27	20 93	44 22	33 62	28 50	27	104 16	66 29	47 98
28	21 48	45 10	34 31	29 09	28	104 29	66 44	48 15
29	22 07	46 02	35 02	29 71	29	104 43	66 60	48 33
30	22 70	46 97	35 76	30 36	30	104 58	66 77	48 53
31	23 35	47 98	36 54	31 03	31	104 75	66 96	48 74
32	24 05	49 02	37 35	31 74	32	104 92	67 16	48 97
33	24 78	50 10	38 20	32 48	33	105 11	67 36	49 22
34	25 56	51 22	39 09	33 26	34	105 31	67 60	49 49
35	26 38	52 40	40 01	34 08	35	105 53	67 85	49 79
36	27 25	53 63	40 98	34 93	36	105 75	68 12	50 11
37	28 17	54 91	42 00	35 83	37	106 00	68 41	50 47
38	29 15	56 24	43 06	36 78	38	106 28	68 73	50 86
39	30 19	57 63	44 17	37 78	39	106 58	69 09	51 30
40	31 30	59 09	45 33	38 83	40	106 90	69 49	51 78
41	32 47	60 60	46 56	39 93	41	107 26	69 92	52 31
42	33 72	62 19	47 84	41 10	42	107 65	70 40	52 89
43	35 05	63 84	49 19	42 34	43	108 08	70 92	53 54
44	36 46	65 57	50 61	43 64	44	108 55	71 50	54 25
45	37 97	67 37	52 11	45 03	45	109 07	72 14	55 04

1. The above table represents the maximum rates of the leading New York companies. Surplus premiums or dividends are returned annually commencing at the payment of the second premium.

2. Policies which do not share in the dividends of the company, are issued at fixed rates 15 to 20% less than the above.

3. The above rates are for annual payments only. To obtain semi-annual payments, add 4% and divide by 2. To obtain quarterly payments, add 6% and divide by 4.

EXAMPLES.

482. 1. Find the amount of premium for an ordinary life policy (**476, 481**) of \$5000, issued to a person 35 years of age.

2. What is the first annual premium of a life policy of \$6000, issued to a person 30 years old, \$1.00 being charged for the policy?

3. Find the annual premium for a 20-payment life policy (**477, 481**) of \$4000, issued to a person 28 years old.

4. What annual premium must be paid for a 20-year endowment policy (**478**) of \$8000, age of the insured at nearest birthday, 40 years? If the insured dies during the tenth year, how much more would have been paid than if he had been insured on the ordinary life plan?

5. What is the average daily cost of a life policy for \$1000, no allowance being made for probable dividends, insurance commencing at age 25? At 35? At 45?

6. How much must a person, aged 35, lay aside weekly to secure a life policy of \$1000, payable in 20 annual payments?

7. When 40 years old, a person took out a 20-year endowment policy of \$10000. He survived the endowment period. How much less did he receive than he paid as premiums, not reckoning interest?

8. Mr. A. when 26 years old took out an ordinary life policy of \$20000. He died aged 41 years 2 months. How much more did his heirs receive than had been paid as premiums, no allowance being made for interest?

9. Supposing money to be worth 6% (simple interest), what was the net gain of the above insurance?

10. The annual premium, without profits, on a life policy of \$10000 at age 35 is \$222. How much would it be necessary to invest at 6% interest to secure the payment of the annual premium? How much would the insured leave his family at his death?

11. A gentleman, age 30, insures his life for \$20000, ordinary life plan. How much must be placed in trust so that the interest at 5% will be sufficient to pay the annual premiums? At his death, how much does he leave his family?

12. Mr. C. when 25 years of age secured a 20-year endowment policy of \$6000; when he was 30 years of age, he obtained an ordinary life policy of \$4000; when 35 years of age, he took out a 20-payment life policy of \$10000. What was the total annual premium after taking the last policy?

13. Suppose Mr. C. had died at the age of 40½ years, how much more would his heirs receive than had been paid as premiums?

14. A single premium for an assurance of \$1000, without profits, for a person 32 years of age, is \$300. What would be the excess of the assurance over the amount produced by placing the money at compound interest (314) at 4%, supposing the insured to live 20 years?

15. Mr. A., aged 30, secures an ordinary life policy, annual premium \$100. How much more would his heirs receive from the insurance company than from the money at compound interest at 5%, should he die at the age of 32? Of 40? Of 50?

16. What is the semi-annual premium (481, 3) on a 20-year endowment policy for \$6000, age 32?

17. Mr. A., who will be 35 years of age July 1, takes out Apr. 1 a 20-payment life policy for \$10000, premium payable semi-annually. Mr. B., of the same age, takes out Apr. 1 the same kind of policy for \$5000, and Oct. 1. another policy of the same kind for \$5000, premium payable annually. How much less does Mr. B. pay as premium each year than Mr. A.? (481, 3.)

18. An ordinary life policy issued at age 35 for \$10000 has, at age 45, a 4% reserve of \$1262.60. How much non-participating paid-up insurance will this amount purchase, the single premium rate per \$1000 at age 45 being \$475.44?

MISCELLANEOUS EXAMPLES.

483. 1. Add $17\frac{1}{3}$, $28\frac{3}{4}$, $36\frac{1}{2}$, $44\frac{5}{8}$, $89\frac{7}{12}$, and $76\frac{4}{5}$; multiply the sum by 87; subtract $1022\frac{1}{4}$ from the product; and divide the remainder by $234\frac{2}{3}$.

2. Divide eighty-three, and seventy-five hundredths by one hundred and twenty-five ten-thousandths; add to the quotient sixty-eight, and six hundred and twenty-five thousandths; and multiply the sum by three, and two tenths.

3. How many minutes in the month of February, 1900?

4. Find the cost of 7312 pounds of meal at \$2.25 per *cwt.*

5. The difference in the local time of two places 1 *hr.* 7 *min.* 13 *sec.*; what is the difference in longitude?

6. Find the number of square yards of paving in a street, 3000 *ft.* long and 50 *ft.* wide.

7. What is the charge for packing, marking, and shipping 251 bales merchandise at 5s. 6*d.* per bale?

8. If 46 *T.* 12 *cwt.* of coal are worth \$174.75, what is the value of 37 *T.* 8 *cwt.*?

9. How many square yards of linoleum would cover a floor 22 *ft.* 6 *in.* by 15 *ft.* 4 *in.*? Find its value at 63¢ per *sq. yd.*

10. What is the freight of 5 *T.* 9 *cwt.* 2 *qr.* 8 *lb.*, at 70 shillings per ton (2240 *lbs.*)?

11. What is the weight in grams of the U. S. gold dollar?

12. What is the value of a *Lac* (100,000) of rupees in U. S. money? (See Art. 187, India.)

13. A bank collected a draft of \$9375.16. What were the proceeds, the charge for collection being $\frac{1}{4}\%$?

14. What is the cost of insuring \$18000 at 75¢ per \$100?

15. What is the cost of 250 *ft.* hose, at 60 *cts.* per foot, less and 10%, and 5 sets couplings at \$1.50 each?

16. What is $2\frac{1}{2}\%$ of £159 13s. 10*d.*?

17. A's property is assessed at \$7500, and the rate of taxation is \$2.165 on \$100. What is his tax, including a commission of 1%?

18. What is the duty at 60% on an invoice of silk amounting to 36475 francs?

19. A merchant buys a bill of dry goods, Apr. 16, amounting to \$6377.84, on the following terms: 4 months, or less 5% 30 days. How much would settle the account May 16? The above discount is equivalent to what rate per cent. per annum?

20. Mr. B. purchased 36150 pounds of hay at \$16.50 per ton, and 16438 pounds of oats at 70 cents per bushel. He sold the hay at a gain of 16%, and the oats at a loss of 8%. What were the proceeds?

21. A merchant buys goods at a discount of 40 and 20% from the list price, and sells at a discount of 30 and 10%. What is the gain per cent.?

22. Mar. 16, a merchant buys a bill of goods amounting to \$2475 on the following terms: 4 months, or less 5% if paid in 30 days. Apr. 15 he makes a payment of \$1000, with the understanding that he is to have the benefit of the discount of 5%. With what amount should he be credited on the books of the seller? How much would be due July 16, the expiration of the 4 months?

23. May 10, A buys a bill of goods amounting to \$5000 on the following terms: 60 days, or 1% discount in 30 days, or 2% discount in 10 days. May 20 he makes a payment of \$2000, and June 9, of \$2500. How much would be due July 9, the end of the 60 days' credit?

24. Oct. 16, B bought a bill of merchandise amounting to \$2000 on the following terms: 4 months, or 5% discount in 30 days, or 6% discount in 10 days. Oct. 26 he made a payment of \$1000. How much would settle the bill Nov. 15?

25. What is the freight of 39445 pounds of merchandise at 64s. per ton of 2240 pounds?

26. Find the total freight on 68 *ft.* mdse. at 35 shillings per ton (40 *cu. ft.*), and 123 *ft.* at 40 shillings per ton, plus 10% primage on each item.

27. Total imports and exports carried in foreign vessels for the fiscal year, 1879, were valued at \$911,269,232; in American vessels for the same time, \$272,015,697. What per cent. were carried in American vessels?

28. Total imports and exports carried in foreign vessels during the fiscal year 1858, were valued at \$160,666,267; in American vessels for the same time, \$447,191,304. What per cent. were carried in American vessels? In foreign vessels?

29. Bought coal by the long ton at \$3.64, and sold by the short ton at \$4.25. What was the gain per cent.?

30. A commission merchant in Chicago sells for me 12 bales brown sheeting, each bale containing 800 yards, at 7 *cts.* per yard; pays transportation and other charges amounting to \$72; and invests the proceeds in flour at \$4.80 per barrel. If he charges $2\frac{1}{2}\%$ for selling and $1\frac{1}{4}\%$ for purchasing, how many barrels of flour does he send me?

31. Find the date of maturity and the net proceeds of a note for \$5000, dated May 16, payable 4 months after date, and discounted July 21 at 6%.

32. When the above note became due, its maker had discounted at 6% a new note, payable 90 days after date, whose proceeds were sufficient to pay the first note. What was the face of the new note?

33. Apr. 1, a merchant buys a quantity of coffee on 90 days' credit, with privilege of discounting within 30 days from date of purchase at the rate of 6% per annum for the unexpired time. Apr. 16 he makes a payment of \$28000 on account, no actual invoice having been rendered. May 1 he receives the invoice, amounting to \$29215, and on the same date full settlement is made. What amount was required to cancel the bill? (Exact days, 360 days to the year.)

34. Find the amount due on the following note Jan. 1, 1883, by the United States Rule :

\$5000 $\frac{90}{100}$.

DAVENPORT, IOWA, May 1, 1878.

On demand, I promise to pay EDWIN D. MORGAN, or order, Five thousand dollars, with interest at ten per cent. for value received.

E. H. CONGER.

On this note the following payments were indorsed :

Received Jan. 16, 1879, \$400. Received Dec. 12, 1880, \$150.
Received Sept. 7, 1879, \$100. Received Aug. 18, 1881, \$850.
Received May 1, 1880, \$500. Received Apr. 23, 1882, \$100.

35. What is the value of a draft on Hamburg of 17468 marks at 95 $\frac{3}{4}$?

36. C. of London owes me for goods sold on my account, £129 18s. 7d. How much do I receive in payment, if I draw a bill of exchange for the amount and sell it at 4.85 $\frac{1}{2}$?

37. My agent in Paris buys an invoice of merchandise amounting to 12488 francs, at a commission of 2 $\frac{1}{2}$ %. What is the cost of the draft which I remit in payment, the rate of exchange being 5.17 $\frac{3}{4}$?

38. An exporter sold the following bills of exchange through a broker: 10000 francs on Paris at 5.16 $\frac{3}{4}$, £375 16s. 8d. on London at 4.83 $\frac{1}{2}$, 16480 marks on Hamburg at 94 $\frac{1}{2}$, 5287 guilders on Amsterdam at 41 $\frac{1}{8}$. What were the proceeds, brokerage $\frac{1}{8}$ %?

39. A commission merchant at New York sells goods for A. of Havre to the amount of \$3435.27, and charges a commission of 2 $\frac{1}{2}$ % for selling. What is the face of the draft which he purchases and remits in settlement, exchange being 5.27?

40. My agent in London has purchased for me, at a commission of 2 $\frac{1}{2}$ %, 375 dozen kid gloves at 49d. per dozen, and 636 yards silk at 9s. 6d. per yard. When exchange is \$4.86 $\frac{3}{4}$, what will be the cost of the draft which I remit to him in settlement?

41. What per cent. is gained by selling pans at 21 cents apiece, that cost \$2.56 per dozen less 20 and $12\frac{1}{2}\%$?

42. The estimated sound value of a quantity of merchandise, damaged at sea, was \$328.55, and the proceeds when sold at auction, \$299.35. How much of the loss was shared by the Insurance Co., the insurance having been \$315.33?

43. I purchase books at \$2 each, less $33\frac{1}{3}\%$, and 5% for cash. What was the net cost, and what per cent. discount may be given on the list price to produce a net profit of 10%?

44. A synopsis of the report of the N. Y. C. & H. R. R. R. for its fiscal year ended Sept. 30, 1881, is as follows: Gross earnings from passengers, \$6,958,038; from freight, \$20,736,749; from miscellaneous, \$4,653,608; expenses, \$19,464,786; interest, rentals, and taxes, \$4,990,783. What was the surplus for the year after the declaration of a dividend of 8% on a capital stock of \$89,229,300?

45. If a merchant buys goods at a certain price 10 and 5 off, and sells them at the same price, 5 off, what per cent. profit does he make?

46. What must be the asking price for books that cost \$1.60, in order to abate 20%, and still make a profit of 25%?

47. M, the owner of a mill, employs S, a miller, under the following conditions: M is to furnish the requisite capital, and S to receive, in lieu of salary, $\frac{1}{3}$ of the profits. M has a store connected with the mill, on the books of which are entered all time sales of mill products. The grain, etc. for the mill is furnished by M. At the beginning of the year the value of the grain, flour, feed, etc. is \$1727. During the year M's purchases for the mill amount to \$19275. S has received for cash sales \$16337, of which he has paid over to M \$15550. The sales on account, as shown on M's books, amount to \$8375; and the value of the products on hand is \$2828. During the year S has purchased goods at M's store to the amount of \$837.65. How much is owing to S at the expiration of the year?

48. Average the following account :

Nov. 13, 1881,	a bill of \$138.42	on 30 days' credit.
" 30, " "	416.10	" 60 " "
Dec. 16, " "	324.70	" 30 " "
Jan. 5, 1882,	" 586.85	" 4 months' "
" 26, " "	234.38	" 60 days' "
Feb. 12, " "	93.60	" 4 months' "
" 23, " "	618.75	" 30 days' "
Mar. 5, " "	374.36	" 60 " "

49. Average the following account. What will be the amount due Jan. 1, 1883 ?

<i>Dr.</i> ARNOLD, CONSTABLE & Co.				<i>Cr.</i>		
1882.				1882.		
Apr. 4	Mdse. 4 mo.	426	32	Apr. 25	Cash.	375
" 20	" Cash.	387	40	June 30	"	600
May 13	" 60 da.	622	39	July 31	Note 60 da.	600
" 27	" 30 da.	584	75	Aug. 15	Cash.	500
July 5	" 4 mo.	224	50	Oct. 31	"	400
" 16	" 4 mo.	838	95			

50. Prepare an account current, including interest at 6% to Jan. 1, 1883, from the above ledger account, according to the form and method of Art. 409.

51. Sold five \$1000 bonds at $116\frac{3}{8}$, and invested the proceeds in railroad stock at $92\frac{7}{8}$, which I sold at $98\frac{1}{4}$. What was the gain on the stock, allowing usual brokerage ?

52. Sold Aug. 11, 1879, 500 shares Chicago & Alton, s. 10, at $94\frac{1}{2}$, and covered my short sale Aug. 16, 1879, at 91. What was my profit, allowing the usual brokerage ?

53. What annual income will be obtained by investing \$9923.75 in bonds, bearing 5% interest, and purchased at $116\frac{3}{4}$?

54. The duty on spool thread of cotton, containing 100 yds. to the spool, is 6¢ per dozen spools and 30% ad valorem. Find the duty on 11160 spools valued at 3¢ a spool.

APPENDIX.

DRILL EXERCISES.

484. Exercise in Addition.—Take any number less than 1000; repeat the number; add the two numbers; add the three numbers; add the last three numbers, and so continue until there are twelve numbers. The numbers expressed by the three right hand figures of the fourth and twelfth numbers will be the same, if the original number is even, and will differ by 500 if the original number is odd. Add all the numbers. The sum will equal 1104 times the original number. (See Ex. 3.)

Hints in Addition.—1. Write the numbers in vertical lines. Irregularity in the placing of figures is the cause of many errors.

2. Think of results and not of the numbers themselves. Thus, in Ex. 1, Art. **487**, do not say 3 and 4 are 7 and 9 are 16, etc., but 7, 16, 26, etc.

3. Make combinations of 10 or other numbers as often as possible, and add them as single numbers. When a figure is repeated several times, multiply it instead of adding.

Add 9 and 1, 8 and 2, 6 and 4, 5 and 5, 4, 3 and 3, etc., as 10; 7 and 2, 6, 2, and 1, etc., as 9; 2 and 3, 4 and 1, 2, 2 and 1, as 5; etc., etc.

4. To avoid repeating the work, in case of interruption, write the figures to be carried in pencil underneath, as in Ex. 3.

5. In adding long columns, prove the work by adding each column separately in the opposite direction, before adding the next column.

485. Exercise in Subtraction.—Take any number less than 1000; subtract it from 1000; subtract the remainder from the last number, omitting the fourth figure and borrowing from the fourth place when necessary; so continue until sixteen subtractions have been made. The seventh and sixteenth remainders will be the same. Add the numbers. The three right-hand figures of the sum

will be the same as the three right-hand figures of the product obtained by multiplying the original number by 391. (See Ex. 4.)

486. Exercise in Multiplication and Division.—Take any number; find the continued product of it and any set of numbers. Use the last product as a dividend, and divide it by the same numbers in the same order, using each quotient as a dividend for the next division. The last quotient will be the original number. (See Ex. 5.)

NOTE.—In the drill exercises in addition, multiplication, and division, if the original number is a multiple of 9, each number and result will be a multiple of 9, and therefore the sum of the digits of each number will be a multiple of 9. This property of 9 may be used in the detection of errors.

487.**EXAMPLES.**

(1.)	(2.)	(3.)	(4.)	(5.)
Add	Add	Add	1000	87×2
3456	37.16	347	517	174×3
9716	875.25	347	483	522×4
2356	412.75	694	034	2088×5
7327	734.	1388*	449	10440×6
2468	147.03	2429	585	62640×7
7535	948.26	4511	864	438480×8
2845	272.72	8328	721*	3507840×9
9610	371.59	15268	143	2) 31570560
2581	87.20	28107	578	3) 15785280
1473	3.16	51703	565	4) 5261760
7812	27.84	95078	013	5) 1315440
1593	375.13	174888*	552	6) 263088
4826	617.37	383088	461	7) 43848
7374	583.14	24557	091	8) 6264
3259	27.48		370	9) 783
4374	344.22		721*	87
3213	5.76		8147	

SQUARE ROOT.

488. The Square Root of a number is one of the two equal factors of a number. Thus, the square root of 25 is 5. $5 \times 5 = 25$.

489. To find the square root of a number.

490. RULE.—*Beginning at units' place, separate the given number into periods of two figures each.*

Find the greatest square in the left-hand period, and write its root at the right in the form of a quotient in division. Subtract this square from the left-hand period, and to the remainder annex the next period to form a dividend.

Double the part of the root already found for a trial divisor. Find how many times this divisor is contained in the dividend, exclusive of the right-hand figure, and write the quotient as the next figure of the root. Annex this quotient to the right of the trial divisor to form the complete divisor. Multiply the complete divisor by the last figure of the root, and subtract the product from the dividend.

To the remainder annex the next period, and proceed as before.

NOTE.—When the given number is a decimal, separate the number into periods of two figures each, by proceeding in both directions from the decimal point.

EXAMPLES.

491. 1. Find the square root of 1156.

OPERATION.		ANALYSIS.—Beginning at units' place, separate the number into periods of two figures each. The greatest square in the left hand period (11) is 9, and its root is 3, which is written in the quotient. By subtracting this square (9) from the left-hand period (11) and annexing to the remainder (2) the next period (56), we form the dividend, 256. By taking twice the root already found (3), we have 6 as a
64	11'56 (34	
	9	
	256	
	256	
	0	

trial divisor, which is contained in the dividend (25), exclusive of the last figure, 4 times. Write 4 in the quotient, and also to the right of the trial divisor, forming the complete divisor, 64. Multiplying the complete divisor, 64, by 4, the last figure of the root, and subtracting the product (256) from the dividend (256), there is no remainder. 34 is the required root.

Find the square root of

- | | | | |
|-----------|-----------|---------------|----------------|
| 2. 1089. | 6. 47524. | 10. 119025. | 14. 3976036. |
| 3. 14641. | 7. 65025. | 11. 1406.25. | 15. 431.8084. |
| 4. 18225. | 8. 86436. | 12. 512656. | 16. 7463824. |
| 5. 46656. | 9. 97344. | 13. 232.5625. | 17. 387420489. |

18. Find the square root of $\frac{256}{6}$. Of $1\frac{369}{600}$. Of $\frac{5625}{12321}$.

NOTE.—In finding the square root of a fraction, extract the square root of the numerator and denominator separately.

CUBE ROOT.

492. The **Cube Root** of a number is one of the three equal factors of that number. Thus, the cube root of 8 is 2
 $2 \times 2 \times 2 = 8$.

493. To find the cube root of a number.

494. RULE.—*Beginning at units' place, separate the given number into periods of three figures each.*

Find the greatest cube in the left-hand period, and write its root at the right in the form of a quotient in division. Subtract this cube from the left-hand period, and to the remainder annex the next period to form a dividend.

Multiply three times the square of the root already found by 100 for a trial divisor. Find how many times this divisor is contained in the dividend, and write the quotient as the next figure of the root. Add to the trial divisor three times the product of the last figure of the root, with a cipher annexed, and the other figures of the root, and the square of the last

figure to form the complete divisor. Multiply the complete divisor by the last figure of the root, and subtract the product from the dividend.

To the remainder annex the next period, and proceed as before.

EXAMPLES.

495. 1. Find the cube root of 39304.

OPERATION.

	39'304 (34	
	27	
Trial divisor. $3 \times 3^2 \times 100 = 2700$	12 304	
$3 \times 3 \times 4 \times 10 = 360$		
$4^2 = 16$		
Complete divisor, 3076	12 304	
	0	

ANALYSIS.—Beginning at units' place, separate the number into periods of three figures each. The greatest cube in the left-hand period (39) is 27, and its root is 3, which is written in the quotient. By subtracting this cube (27) from the left hand period (39), and annexing to the remainder (12) the next period (304), we form the dividend, 12304. By multiplying 3 times the square of the root already found by 100 (to multiply by 100 add two ciphers), we form the trial divisor, 2700 ($3 \times 3^2 \times 100 = 2700$). The trial divisor, 2700, is contained in the dividend, 12304, 4 times. Write 4 as the next figure of the root. To form the complete divisor, add to the trial divisor (2700) 3 times the product of the last figure (4) of the root and the other figure (3) with a cipher annexed ($3 \times 3 \times 4 \times 10 = 360$), and the square of the last figure ($4^2 = 16$). Multiplying the complete divisor, 3076, by 4, the last figure of the root, and subtracting the product (12304) from the dividend (12304), there is no remainder. 34 is the required cube root.

Extract the cube root of

2. 5832.	6. 551368.	10. 98611128.
3. 10.648.	7. 7529536.	11. 279.726264.
4. 39304.	8. 9.663597.	12. 435519.512.
5. 474552.	9. 13651.919.	13. 676836152.

ANSWERS

Page 9.

1. 1614.
2. 1654.
3. 19380.
4. 23243.
5. 26162.
6. 35130.
7. 4566.
8. 3722.
9. 53609.
10. 44601.
11. 50480.
12. 34914.

Page 11.

1. 50.
2. 556.
3. 629.
4. 4915.
5. 4857.
6. 394.
7. 376.
8. 321.
9. 435.
10. 508.
11. 3642.
12. 3645.
13. 3755.

Page 12.

14. 54877.
15. 44444.
16. 41568.
17. 36311.
18. 84839.
19. 139059.
20. 10,078,521.
21. 561.
22. 3363.
23. 1308.
24. 7745.
25. 297.

26. 6822.
27. 1711.
28. 1440.
29. 7529.

Page 13.

30. 14152.
31. 442,254,988.
32. \$1,177,761.-
723.
33. 5,302,516.
34. 324,423,840.
35. \$9858,94.

Page 15.

1. 4337.
2. 907823.
3. 40865.
4. 110547.
5. 8,495,098.
6. 853759.
7. 999895.
8. 1,109,975.
9. 6,419,754.
10. 72540.
11. 57,249,251.
12. 10,648,519.
13. 113,558,829.
14. 15,562,130.
15. 74,299,273.
16. 5,654,786.
17. 90,119,023.
18. 122882.
19. 921294.
20. 19,212,939.
21. 1235.
22. 11,594,495.
23. 193,941,760
acres.

Page 16.

24. \$93,309,621.
25. 3025 lbs.

26. 5,204,822;
1,354,008;
6,558,830.
27. 3,850,814.
28. 388,904.
29. 1106.

Page 18.

1. 164192;
187648.
2. 340236;
226824.
3. 49374;
98748.
4. 110775;
295400.
5. 243580;
438444.
6. 817281;
726472.
7. 130240;
182336.
8. 1,578,246;
2,367,369.
9. 494268;
617835.
10. 4,690,158;
7,035,237.
11. 3,336,072;
2,919,063.
12. 4,072,384;
3,563,336.
13. 3,824,910;
5,737,365.
14. 5,240,172;
3,742,980.
15. 58080.
16. 2016.
17. 24256.
18. 63360.

Page 19.

1. 63936;
831168.

2. 75218;
1,463,858.
3. 70272;
2,436,096.
4. 209387;
1,915,125.
5. 358661;
1,264,432.
6. 544375;
4,606,875.
7. 6,896,064;
87,772,352.
8. 5,817,408;
195,035,421.
9. 5,761,888;
195,249,432.
10. 18,413,409;
556,524,675.
11. 8,326,575;
173,434,110.
12. 25,930,788;
317,327,062.
13. 5250.
14. 506880;
1,098,240.

Page 20.

15. 26376.
 16. 106515;
153720.
 17. 4608.
 18. 5016.
-
1. 144000;
1,080,000.
 2. 138,240;
864000.
 3. 241,920;
1,451,520.
 4. 185500;
1,335,600.
 5. 120000;
3,200,000.

6. 252,000;
1,036,000.
7. 84000;
2,100,000.
8. 9,680,000;
67,760,000.
9. 18,500,000;
92,500,000.
10. 7,407,000;
44,442,000.
11. 11,760,000;
131,600,000.
12. 6,698,000;
114,260,000.
13. 80,192,000;
98,808,000.
14. 86,400,000;
460,800,000.

Page 22.

1. 39,456,174;
26,304,116.
2. 24,413,116;
16,275,410 $\frac{2}{3}$.
3. 3,265,524;
2,721,270.
4. 19,517,701;
11,152,972.
5. 2,057,613;
1,371,742.
6. 197,730,864;
123,581,790.
7. 58,642,209;
26,063,204.
8. 178,606,127;
51,030,322.
9. 51,525,354 $\frac{1}{2}$;
20,610,141 $\frac{1}{2}$.
10. 71,387,270;
35,693,635.
11. 106,315,682;
60,751,818 $\frac{2}{3}$.
12. 41,152,263;
18,432,098 $\frac{1}{2}$.
13. 41,133,539;
22,436,475 $\frac{9}{11}$.
14. 112,731,950;
78,912,365.
15. 57,447,290;
23,936,379 $\frac{1}{2}$.
16. 29,351,981 $\frac{1}{2}$;
16,010,171 $\frac{7}{11}$.
17. 4628 sq. yd.

18. 10388s.
19. 89 doz.;
\$20.47.
20. 5280 ft.
21. 20006 gal.

Page 24.

1. 11840 $\frac{5}{25}$;
2152 $\frac{109}{143}$.
2. 13100 $\frac{11}{10}$;
9991 $\frac{66}{249}$.
3. 9879 $\frac{28}{88}$;
1001 $\frac{53}{77}$.
4. 10028 $\frac{25}{44}$;
4149 $\frac{85}{115}$.
5. 16948 $\frac{6}{25}$;
2037 $\frac{10}{205}$.
6. 17022 $\frac{6}{18}$;
714 $\frac{26}{42}$.
7. 20144 $\frac{47}{44}$;
227 $\frac{25}{56}$.
8. 97138 $\frac{1}{3}$;
1641 $\frac{1}{3}$.
9. 119348 $\frac{66}{33}$;
10663 $\frac{78}{17}$.
10. 17008 $\frac{1}{2}$;
3914 $\frac{106}{355}$.
11. 41933 $\frac{5}{22}$;
20093 $\frac{37}{144}$.
12. 526,009 $\frac{2}{14}$;
57532 $\frac{32}{125}$.
13. 30366 $\frac{19}{70}$;
6642 $\frac{199}{30}$.
14. 58694 $\frac{9}{4}$;
16261 $\frac{34}{233}$.
15. 116213 $\frac{23}{20}$;
11462 $\frac{61}{385}$.
16. 39625 $\frac{9}{22}$;
52181 $\frac{50}{220}$.
17. 365 $\frac{6}{24}$.
18. 1381 $\frac{8}{4}$.
19. 677 $\frac{10}{32}$.
20. 356 $\frac{3}{22}$.
21. 498299 $\frac{56}{11}$.
22. 368 $\frac{60}{28}$.
23. 153 $\frac{20}{10}$.
24. \$786395 $\frac{7}{15}$.
25. 2222.
26. 3,969,568.

Page 27.

1. \$2837.46.
2. \$1022.25.

3. \$2775.87.
4. \$3383.08.
5. \$14.91.
6. \$18.51.
7. \$569.25.
8. \$4839.75.

1. \$97.44.
2. \$51.
3. \$273.53.
4. \$677.
5. \$3306.
6. \$187.60;
\$276.48.
7. \$86.73;
\$939.12.
8. \$400.80;
\$2164.32.
9. \$3168;
\$25344.
10. \$1305;
\$4241.25.
11. \$2313.12;
\$13878.72.
12. \$3756;
\$9615.36.

Page 28.

1. \$3.45.
2. 15 lbs.

Page 29.

3. \$.53; \$.24.
4. \$19.04;
\$13.60.
5. \$.48; \$.36.
6. \$.87; \$.84.
7. \$1.48; \$.185.
8. \$.25.12;
\$1,227.
9. 53; 123.
10. 414; 72.
11. 217; 36.
12. 2304; 256.
13. 1540.44;
256.
14. 4088; 280.
15. 876 lb.

Art. 20.

1. 3², 5, 7, 11.
2. 3, 7, 11, 13.

3. 2, 3³, 7, 11.
4. 2, 3², 5², 7².
5. 3, 5², 7².
6. 2, 3, 5, 7, 11.
7. 2³, 3², 7, 13.
8. 2, 7, 13, 43.
9. 2, 3, 7, 11, 13
10. 2³, 11, 61.
11. 3², 5², 17.
12. 2², 11².

Art. 26.

1. 12. 9. 36.
2. 7. 10. 48.
3. 14. 11. 55.
4. 15. 12. 72.
5. 32. 13. 81.
6. 66. 14. 126.
7. 30. 15. 125.
8. 42. 16. 144.

Art. 30.

1. 29.
2. 37.
3. 31.
4. 43.
5. 47.
6. 6.
7. 23.
8. 1.
9. 252.
10. 17.

Art. 37.

1. 60. 6. 360.
2. 60. 7. 504.
3. 360. 8. 480.
4. 30. 9. 480.
5. 84. 10. 480.

Art. 41.

1. 1.
2. 14 $\frac{12}{100}$.
3. 4 $\frac{20}{100}$.
4. 6.
5. 41 $\frac{10}{100}$.
6. 51 $\frac{1}{10}$.
7. 2.
8. 1008.
9. 750 yd.

10. \$31.50.
11. 36 cows.
12. \$16.92.
13. \$79.
14. \$21.
15. 56 lbs.
16. 10.
17. 90.

Art. 42.

1. 10,078,521.
2. 3,194,084.
3. 1,042,916-880.
4. \$4799.50.
5. 9,741,862.
6. \$18487.
7. 251.
8. 433.
9. 4653.
10. 862.72.
11. 16098.
12. 8899.35.
13. 1145760.
14. 92250.
15. 456.
16. 4745.
17. 64535.
18. 45009.
19. 27369.
20. 41976.
21. 2², 3³, 7, 11, 13.
22. 1802.
24. \$458.
25. 1440.
26. 3582.
27. 132 acres.
28. 2520.
29. 2006.
30. \$4.50.
31. 6.
32. \$480.
33. \$126.28.
34. \$4664.24.
35. \$46.
36. 339 head.
37. \$9292.80.
38. 750 lbs.
39. 3000 lbs.
40. \$1044.
41. \$203.22.

42. A, \$124;
B, \$125.
43. \$7.75.

Art. 63.

- | | |
|---------------------|----------------------|
| 1. $\frac{23}{33}$ | 15. $\frac{3}{16}$ |
| 2. $\frac{1}{11}$ | 16. $\frac{6}{91}$ |
| 3. $\frac{3}{4}$ | 17. $\frac{41}{128}$ |
| 4. $\frac{7}{9}$ | 18. $\frac{13}{30}$ |
| 5. $\frac{9}{16}$ | 19. $\frac{3}{8}$ |
| 6. $\frac{12}{17}$ | 20. $\frac{5}{16}$ |
| 7. $\frac{23}{35}$ | 21. $\frac{1}{11}$ |
| 8. $\frac{1}{8}$ | 22. $\frac{1}{5}$ |
| 9. $\frac{8}{9}$ | 23. $\frac{1}{8}$ |
| 10. $\frac{11}{14}$ | 24. $\frac{49}{80}$ |
| 11. $\frac{7}{8}$ | 25. $\frac{9}{16}$ |
| 12. $\frac{4}{5}$ | 26. $\frac{3}{5}$ |
| 13. $\frac{4}{5}$ | 27. $\frac{71}{193}$ |
| 14. $\frac{11}{25}$ | 28. $\frac{71}{193}$ |

Art. 66.

- | | |
|----------------------|--------------------------------------|
| 1. $\frac{36}{48}$ | |
| 2. $\frac{36}{48}$ | |
| 3. $\frac{36}{48}$ | $\frac{40}{48}$, $\frac{27}{48}$ |
| 4. $\frac{75}{105}$ | $\frac{63}{105}$, $\frac{147}{105}$ |
| 5. $\frac{16}{25}$ | $\frac{28}{25}$, $\frac{28}{25}$ |
| 6. $\frac{42}{96}$ | $\frac{88}{96}$, $\frac{578}{96}$ |
| 7. $\frac{315}{360}$ | $\frac{320}{360}$, $\frac{10}{360}$ |
| 8. $\frac{51}{72}$ | $\frac{40}{72}$, $\frac{4}{72}$ |
| 9. $\frac{48}{108}$ | $\frac{80}{108}$, $\frac{153}{108}$ |

Art. 71.

- | | |
|-----------------------|--|
| 1. $\frac{12}{30}$ | $\frac{3}{30}$, $\frac{8}{30}$ |
| 2. $\frac{42}{45}$ | $\frac{21}{45}$, $\frac{30}{45}$ |
| 3. $\frac{45}{60}$ | $\frac{32}{60}$, $\frac{152}{60}$ |
| 4. $\frac{42}{48}$ | $\frac{104}{48}$, $\frac{24}{48}$ |
| 5. $\frac{80}{84}$ | $\frac{75}{84}$, $\frac{66}{84}$ |
| 6. $\frac{148}{180}$ | $\frac{38}{180}$, $\frac{53}{180}$ |
| 7. $\frac{20}{35}$ | $\frac{18}{35}$, $\frac{288}{35}$ |
| 8. $\frac{65}{72}$ | $\frac{65}{72}$, $\frac{688}{72}$ |
| 9. $\frac{102}{72}$ | $\frac{117}{72}$, $\frac{80}{72}$ |
| 10. $\frac{65}{100}$ | $\frac{61}{100}$, $\frac{80}{100}$ |
| 11. $\frac{117}{110}$ | $\frac{51}{110}$, $\frac{94}{110}$ |
| 12. $\frac{110}{225}$ | $\frac{144}{225}$, $\frac{1144}{225}$ |

Art. 74.

1. $\frac{243}{8}$
2. $\frac{28}{3}$
3. $\frac{203}{8}$

4. $\frac{151}{33}$, $\frac{159}{33}$
5. $\frac{170}{127}$, $\frac{299}{127}$
6. $\frac{751}{223}$, $\frac{249}{16}$
7. $\frac{179}{4}$, $\frac{449}{12}$
8. $\frac{25}{2}$, $\frac{967}{20}$, $\frac{545}{12}$
9. $\frac{545}{159}$, $\frac{393}{2}$
10. $\frac{2911}{255}$, $\frac{5020}{11}$
11. $\frac{339}{14}$, $\frac{2877}{7071}$
12. $\frac{68}{30}$, $\frac{5885}{12}$
13. $\frac{3077}{5635}$, $\frac{647}{1377}$
14. $\frac{1377}{8}$, $\frac{1735}{8}$
15. $\frac{1949}{9}$, $\frac{2337}{4}$

Art. 77.

1. 39 $\frac{5}{8}$.
2. 16.
3. 93 $\frac{3}{4}$; 52.
4. 69; 125 $\frac{1}{8}$.
5. 57 $\frac{5}{8}$; 88 $\frac{7}{8}$.
6. 181 $\frac{5}{16}$; 35 $\frac{7}{8}$.
7. 36 $\frac{2}{5}$; 218 $\frac{1}{4}$.
8. 321 $\frac{3}{4}$; 78.
9. 494; 161 $\frac{7}{8}$.
10. 24 $\frac{3}{4}$; 13 $\frac{259}{370}$.
11. 201 $\frac{1}{5}$; 15 $\frac{77}{30}$.
12. 27 $\frac{2}{5}$; 38 $\frac{1}{3}$.
13. 24 $\frac{3}{8}$; 44 $\frac{1}{4}$.
14. 64 $\frac{5}{8}$; 48 $\frac{5}{7}$.

Art. 81.

1. 24.
2. 31.
3. 75 $\frac{3}{5}$.
4. 143.
5. 1403 $\frac{1}{2}$.
6. 304 $\frac{5}{8}$.
7. 382 $\frac{4}{7}$.
8. 1105 $\frac{1}{10}$.
9. 378 $\frac{6}{15}$.
10. 270 $\frac{5}{8}$.

11. 1640 $\frac{1}{8}$.
12. 1514 $\frac{1}{4}$.
13. 463 $\frac{7}{10}$.
14. 2291 $\frac{1}{10}$.
15. 1041 $\frac{9}{10}$.
16. 275 $\frac{2}{5}$.
17. 349 $\frac{1}{10}$.
18. 1586 $\frac{7}{10}$.
19. 278 $\frac{3}{5}$.
20. 370 $\frac{1}{10}$.

Art. 85.

1. $\frac{7}{36}$.
2. $\frac{11}{24}$.
3. $\frac{5}{16}$.
4. $\frac{1}{16}$.
5. $\frac{1}{6}$.
6. $\frac{4}{35}$.
7. 84.
8. 88 $\frac{3}{4}$.
9. 121 $\frac{5}{12}$.
10. 130 $\frac{1}{10}$.
11. 367.
12. 458 $\frac{2}{5}$.
13. 86 $\frac{1}{2}$.
14. 68 $\frac{1}{2}$.
15. 363 $\frac{1}{2}$.
16. 138 $\frac{1}{10}$.
17. 70 $\frac{1}{2}$.
18. 270 $\frac{1}{2}$.
19. 119 $\frac{1}{2}$.
20. 389 $\frac{1}{10}$.
21. 388 $\frac{1}{10}$.
22. 891 $\frac{1}{10}$.
23. 472 $\frac{7}{10}$.
24. 88 $\frac{2}{5}$.
25. 126 $\frac{3}{5}$.
26. 299 $\frac{1}{2}$.
27. 121 $\frac{2}{5}$.

Art. 88.

1. \$17 $\frac{1}{2}$.
2. 922 $\frac{1}{2}$ bu.
3. \$368 $\frac{1}{2}$.
4. \$21.25 $\frac{1}{2}$.
5. 31 $\frac{5}{8}$.
6. 5 $\frac{1}{2}$.
7. 2 $\frac{1}{2}$.
8. 1326.
9. 1884.
10. 745 $\frac{1}{2}$.

11. 440.

12. $1379\frac{1}{2}$.13. $3364\frac{1}{2}$.14. $4615\frac{3}{4}$.

15. 18024.

16. $19381\frac{1}{8}$.

17. 5955.

18. $36706\frac{1}{2}$.19. $50383\frac{1}{8}$.20. $2391\frac{1}{2}$.21. $13136\frac{3}{4}$.22. $10158\frac{3}{4}$.

23. 14865.

24. $77893\frac{3}{4}$.25. $204837\frac{3}{4}$.26. $252037\frac{1}{2}$.

27. 74571.

28. $289790\frac{2}{3}$.29. $322887\frac{1}{2}$.30. $389291\frac{1}{2}$.31. $79032\frac{5}{8}$.32. $259546\frac{3}{8}$.33. $183288\frac{1}{2}$.34. $486354\frac{1}{16}$.

35. 340184.

36. $342468\frac{3}{4}$.37. $413568\frac{3}{4}$.**Art. 92.**1. $\$2.18\frac{3}{4}$.2. $\$11.14\frac{1}{2}$.3. $\$36.65\frac{5}{8}$.

4. 63.

5. 72.

6. $30\frac{5}{8}$.

7. 1216.

8. 1287.

9. $2010\frac{1}{2}$.

10. 3105.

11. $9328\frac{1}{8}$.

12. 8500.

13. $4197\frac{1}{2}$.14. $3091\frac{1}{2}$.

15. 16061.

16. 50570.

17. $19017\frac{1}{2}$.18. $53462\frac{3}{4}$.19. $57734\frac{1}{2}$.

20. 70977.

21. $47078\frac{1}{2}$.22. $64171\frac{1}{2}$.23. $53253\frac{3}{8}$.24. $34246\frac{1}{2}$.**Art. 95.**1. $\frac{3}{20}$.2. $\frac{4}{21}$.3. $\frac{5}{18}$.4. $\frac{4}{7}$.5. $1\frac{1}{2}$.

6. 16.

7. 3.

8. $55\frac{5}{9}$.9. $\frac{1}{3}$.10. $12\frac{1}{2}$.11. $11\frac{1}{2}$.12. $9\frac{1}{3}$.

13. 1.

14. $1\frac{7}{105}$.15. $1\frac{1}{6}$.16. $3\frac{1}{2}$.17. $13\frac{1}{3}$.18. $3\frac{1}{4}$.19. $3\frac{3}{2}$.20. $3\frac{5}{6}$.21. $9\frac{7}{2}$.22. $2\frac{1}{10}$.23. $2\frac{1}{7}$.

24. 20.

25. $2\frac{5}{20}$.26. $115\frac{1}{2}$.27. $281\frac{1}{2}$.

28. 756.

29. 625.

30. $1\frac{3}{4}$.31. $1\frac{1}{6}$.32. $1\frac{5}{8}$.33. $2\frac{1}{2}$.34. $2\frac{5}{16}$.**Art. 98.**1. $\frac{1}{8}$.2. $\frac{1}{8}$.3. $\frac{1}{2}$.4. $\frac{1}{8}$.5. $2\frac{7}{8}$.6. $3\frac{7}{10}$.7. $5\frac{1}{2}$.8. $145\frac{7}{8}$.9. $116\frac{1}{2}$.10. $81\frac{1}{8}$.

11. 703.

12. $72\frac{1}{2}$.13. $214\frac{7}{8}$.14. $576\frac{1}{2}$.15. $483\frac{1}{2}$.

16. 703.

17. $809\frac{7}{8}$.18. $1593\frac{2}{3}$.19. $2143\frac{7}{10}$.

20. 4101.

21. $1329\frac{1}{6}$.22. $1428\frac{7}{10}$.23. $576\frac{7}{8}$.24. $194\frac{1}{3}$.25. $1460\frac{7}{12}$.26. $923\frac{7}{8}$.27. $472\frac{7}{8}$.28. $1019\frac{1}{4}$.29. $810\frac{3}{5}$.30. $396\frac{3}{5}$.**Art. 102.**

1. 20.

2. $37\frac{1}{2}$.

3. 63.

4. $117\frac{1}{3}$.5. $1\frac{1}{2}$.6. $1\frac{1}{2}$.7. $1\frac{1}{2}$.8. $1\frac{1}{2}$.9. $6\frac{2}{3}$.10. $6\frac{2}{3}$.

11. 9.

12. $11\frac{2}{3}$.13. $6\frac{3}{4}$.14. $2\frac{1}{3}$.15. $2\frac{1}{3}$.16. $1\frac{2}{3}$.17. $1\frac{2}{3}$.18. $1\frac{2}{3}$.19. $1\frac{2}{3}$.20. $1\frac{2}{3}$.21. $1\frac{2}{3}$.22. $1\frac{2}{3}$.23. $1\frac{2}{3}$.24. $1\frac{2}{3}$.25. $1\frac{2}{3}$.26. $1\frac{2}{3}$.27. $1\frac{2}{3}$.28. $1\frac{2}{3}$.29. $1\frac{2}{3}$.30. $1\frac{2}{3}$.31. $1\frac{2}{3}$.32. $1\frac{2}{3}$.33. $1\frac{2}{3}$.34. $1\frac{2}{3}$.35. $1\frac{2}{3}$.36. $1\frac{2}{3}$.37. $1\frac{2}{3}$.38. $1\frac{2}{3}$.39. $1\frac{2}{3}$.40. $1\frac{2}{3}$.41. $1\frac{2}{3}$.42. $1\frac{2}{3}$.**Art. 105.**1. $\frac{9}{16}$.2. $\frac{1}{16}$.3. $\frac{1}{16}$.4. $1718\frac{7}{8}$.5. $193\frac{1}{4}$.6. $862\frac{1}{2}$.7. $3\frac{1}{2}$.8. $302356\frac{1}{4}$.9. $303520\frac{1}{4}$.

10. 10.

11. 12.

12. 21.

13. $1763\frac{3}{4}$.14. $352\frac{3}{4}$.15. $\$186,000\frac{1}{4}$.16. $145\frac{3}{4}$.17. $\$2448$.18. $\$280.59$.19. $\$48492$.20. $95\frac{1}{2}$ bu.21. $\$22,90\frac{1}{2}$.

22. 8951.

23. A. $\$648$;B. $\$1080$.24. $4360\frac{7}{8}$.

25. 336.

26. $\$2475$.27. $\$262\frac{1}{2}$.

28. 941.

29. $96\frac{1}{2}$.30. $36\frac{1}{2}$.31. H. $\$705$;C. $\$140\frac{1}{2}$.32. $\$2003\frac{1}{2}$.33. $\$157.67$.

34. 501.

35. $\$9.46$.36. $123\frac{1}{2}$ gal.

37. Widow.

38. $\$2876.12$;

Each child,

 $\$1438.06$.

39. 1403.

40. $\$192$.41. $\$4600$.42. $\$198.31$.

43. 19744.

44. $\$4124$.45. $\$27.80$.46. $\$63.76$.

47. 320 rods.

48. Gained 2¢.

49. $\$629.30$.50. $\$5487.98$.

51. 110 bu.

52. $\$35.46$.53. $\$115.30$.54. $\$316.74$.

Art. 124.

1. .5.
2. .875.
3. .75.
4. .375.
5. .4375.
6. .78125.
7. .425.
8. .66 $\frac{2}{3}$.
9. .83 $\frac{1}{3}$.
10. .583 $\frac{1}{3}$.
11. .714285 $\frac{1}{7}$.
12. .444 $\frac{4}{9}$.
13. 16.625.
14. 27.923076 $\frac{1}{13}$.
15. 36.9583 $\frac{1}{3}$.

Art. 127.

- | | |
|--------------------------|-----------------------|
| 1. $\frac{1}{4}$. | 13. $\frac{1}{8}$. |
| 2. $\frac{3}{4}$. | 14. $\frac{1}{6}$. |
| 3. $\frac{5}{8}$. | 15. $\frac{1}{3}$. |
| 4. $\frac{6}{8}$. | 16. $\frac{5}{12}$. |
| 5. $\frac{7}{8}$. | 17. $\frac{1}{5}$. |
| 6. $\frac{1}{8}$. | 18. $\frac{3}{8}$. |
| 7. $\frac{9}{12}$. | 19. $\frac{5}{8}$. |
| 8. $\frac{10}{12}$. | 20. $\frac{7}{8}$. |
| 9. $\frac{9}{6250}$. | 21. $\frac{3}{560}$. |
| 10. $\frac{64}{125}$. | 22. $\frac{4}{9}$. |
| 11. $\frac{9}{16}$. | 23. $\frac{1}{4}$. |
| 12. $\frac{3}{4}$. | 24. $\frac{1}{12}$. |
| 25. 28 $\frac{3}{40}$. | |
| 26. 107 $\frac{1}{6}$. | |
| 27. 175 $\frac{1}{25}$. | |
| 28. 6 $\frac{7}{10}$. | |

Art. 130.

1. 492.319787.
2. 176.3380807.
3. 7162.31526.
4. 1.7591587.
5. 9710.27879.
6. 1.83586255.
7. \$776.42 $\frac{1}{2}$.
8. 21.9026730 $\frac{5}{8}$.

Art. 133.

1. 3.9803.
2. 26971.
3. 8999.1.
4. 4648.
5. 16.6736.

6. .010102.
7. \$86.17.
8. 2.126155.
9. 1.728 $\frac{1}{4}$.
10. \$121.14 $\frac{1}{2}$.
11. \$1727.93 $\frac{1}{4}$.
12. .924 $\frac{2}{3}$.
13. 31.0458 $\frac{1}{5}$.
14. 53.39 $\frac{1}{4}$.
15. 68.04 $\frac{11}{12}$.

Art. 136.

1. .0004128.
2. .80448.
3. .00010112.
4. \$43.216;
f. 182.2875.

5. 45.77125;
55.02291 $\frac{2}{3}$.
6. .273735;
1.3136.
7. 3.39924;
.409652.
8. .00540625;
.8455375.
9. 11.208704;
.0100672.
10. 5.705483 $\frac{1}{2}$;
34.01345 $\frac{1}{10}$.
11. 28.648083 $\frac{1}{2}$;
21.984375.
12. .288;
44.0928.
13. 93.0568 $\frac{1}{2}$;
4.0203 $\frac{3}{8}$.
14. 115.6666 $\frac{2}{3}$;
500.4029 $\frac{1}{8}$.
15. .51153;
3.8533 $\frac{1}{4}$.
16. 82.0166 $\frac{2}{3}$;
1061.1796 $\frac{3}{4}$.
17. 170845.86.

Art. 139.

1. .048.
2. 250.
3. 104; 8.625.
4. 1.914; 2.82.
5. .875; 100.8.
6. 481.5; 385.2.
7. 4.25; 6.2.
8. 15.24706;
2.25.

9. .49; 82.6875.
10. .5694; 39.
11. 18.66;
10.30152+.
12. 2722.02; 42.
13. 86.40; 69.12;
51.84; 138.24;
25.92.
14. 1800.
15. 3720.
16. 12.
17. 2.525.
18. 293.040015.
19. 3 $\frac{1}{2}$.
20. 158 $\frac{1}{2}$.

Art. 140.

1. .1651386.
2. 289.3624 $\frac{2}{3}$.
3. .8125.
4. $\frac{1}{12}$.
5. 158.916 $\frac{2}{3}$.
6. 176.277 $\frac{2}{3}$.
7. $\frac{6}{80}$.
8. 863.68964.
9. 3500.
10. .04.
11. 2.
12. 117.16 $\frac{2}{3}$.
13. 61.875.
14. 576.
15. 3 $\frac{1}{2}$.
16. 331.20.
17. \$56.16.
18. \$1575.
19. 1100.
20. \$108.99.
21. \$143.06.
22. 21200 bu.
23. \$33107.12.
24. \$10542.48, B;
\$7028.32, C;
D, E.
25. \$31444.87.
26. 203 $\frac{2}{80}$ Acres;
36 $\frac{3}{4}$ to B;
67 $\frac{3}{80}$ to C;
\$4575.65.
27. 7040 ft.
28. \$500.
29. \$48500, wife;
\$36375, son;
\$24250,
daughter;
145500, total.

30. 15 $\frac{1}{2}$ hours.
31. \$1065.
32. \$4.63.

Art. 191.

1. 2326 d.
2. 220 gill-.
3. 108404 f.
4. 13265 lb.
5. 8784 hr.
6. 525600 m.
7. 235923 f.
8. 1566 qt.
9. 8760 hr.
10. 4480 pwt.
11. 27005 ft.
12. 2407680 ft.
13. 66 in.
14. 28675 l.
15. 2460 rd.
16. 7895 $\frac{1}{2}$ sq. ft.
17. 2934 $\frac{1}{4}$ sq. yd
18. 3200 a.
19. 16000 a.
20. 2160 cu. ft.
21. 32722 lb.
22. 35942 lb.
23. 474 qt.
24. 133000 lb.
25. \$345.60.
26. 6574 d.
27. 143340 lb.
28. \$182494;
\$452.585.

Art. 194.

1. £35 6s. 3d.
2. 75 C. 83 cu. ft.
3. 117 bu. 2 pk.
4. 38 bbl. 30 gal.
1 pt.
5. 1 mi.
6. 252 sq. rd.
10 sq. yd.
7. 23 y. 42 d.
8. 37 wk. 6 d.
15 hr.
9. 2 hr. 38 m.
57 s.
10. 31 rd. 2 yd.
2 ft. 5 in.

11. £55 15s. 7d.
12. 1704 f.
13. 604.27 ch.
14. 247 ch. 36 ft.
15. 1346 ch.
16. 255 rd.
17. 33 A. 36 sq. rd.
18. £19 9s. 7d.
19. 6m. 25sq. ch.
20. 48 A. 5 p.
21. £18 8s. 2d.
22. \$564.29.
23. \$260.54.
24. \$436.95.
25. \$326.88.
26. \$85.48.
27. \$10.06.
28. 10 m. 27 d.

Art. 197.

1. 150 d.
2. 17s. 6d.
3. 135d.
4. 11s. 3d.
5. 2 y. 4 m.
6. £16 9s. 4d.
7. £2053 7s. 9d.
8. 2 y. 5 m.
9. 36 cu. yd.
3 cu. ft.
10. 3 m. 3 d.
11. £14 17s. 6d.

Art. 199.

1. £ $\frac{1}{344}$.
2. £.04375.
3. 6 T. 4. $\frac{3}{8}$ oz.
5. £.003125.
6. £.925.
7. £.4875.
8. .90625 cd.
9. £247.7375.
10. .675 A.
11. $\frac{5}{8}$ mi.
12. £27.525.
13. \$42.65.
14. \$166.56.
15. \$138.01.
16. \$146.56.
17. \$6022.92.
18. £1 5s. 7 $\frac{1}{2}$ d.

Art. 201.

1. £85 7s. 10d.
2. 69 T. 15 cwt.
20 lb.
3. £460 7s. 6d.
4. 12 yr. 2 mo.
16 da.
5. 21 cd. 34
cu. ft.
6. 26 hr. 46 min.
7. 152° 4' 2".
8. 117.15 ch.
9. 10 lb. 2 pwt.
20 gr.
10. 124 gal. 2 qt.
11. £205 12s. 5d.
12. £546 1s. 7d.

Art. 203.

1. £88 4s. 2d.
2. £21 4s. 4d.
3. £3 14s. 3d.
4. £23 13s. 10d.
5. 4 yr. 3 mo.
2 da.
6. 4 yr. 4 mo.
22 da.
7. 6 mo. 10 da.
8. 2 yr. 6 mo.
24 da.
9. 1 hr. 58 min.
4 sec.
10. 17° 53' 56".

Art. 206.

1. 7 mo. 18 da.
2. 8 mo. 18 da.
3. 1 yr. 8 mo.
11 da.
4. 2 yr. 5 mo.
7 da.
5. 1 yr. 10 mo.
8 da.
6. 9 mo. 12 da.
7. 9 mo. 12 da.
8. 1 yr. 4 mo.
15 da.
1. 230 days.
2. 263 days.
3. 619 days.
4. 888 days.
5. 678 days.
6. 286 days.

7. 286 days.
8. 504 days.

Art. 208.

1. £157 16s.;
£192 17s. 4d.;
£263.
2. 34 cd. 16
cu. ft.
3. £28 2s. 6d.
4. £6 7s. 10d.;
£7 5s. 9d.
5. 3 lb. 3 oz.
12 pwt.
6. 24 hr. 35 min.
30 sec.;
19 hr. 40 min.
24 sec.;
29 hr. 36 min.
36 sec.
7. 55 da. 13 hr.
8. £3 9s. 2d.;
£8 6s. 1d.;
£9 13s. 10d.
9. £20 12s. 6d.
10. 167 bu. 1 pk.
11. 44 mi. 6 fur.;
71 mi. 4 fur.
32 rd.

Art. 210.

1. £1 4s. 9d.;
£1 10s. 3d.;
8s. 3d.
2. 192 yd.;
324 yd.
3. £17 5s.;
£9 11s. 8d.;
£5 15s.
4. 44 yd.
5. 5 41' 14";
4 44' 21 $\frac{1}{2}$ ";
3° 9' 34 $\frac{1}{2}$ ".
6. £41 18s. 11d.
7. 32 A. 41 sq. rd.
24 A. 130 sq.
rd.
- 20 A. 25 $\frac{1}{8}$ sq.
rd.
8. 99 ft.
9. 3 oz. 5 $\frac{1}{8}$ pwt.
10. 9s. 11d.;
£5 19s.;
£9 18s. 4d.

Art. 215.

1. 73° 54' 25"
2. 73° 23' 52".
3. 34° 49'.
4. 178° 34' 17".
5. 88° 33' 45".
6. 149° 14' 13".
7. 4 hr. 56 min.
 $\frac{1}{2}$ sec.
8. 54 m. 30 $\frac{3}{5}$ s.
9. 11 h. 2 m.
59 $\frac{2}{5}$ sec.
10. 50 m. 11 $\frac{7}{15}$ s.
11. 5 h. 4 m.
20 $\frac{3}{5}$ s.
12. 11 h. 46 m.
24 $\frac{1}{5}$ s.
13. 74° 0' 3".
14. 121 $\frac{1}{5}$ °.
15. Add 5 hr.
5 m. 21 $\frac{2}{5}$ s.
16. 154° 8' 30"
W.
17. 5 h. 8 m.
42 s. P. M.
18. 19° 31'.
19. 10 h. 54 m.
27 s. A. M.

Art. 221.

3. 38364 cm.;
38364 Km.
4. .0017516 Km.
- 1.7516 m.
5. 8742.57 m.
6. 119 Km.
7. \$98.70.
8. 2306.8 m.

Art. 224.

2. 83.09 sq. m.
3. 4700 sq. m.
4. 602500 cu.
5. 256 sq. m.
6. 400.
7. 166.4 H.
8. \$12.44.
9. 40.
10. 96 fl.
11. \$22.
12. \$44.88.
13. 1024 m.

Art. 227.

2. 17.218027.
cu. m.
3. 28 cu. m.
4. 3.3 cu. m.
5. 4.81208
cu. m.
6. 2051.28
cu. m.
7. \$109.69.
8. 82.5 sters;
43.2 m.
9. \$221.40.
10. 1.28 cu. m.

Art. 229.

3. \$1.28.
4. 72 Hl.
5. 168 Hl.
7. \$1498.
9. 180 bags.
10. 10080 Dl.
11. \$14.
12. 6.25 m.
13. 3360 l.
14. 55296 l.

Art. 231.

2. 16.816 T.
3. 80.
4. 30.
5. 750 g.
6. \$13.608.
7. 1200.
8. 9300 Kg.

Art. 233.

1. 246.06 yd.;
8858.25 in.
2. 9.6558 Km.
3. 259.008 Hl.
4. 32808.3 ft.;
6.2137 mi.
5. 828.04776 lb.
6. 204.12.
7. 26.73 grams;
27.216 g.
8. 1762 Hl.
9. 668.9375
c. m.
10. 1308 cu. yd.
11. 6540.48 l.

12. 291.824 sq.
yd.
2626.416 sq.
ft.
13. 6237 g.;
6.237 Kg.
14. 2.72595 m.

Art. 235.

1. 202; 178.
2. 1 y. 4 m. 22 d.
3. 4; 12; 4.
4. 2 y. 4 m. 15 d.
5. $69\frac{1}{2}\%$.
6. 66 ch. 20 ft.;
52512 in.
7. 99 ch.;
6534 ft.
8. 2478.96 ft.;
150.24 rd.
9. $6\frac{1}{2}$ in. $8\frac{1}{2}$ in.
 $11\frac{1}{2}$ in.
11. 2999.875 sq.
ft.
12. 132.183 A.
13. $23\frac{3}{4}$ sq. yd.
14. \$68.27.
15. \$7200.
16. 990 ft.
17. \$72.
18. 8 ft.
19. 240 ft.
20. 240 ft.
21. 2400 ft.
22. 16500 ft.
23. \$327.78.
24. 1400.
25. 76.8 oz. av.
26. 57.6 lb. av.
27. \$141.87.
28. \$84.31.
29. \$172.36.
30. \$127.49.
31. $487\frac{1}{2}$ bu.;
\$237.81.
32. \$237.94.
33. $229\frac{1}{2}$ bu.
34. \$158.54.
35. \$4.92.
36. \$5.09.
37. \$6.98.
38. \$7.20.
39. \$0.687;

- \$0.531;
\$1.02.
40. $455\frac{1}{2}$;
 $435\frac{1}{2}$;
 $506\frac{2}{3}$;
 $446\frac{1}{4}$;
 $511\frac{1}{2}$;
41. 90; $155\frac{2}{3}$;
 $143\frac{2}{3}$;
 $110\frac{2}{3}$;
42. \$3.091;
\$1.362;
\$1.523;
\$1.812.
43. \$0.762;
\$0.741.
44. \$3.889;
\$14.25.
45. \$430.72.
46. \$1634.09.
47. \$808.65.
48. 2400.
49. 162145.26 g..
gold;
 18016.14 g..
alloy.
50. 1546875 oz..
silver;
 171875 oz..
copper.
51. \$18.605;
\$16.957.
52. $40.608.021$
pieces;
\$88.312.142-
50.
53. £135 1s. 10d.
54. £49 7s.
55. £219375;
\$23.72;
\$1067588.44.
56. 40 yd.
57. £19.8375.
58. \$1100.25.
59. £10 10s.
60. 5.1813 f.
61. \$3377.50.
62. 412.50 f.
63. \$5712.
64. \$3018.49;
\$7977.96.
65. 43 r. 9 a. 11 p.
66. $55^{\circ} 48'$.

67. \$59.33.
68. \$3.10.
69. 4 h. 23 m.
 $47\frac{1}{2}$ s., P. M.
70. 160.71 bu.
exact;
160 bu. app.
71. 320 Hl.
72. $4189\frac{1}{4}$ gal.
73. 165000 l.
74. 56.867 yd.
75. \$0.526.
76. 46.45 sq. dm
77. \$6.889.
78. \$0.528.
79. \$0.055.

Art. 248.

1. 432.
2. 614.
3. 1320.
4. 2106.
5. \$1.25.
6. 60.
7. 1.94.
8. 21.90.
9. \$125.85.
10. \$159.56 $\frac{1}{4}$.
11. \$23.75.
12. \$112.68.
13. \$2731.25.
paid over.
14. \$225.
15. \$832.58.
16. 2500.
17. 2400.
18. \$361.60.
19. \$10800.
20. \$1502.40.
21. \$72.
22. 495 ft.
23. £7200.
24. \$200.
25. 12000.
26. \$15627.50.
27. \$2058.75.
28. \$2.94.
29. 2200.
30. \$65500.
31. \$1612.50, A
\$2700.40, B.
32. .01 $\frac{1}{4}$.
33. $33\frac{1}{2}\%$.

34. 20%.
 35. $8\frac{1}{2}\%$.
 36. $2\frac{1}{2}\%$.
 37. $3\frac{1}{2}\%$.
 38. $1\frac{1}{2}\%$.
 39. $6\frac{1}{4}\%$.
 40. $2\frac{1}{2}\%$.
 41. $1\frac{1}{2}\%$.
 42. 25%.
 43. $\frac{3}{5}\%$.
 44. \$66.
 45. $12\frac{1}{2}\%$.
 46. \$2325.37.
 47. \$564.
 48. 400.
 49. \$930.
 50. \$444.
 51. \$324.
 52. 6210.
 53. \$456.80.
 54. \$2555.91.
 55. 120. 56. 275.
 57. \$531.92.
 58. \$1392.
 59. 125%.
 60. \$1197.
 61. \$7500.
 62. £18 15s.
 63. £1 12s. 5d.
 64. £3 0s. 6d.
 65. £3 14s. 10½d.
 66. £32.
 67. £35 8s. 4d.

Art. 251.

1. \$78.55.
 2. \$31.50.
 3. \$76.58.
 4. \$40.30.
 5. \$18.24.
 6. $23\frac{1}{2}\%$;
 $50\frac{1}{2}\%$;
 30% ; 64% .
 7. \$102.82.
 8. \$420.26.
 9. \$535.43.
 10. \$147.67.
 11. \$40.83.
 12. \$26.25.
 13. \$119.76.
 14. \$905.23;
 \$896.08.
 15. \$446.41.

Art. 253.

1. \$35.60.
 2. \$883.37.
 3. \$135.73.
 4. \$118.88.
 5. \$260.07.
 6. \$533.22.
 7. \$65.64.
 8. \$113.
 9. \$845.65.
 10. \$131.44.
 11. \$1082.55.
 12. \$326.93.
 13. \$92.05.

Art. 261.

1. \$21.60.
 2. \$200000.
 3. \$212.50.
 4. \$2256.25.
 5. \$2295.
 6. 2%. 7. 480.
 8. \$3099.37.
 9. \$12.61.
 10. \$81.75.
 11. \$2161.17, net
 amount.
 12. \$65.63.
 13. \$869.60, com.
 14. \$125.
 15. 5%.
 16. \$5091.
 17. \$2554.75.
 18. \$43500.
 19. \$1773.50.
 20. \$2782.30.
 21. \$7872.07.
 22. \$18909.18.
 23. \$2748.31.
 24. \$123.31.

Art. 264.

1. \$40.
 2. \$364.
 3. \$900.
 4. \$4.80.
 5. \$4726.
 6. \$7.50. 7. \$3.
 8. \$10179.
 9. 21c.
 10. \$1048.20.
 11. 72¢.

12. \$2425.
 13. $12\frac{1}{2}\%$.
 14. 16%.
 15. $13\frac{2}{3}\%$.
 16. $12\frac{1}{2}\%$.
 17. \$1775.
 18. \$608.
 19. \$11225.
 20. \$520.
 21. 44¢. 22. \$72.
 23. $23\frac{1}{2}\%$.
 24. \$400.
 25. $57\frac{1}{2}\%$.
 26. 12%.
 27. 10%.
 28. 4%.
 29. \$210.
 30. \$50.
 31. \$7.50.
 32. 36¢.
 33. $42\frac{6}{7}\%$.
 34. 150%.
 35. $33\frac{1}{3}\%$; 50%;
 $53\frac{1}{3}\%$;
 $40\frac{2}{3}\%$;
 $42\frac{6}{7}\%$.
 36. $6\frac{2}{3}\%$, gain;
 \$3.80.

Art. 278.

1. \$122.50.
 2. \$115.65.
 3. \$484.80.
 4. \$632.26.
 5. \$898.99.
 6. \$214.31.
 7. \$561.
 8. \$397.49.
 9. \$102.77.
 10. \$3138.83.
 11. \$1059.71.
 12. \$470.63.
 13. \$83.55.
 14. \$18.81.
 15. \$1791.94.
 16. \$269.45.
 17. \$20.58.
 18. \$56.68;
 \$5.57.
 19. \$180.10;
 \$120.07.
 20. \$11.43;
 \$17.14.
 21. \$15.50;
 \$18.08.
 22. \$29; \$33.83
 \$33.17;
 \$44.23.
 23. \$9.19; \$7.66
 \$16.38;
 \$10.92.
 24. \$39.45;
 \$46.03.
 25. \$19.79;
 \$23.09.
 26. \$2.85; \$2.37
 \$13.75;
 \$16.04.
 27. \$106.66;
 \$142.22.
 28. \$137.72;
 \$114.77.
 29. \$107.36;
 \$80.52.
 30. \$93.53;
 \$140.30.
 31. \$19.27;
 \$14.46.
 32. \$494.15;
 \$247.07.
 33. \$418.64;
 \$537.29.
 34. \$17.85;
 \$20.83.
 35. \$158.29;
 \$211.05.
 36. \$48.25;
 \$40.21.
 37. \$53.51;
 \$35.67.
 38. \$188.40;
 \$282.60.
 39. \$104.93;
 \$78.69.
 40. \$5.64; \$2.82
 \$154.87;
 \$309.74.
 41. \$717.50;
 \$657.71.

Art. 289.

1. \$16.28.
 2. \$64.76.
 3. \$36.85;
 \$42.99.
 4. \$24.70;

33. \$34.96;
\$33.21.
33. \$35.65;
\$33.87.
34. \$14.27;
\$18.73.
34. \$14.47;
\$18.69.
35. \$26.73;
\$19.09.
35. \$27.29;
\$19.49.
36. \$193.96;
\$113.14.
36. \$195.15;
\$113.83.
37. \$309.07;
\$347.70.
37. \$310.14;
\$348.90.
38. \$57.27;
\$81.81.
38. \$57.82;
\$82.60.
39. \$75.59;
\$10.12.
40. \$25.95;
\$11.53.
41. \$111;
\$27.75.
42. \$76.50;
\$60.56.
42. \$77.40;
\$61.28.
43. \$16.28;
\$14.65.
43. \$16.57;
\$14.92.
44. \$10658.20.
45. \$1050.
46. \$1556.66.
47. \$27.84.

Art. 293.

1. \$1.79.
2. \$116.47.
3. \$11.54.
4. \$5.28.
5. \$3.92.
6. \$15.12.
7. \$5.75.
8. \$6.16.
9. \$42.18.

10. \$14.83.
11. \$4.44.
12. \$10.44.
13. \$39.35.
14. \$246.89.
15. \$58.97.
16. \$27.74.
17. \$41.64.
18. \$7.58.
19. \$2.14.
20. \$30.21.
21. \$5.
22. \$8.64.

Art. 296.

1. 7%. 6. $3\frac{1}{2}\%$.
2. 7%. 7. 8%.
3. 6%. 8. $4\frac{1}{2}\%$.
4. 5%. 9. 4%.
5. 7%.
10. 6%.
11. $8\frac{1}{2}\%$.
12. 9%.
13. $10\frac{1}{2}\%$.
14. 30%.

Art. 299.

1. 6 m. 6 d.
2. 1 y. 4 m. 20 d.
3. 2 y. 2 m. 2 d.
4. 2 m. 15 d.
5. 6 m. 14 d.
6. 1 y. 10 m. 22 d.
7. 4 y. 9 m. 15 d.
8. 4 y. 8 m. 24 d.
9. 9 m. 20 d.
10. 11 m. 21 d.
11. 1 y. 5 m. 15 d.
12. 1 y. 5 m. 18 d.
13. 16 y. 8 m.

Art. 302.

1. \$12107.84.
2. \$871.31.
3. \$2241.
4. \$7719.16.
5. \$1997.87.
6. \$3000.
7. \$3228.33.
8. \$29419.35.
9. \$30612.25.
10. \$31746.03.
11. \$11973.33.

12. \$14370.69.
13. \$3436.99.

Art. 305.

1. \$1234.
2. \$5280.
3. \$3456.
4. \$375.60.
5. \$12375.
6. \$1728.
7. \$723.01.
8. \$879.54.
9. \$1511.67.
10. \$2309.28.
11. \$3770.52.
12. \$5307.72.
13. \$1642.31.
14. \$2138.94.

Art. 309.

1. \$892.86;
\$107.14.
2. \$438.60;
\$61.40.
3. \$547.95;
\$52.05.
4. \$762.63;
\$37.37.
5. \$283.35;
\$41.65.
6. \$0.68.
7. \$3629.03.
8. \$204.29.

Art. 310.

1. \$42.32.
2. \$1843.93.
3. 5%.
4. 1 y. 10 m. 28 d.
5. \$1722.02.
6. \$10000.
7. $14\frac{2}{3}\%$.
8. latter $1\frac{1}{2}\%$ better.
9. Oct. 3.
10. 7%.
11. \$19230.77.
12. \$28.33.
13. June 11, '74.
14. \$129;
\$131.50;
\$129.70.
15. \$2376.71.

16. \$7.16.
17. \$17 $\frac{1}{2}\%$.
18. \$650.
19. \$1483.58.
20. \$4293.04;
\$4342.65.
21. \$86.07.
22. \$89.17.
23. \$288.39.
24. £198 16s. 5d
25. £8 10s.
26. £5 8s. 4d.
27. £9 5s. 10d.
28. £2 12s. 11d.
29. £3 17s. 3d.
30. £10 9s. 8d.
31. \$6200.83.
32. $58\frac{1}{3}\%$.
33. $31\frac{1}{15}\%$.

Art. 315.

1. \$526.44;
\$506.48.
2. \$45.18;
\$37.37.
3. \$83.27.
4. \$1909.63;
\$2104.72.
5. \$1211.
6. \$142.82;
\$155.97.
7. \$2794.32;
\$7798.54.
8. \$993.03.
9. \$4445.17.
10. \$747.27.
11. \$4172.57.
12. \$405.34.
13. \$13363.84.

Art. 326.

1. \$1022.
2. \$911.04;
\$919.21;
\$917.46.
3. Mar. 7, 1882.
\$6022.10.
5. \$431.10.
6. Aug. 21, '81.

Art. 331.

2. \$7937.33, or
\$7938.19.

3. Apr. 27; \$1181.40, or \$1181.65.	23. $5\frac{1}{2}\%$.	5. \$833.87; \$920.94.	17. \$16.20; \$27.
4. Aug. 21; \$5196.40, or \$5197.55.	24. \$15.87.	6. \$3067.14. \$3347.31.	18. 40%.
5. Nov. 2; \$2524.16, or \$2524.65.	25. \$3012.09, or \$3012.63.	7. \$549.89; \$764.06.	19. \$1440.
6. Oct. 7; \$3664.71, or \$3665.96.	26. Aug. 30; \$3737.21, or \$3738.32.	8. \$3260.51; \$4594.82.	20. \$792.
7. Nov. 18; \$6395.55, or \$6395.96.	27. May 19; \$1641.17, or \$1641.51.	9. \$2849.04; \$3100.52.	21. M, \$1761.36; P, \$1409.09; T, \$880.68.
8. Sept. 16; \$8135.73, or \$8139.01.	28. Apr. 15; \$882.22, or \$882.56.	Art. 352.	22. A, \$454.54; C, \$568.18.
9. Sept. 1; \$4430, or \$4430.96.	30. \$1523.62.	1. 12. 2. 9.	23. \$134.90.
10. Sept. 28; \$8204.29, or \$8204.78.	31. \$3081.09.	3. 160. 4. \$36.	
11. June 29; \$4276.08, or \$4276.73.	Art. 336.	5. \$50.	
12. Nov. 30; \$4768.85, or \$4770.44.	1. \$678.54.	6. 240 lb.	
13. May 6; \$8895.50, or \$8900.88.	2. \$242.17; \$148.16.	7. \$111.	
14. Jan. 15; \$4909.58, or \$4910.82.	3. \$1102.69; \$1184.38.	8. \$21.875.	
15. Feb. 2; \$5936.20, or \$5937.07.	4. \$1327.21; \$1410.94.	9. 55.50.	
16. June 24; \$9812.71.	5. \$835.74; \$924.38.	10. $85\frac{1}{2}$ yd.	
17. Oct. 11; \$5894.25.	6. \$3073.01; \$3363.56.	11. 396 ft.	
18. May 13; \$5897.87.	7. \$517.82; \$716.61.	12. £25 11s. 9d.	
19. Sept. 6; \$8603.71.	8. \$3260.23; \$4539.19.	13. \$960.	
20. \$5949, or \$5949.70.	9. \$2856.29; \$3118.54.	14. \$2410.71.	
21. Oct. 5; \$4946.67.	Art. 340.	15. A, \$1875.90; B, \$1598.40.	
22. Apr. 4; \$3710.	2. \$440; \$447.16.	16. \$179.56.	
	3. \$223.31; \$214.37.	17. \$98238.64.	
	4. \$651.97; \$753.30.	18. \$126.	
	5. \$104.28; \$133.04.	19. \$3.79.	
	Art. 344.	20. \$137.03.	
	2. \$262.24; \$161.49.	21. 5 rods.	
	3. \$1108.57; \$1192.53.	22. \$153600.	
	4. \$1324.75; \$1406.47.	23. £27 2s. 9d.	
		24. $2355\frac{2}{3}$ ft.	
		Art. 370.	
		1. \$93.75.	
		2. \$68.75.	
		3. \$426.50.	
		4. 80°.	
		5. \$3600.	
		6. \$281.25.	
		7. \$22.50.	
		8. \$56.88.	
		9. \$71.43.	
		10. \$57.60.	
		11. \$4880.	
		12. \$3120.	
		13. \$12.	
		14. \$9000.	
		15. \$9000.	
		16. \$61.25.	
		Art. 379.	
		1. \$8782.81.	
		2. \$8395.94.	
		3. \$5006.25.	
		4. \$4358.59.	
		5. \$8427.52.	
		6. \$9922.37.	
		7. \$4287.11.	
		8. \$3417.18.	
		9. \$2821.96.	
		10. \$9898.30.	
		11. \$1871.48.	
		12. \$2443.86.	
		13. \$3346.56.	
		14. \$2227.28.	
		15. \$7373.16.	
		16. \$9222.61.	
		17. \$6431.57.	
		18. \$9231.46.	
		19. \$23.29;	
		\$9340.10.	
		21. \$2972.25.	
		22. \$11834.	
		23. $\frac{1}{2}\%$ premium	
		24. \$3420.05.	
		Art. 392.	
		1. \$1083.94.	
		2. \$1583.40.	
		3. \$2407.50.	
		4. \$3760.69.	
		5. \$4050.03.	
		6. \$409.34.	
		7. \$2483.15.	
		8. \$4076.72.	
		9. \$3290.93.	
		10. \$1187.98.	
		11. \$4261.23.	
		12. \$4593.93.	
		13. \$2611.06.	
		14. \$8495.46.	
		15. \$8495.46.	

16. \$2373.24.
18. £1225 18s. 6d.
19. 4.837.
20. £1864 6s. 4d.
21. \$1341.32.
22. \$1162.79.
23. \$965.02.
24. \$767.20.
25. \$1631.38.
26. \$1393.72.
27. \$189.60.
28. \$1856.07.
29. \$1538.53.
30. \$2023.10.
31. \$2392.60.
32. \$1688.75.
33. \$1411.11.
34. 80318.70 fr.
35. 17949.60 fr.
36. 5.18^a.
38. \$1692.
39. \$1967.24.
40. \$822.96.
41. \$289.54.
42. \$2359.10.
43. \$1543.24.
44. \$3867.75.
45. \$3102.24.
46. \$1547.25.
47. \$2061.40.
48. \$2359.33.
49. 3467.73 m.
50. 13824 m'ks.
51. 94¹/₂.
52. \$2905.05.
53. \$4982.62.
54. 4562 guilders
55. 7128 guilders
56. 40³/₄.
57. 4.885.
58. \$17366.98.
59. \$3.60.
60. £512 3s. 2d.

Art. 397.

1. Oct. 10.
2. Feb. 11.
3. Aug. 4.
4. June 5, 1882.
5. June 20.
6. July 18;

\$1694.90;

7. Feb. 15, '82.
8. Oct. 6, '81;
9. May 31;
10. July 14.
11. Nov. 15.
12. Sept. 26.
13. Oct. 26, '82.
14. Sept. 25;
15. July 8.
17. Nov. 8, 1881.

Art. 401.

1. Jan. 17, 1882.
2. May 25, 1881.
3. Dec. 3, 1882.
4. May 13, 1882.
5. Feb. 15, '83;

Art. 405.

1. \$2134.06;
2. \$5333.48;
3. \$12505.70.
4. \$4043.09;

Art. 412.

1. \$431.37.
2. \$1694.89.
3. \$518.53.
4. \$986.02.
5. \$3361.51.
6. \$44955.75.
7. \$400.91.
8. Rm. 9997.87.
9. \$1053.22.
10. \$1513.77;

\$32067.54;

- \$1704.29;
- \$47288.32.

Art. 434.

1. \$8750; \$56.
2. 8%; \$200.
3. \$2,500,000.
4. \$20,000,000;
5. \$2,000,000;
6. \$500.
7. \$61250.
8. \$36,745,000.
9. \$60000.
10. 550 shares.
11. \$29043.75.
12. \$220000;
13. \$170,587.50.
14. \$23100.
15. \$36412.50.
16. \$226675.
17. \$17560.
18. \$68625.
19. \$3277.50.
20. \$16200.
21. \$8230.
22. \$80800.
23. 500 shares.
24. \$775.
25. \$8000.
26. \$117645.
27. \$28494.67.
28. \$23541.58.
29. \$325.
30. \$90,448,287-

31. \$16,327,007-
32. \$11,140,834-
33. \$37.87;
34. Latter ¹/₂% better.
35. 7 ³/₁₁%.
36. Chatham \$60 greater.
37. 166²/₃.
38. 6 ¹/₂%; 125;
39. 160; 133¹/₄.
40. \$34137.50.

41. \$2425.
42. \$519.27.
43. \$1128.34.
44. \$1682.91.
45. \$475.
46. \$1450.
47. \$1625.
49. 124.59;
51. 4%.

Art. 440.

1. \$7,690.418-
2. \$11,615,280
3. 3¹/₄ mills;
4. \$13412.19.
5. \$8693.72.
6. Rate 5.8 m
7. \$37.49.
8. \$57.85.
9. \$231.39.
10. \$6557.
12. \$5665.97.

Art. 451.

1. \$9412;
2. \$5647.20.
3. \$97.02.
4. \$816.25.
5. \$85.40.
6. \$135.21.
7. \$723.45.
8. \$950.
9. \$1267.50.
10. \$116.94.
11. \$915.95.
12. \$1004.
13. \$639.03.
14. \$839.40.
15. \$1623.80
16. \$183.75.
17. \$338.24.
18. \$11060.
19. \$43.75.
20. \$208.50.
21. \$439.88.
22. \$57.05.
23. \$37242;
24. \$197.75.
25. \$2483.60
26. \$487.26.

Art. 461.

3. A, \$1960;
B, \$2960.
4. \$833.33, bro.;
\$3366.67.
5. C, \$11431.88;
D, \$11279.75;
E, \$11190.75.
7. M, \$18529.25;
N, \$6389.75.
8. X, \$500;
Y, \$200;
Z, \$700.
10. E, \$2380.83;
F, \$3333.17;
G, \$3809.33;
H, \$4761.67.
11. A, 60 ft.;
B, 80 ft.;
C, 100 ft.
12. A, \$2692.68;
B, \$2468.29;
C, \$1884.88;
D, \$2154.15.
13. A, \$375;
B, \$318.75;
C, \$225;
D, \$187.50;
E, \$131.25.
14. A, Cr.
\$1623.17;
B, Dr, \$164.71.
15. A, \$5004.24;
B, \$2502.12.
16. A, \$6470.24;
B, \$3235.12.
17. P, \$10229.71;
Q, \$10245.54.
18. A, \$3847.56;
B, \$4902.44.
19. \$5877.67.
20. A, \$1533.46;
B, \$1922.54.
21. C, \$819.97;
D, \$745.43.
22. A, \$540;
B, \$560;
C, \$600.
23. A, \$2311.63;
B, \$3581.40;
C, \$4106.97.
24. J, \$1558.97;
K, \$1385.75;

- L, \$1190.88.
25. A, \$2505.74;
B, \$17307.58;
C, \$16723.84.
26. A, \$17527.74;
B, \$20323.76;
C, \$6310.43.

Art. 468.

1. 72%; \$5760.
2. \$702000;
\$35100.
3. \$35625.
4. \$21375;
\$475000.
5. \$3,338,100.
6. \$23,944,096-
.50.
\$1,213,591.50
7. \$1,433,831.70
8. \$1047.54;
\$427.84.
9. \$9028.53.
10. \$59.31.
11. 12%; 10%;
19.56%.

Art. 473.

2. \$374.60;
\$374.93.
3. \$669.35;
\$671.70.
4. \$25.14.
5. \$462.10;
\$464.38.
6. \$348.62;
\$350.50.
7. \$190.91;
\$191.04.
8. \$96.15.
9. \$284.55;
\$284.82.
10. \$121.20;
\$121.94.
11. \$328.60;
\$329.37.
12. \$557.31;
\$557.86.
13. \$705.74;
\$705.94.

Art. 482.

1. \$131.90.

2. \$137.20.
3. \$116.36.
4. \$1638.40.
5. \$0.054;
\$0.072;
\$0.104.
6. \$0.655.
7. \$356.
8. \$13472.
9. \$10469.12.
10. \$3700;
\$13700.
11. \$9080;
\$29080.
12. \$717.68.
13. \$12379.12.
14. \$342.67.
15. \$4190.03;
\$3084.60;
\$933.35.
16. \$152.79.
17. \$13.63.
18. \$2655.69.

Art. 483.

1. 1043.
2. 21659.6.
3. 40320 min.
4. \$164.52.
5. 16 48' 15".
6. 16666 $\frac{2}{3}$ sq. yd.
7. \$69 0s. 6d.
8. \$140.25.
9. 38 $\frac{1}{2}$ sq. yd.;
\$24.15.
10. £19 3s. 6d.
11. 1.67184 gr.
12. \$39000.
13. \$9363.44.
14. \$135.
15. \$102.
16. £3 19s. 10d.
17. \$164.
18. \$4224.
19. \$6058.95;
20%.
20. \$676.77.
21. 31 $\frac{1}{2}$ %.
22. \$1052.63;
\$1422.37.
23. \$433.93.
24. \$889.36.
25. £56 7s.

26. £10 0s. 9d.
27. 22.99%.
28. 73.57%;
26.43%.
29. 30 $\frac{1}{2}$ %.
30. 120 bbbs.
31. Sept. 19;
\$4950.
32. \$5078.72.
33. \$851.96.
34. \$5171.73.
35. \$4165.03.
36. \$630.97.
37. \$2474.07.
38. \$9825.03.
39. \$17651.29.
40. \$1889.22.
41. 40 $\frac{3}{4}$ %.
42. \$28.03.
43. \$1.26 $\frac{3}{4}$;
30 $\frac{1}{2}$ %.
44. \$754482.
45. 11 $\frac{1}{2}$ %.
46. \$2.50.
47. \$554.68.
48. Mar. 21, 1882.
49. Aug. 25, 1882;
\$622.41.
50. \$622.42.
51. \$320.31.
52. \$1625.
53. \$425.
54. \$156.30.

Art. 491.

2. 33. 9. 312.
3. 121. 10. 345.
4. 135. 11. 375.
5. 216. 12. 716.
6. 218. 13. 15. 25.
7. 255. 14. 1994.
8. 294. 15. 20. 78.
16. 2732.
17. 29683.
18. $\frac{1}{2}$; $\frac{1}{10}$; $\frac{7}{11}$.

Art. 495.

2. 18. 8. 2. 13.
3. 2.2. 9. 23.9.
4. 34. 10. 46.2.
5. 78. 11. 6. 51.
6. 82. 12. 75.8.
7. 196. 13. 878.

